

The Semantics of Fuzzy Quantifiers

Qiao Zhang

PhD
The University of Edinburgh
1996



Declaration

I declare that this thesis has been composed by myself and that the research reported therein has been conducted by myself unless otherwise indicated.

Signature:

Qiao Zhang



Acknowledgement

For all the constant guidance, encouragement and stimulating discussions of this thesis given me in the past years, I am immensely indebted to my supervisors, Ronnie Cann and James R. Hurford. They gave generously of their time and ideas and I have benefited greatly from their supervision. Ronnie Cann, especially, has carefully read every section of each earlier draft, and contributed tremendously to shape and reshape this thesis; for that I am most thankful to him. Extensive and invaluable comments on the original version of this thesis have been made by Linda Moxey, to whom I am extremely grateful. Her advice, particularly on handling empirical data, helped me greatly when revising the thesis. All three made numerous suggestions (too many to acknowledge in the text) for the improvements of this thesis that have been incorporated into the final version.

I am deeply grateful to James Miller. He has been extremely helpful in advising the revision of my thesis and in many other ways. He was the very first person who encouraged me to study in the Department and kindly recommended me for the financial support which enabled me to complete my Ph.D studies. I also appreciate what Ron Asher has done for me, who advised me on my thesis and on financial assistance for my studies.

I would like to thank Irene Macleod for her advice in computing, also for her understanding and friendship. Michael Somerville, Luke Minford, Grant Nicholls and Antoinette Fernando gave me much needed help whenever required. To all those mentioned, and to any others whom I may have failed to mention, I extend my gratitude. I owe a special debt to Joanna M. Channell. The English data on fuzzy quantifiers presented in her work provided me with useful information. I am also grateful to those who agreed to act as informants for my Chinese test.

I acknowledge the financial assistance provided by a Van Dunlop Scholarship from the University of Edinburgh (Faculty of Arts Scholarship Fund) and the ORS Awards Scheme during 1989-1992. Subsequently, I have been given financial assistance from the President's Fund of the Edinburgh Association of University Women and the Great Britain-China Educational Trust

in London.

Finally, I owe much of my happy life to my parents—Mr Shimin Zhang and Mrs Lanyin Zhang—and all the love they give me. I thank my husband Zhigang Guan with my whole heart for his love, patience, and support; and my daughter Xiaoqiao Guan for understanding that this thesis has taken so much of my time.

Abstract

The aim of this thesis is to discuss the semantics of FQs (fuzzy quantifiers), formal semantics in particular. The approach used is fuzzy semantic based on fuzzy set theory (Zadeh 1965, 1975), i.e. we explore primarily the denotational meaning of FQs represented by membership functions. Some empirical data from both Chinese and English is used for illustration.

A distinguishing characteristic of the semantics of FQs like *about 200 students* and *many students* as opposed to other sorts of quantifiers like *every student* and *no students*, is that they have fuzzy meaning boundaries. There is considerable evidence to suggest that the doctrine that a proposition is either true or false has a limited application in natural languages, which raises a serious question towards any linguistic theories that are based on a binary assumption. In other words, the number of elements in a domain that must satisfy a predicate is not precisely given by an FQ and so a proposition containing one may be more or less true depending on how closely numbers of elements approximate to a given norm.

The most significant conclusion drawn here is that FQs are compositional in that FQs of the same type function in the same way to generate a constant semantic pattern. It is argued that although basic membership functions are subject to modification depending on context, they vary only with certain limits (i.e. FQs are motivated—neither completely predicated nor completely arbitrary), which does not deny compositionality in any way. A distinctive combination of compositionality and motivation of FQs makes my formal semantic framework of FQs unique in the way that although some specific values, such as a norm, have to be determined pragmatically, semantic and inferential patterns are systematic and predictable.

A number of interdisciplinary implications, such as semantic, general linguistic, logic and psychological, are discussed. The study here seems to be a somewhat troublesome but potentially important area for developing theories (and machines) capable of dealing with, and accounting for, natural languages.

Contents

1	Introduction	1
1.1	Survey of the fundamental notions	3
1.1.1	Fuzzy	3
1.1.2	Fuzzy quantifier	5
1.2	Fuzziness—vagueness—ambiguity	6
1.2.1	Fuzziness	6
1.2.2	Vagueness	9
1.2.3	Ambiguity	12
1.2.4	Distinction between the three concepts	13
1.3	Types of FQs	21
1.3.1	The three types	21
1.3.2	Adaptor and fuzzifier	23
1.4	Causal factors for fuzziness	25

Contents

vii

1.4.1	A language concept	27
1.4.2	Lack of need for preciseness	30
1.4.3	Need for fuzziness	32
1.5	Approach and methodology	36
1.5.1	Denotational approach	37
1.5.2	Membership function	41
1.6	Conclusion	42
2	Previous work on fuzziness	43
2.1	Black's work	43
2.2	The work of Wallsten <i>et al</i>	45
2.3	Mosteller and Youtz's work	51
2.4	Moxey <i>et al</i> 's work	53
2.5	Wachtel's work	57
2.6	Conclusion	60
3	An empirical study of the semantics of FQs	62
3.1	The data	62
3.1.1	Zhang's questionnaire for Chinese FQs	63
3.1.2	Channell's elicitation test for English FQs	69

3.1.3	Remarks on the written instructions	70
3.2	A semantic analysis	72
3.2.1	FQs in Type I	72
3.2.2	FQs in Type II	76
3.2.3	FQs in Type III (semi-FQs)	90
3.3	The numeral and the norm	101
3.4	Function of the size of numerals	104
3.5	Function of approximators	107
3.6	Consistency between the degree of fuzziness, the size of a numeral and the length of a range	108
3.7	Some notes on membership function	111
3.8	Conclusion	118
4	Pragmatics, compositionality and FQs	119
4.1	Pragmatics and FQs	120
4.1.1	Scale effects	121
4.1.2	Other relevant factors	125
4.2	Compositionality of FQs	131
4.2.1	Invariant core meaning/truth condition and variant peripheral meaning	131

4.2.2	Motivation	133
4.2.3	Compositional FQs and propositions containing one . .	135
4.3	Combinative FQs	136
4.3.1	Concentration and dilatation	136
4.3.2	<i>Tebie</i> 'very', <i>hao</i> 'quite' and <i>bu/mei</i> 'not'	139
4.4	Conclusion	145
5	Formal semantics of FQs (I)	147
5.1	Generalized Quantifier Theory	147
5.1.1	Semantic universals	148
5.1.2	Monotonicity	152
5.1.3	Comments on GQT	162
5.2	Fuzzy Set Theory	164
5.2.1	The definition	164
5.2.2	Fuzzy set operation	167
5.2.3	Fuzzy proposition operation	169
5.2.4	FST and prototype theory	171
5.2.5	Fuzzy grammar	175
5.2.6	Development of FST	178

5.2.7	Some questions about FST	180
5.3	Conclusion	183
6	Formal semantics of FQs (II)	184
6.1	Syntactic formation rules	186
6.2	Semantic rules	187
6.3	Fuzzy quantifier	190
6.4	Fuzzy quantification	196
6.5	Compound proposition	200
6.6	Conclusion	201
7	Formal semantics of FQs (III)	203
7.1	GQT models incorporating with membership function	203
7.1.1	Type III FQs (semi-FQs)	204
7.1.2	Type I FQs	206
7.1.3	Type II FQs	208
7.2	An intensional treatment	210
7.3	Entailment with FQs	216
7.3.1	Degree of entailment	216
7.3.2	Entailment pattern with FQs in Type I	218

Contents

xi

7.3.3	Entailment pattern with FQs in Types II and III . . .	219
7.3.4	Deductive or intuitive	221
7.4	Conclusion	224
8	Implications and Conclusions	225
8.1	Semantic implications	226
8.1.1	The properties of compositionality and motivation . . .	227
8.1.2	Internal structure	228
8.1.3	Semantics and pragmatics	229
8.2	Formal semantic and logic implications	230
8.2.1	A generalization of conventional formal theories	230
8.2.2	Core meaning, truth condition and inference pattern .	233
8.3	General linguistic implications	234
8.3.1	Inherent property of language	234
8.3.2	Not necessarily categorical	235
8.4	Psychological implications	237
8.5	Conclusions	241
	References	243

Contents

xii

Appendices

266

Appendix 1: Channell's (1983) elicitation test for English FQs 266

Appendix 2: Zhang's questionnaire for Chinese FQs 277

Appendix 3: Zhang's Questionnaire in Chinese 317

1

Introduction

Suppose this might happen in real life. While preparing a party, Mary asked John to buy *about 20 beers* and *a few apples*. John had to decide exactly how many beers and apples he would buy. In a shop he hesitated for a while then bought 18 beers and five apples. Once Mary saw the things John bought she seemed satisfied. Although this is a hypothetical example of communication with words like *a few* and *about 20*, in fact this kind of communication happens very often in our everyday life. If we closely examine our language, most expressions have a *fuzzy*¹ boundary. For instance, an essay could be *not bad*, a girl may be *rather pretty*, a pile of papers might be *20 or so*, and someone may have *many* friends. This kind of expression enables us to speak about a far greater variety of topics than those precise numbers would allow. The questions that arise are: what kind of linguistic

¹Sainsbury (1991) argues that it is inappropriate to say that some concept has a fuzzy boundary, because the term *boundary* must be understood as a precise one, otherwise there is no boundary at all. Therefore, Sainsbury suggests using *boundariless*. Nevertheless, it seems to me that something with a fuzzy boundary is not the same as something without a boundary at all. Take *about 20 years old* as an example; there is uncertainty about its boundary or there is disagreement towards the precise boundary of the concept. That is, whether or not 16 or 26 is within the boundary is undetermined. However, this does not mean that the concept is boundariless, because we would not agree if one says that a one-month old baby is within the boundary of *about 20 years old* (see page 182 for further discussion on this point).

theory lies behind it, and particularly what kind of formal interpretation can we come up with?

For many years it seems to have been taken for granted that propositions, at least declarative propositions, are either true or false. However, it is found that the conventional two-valued logic approach cannot accurately represent natural languages; not even three or four-valued logic can do the job properly (see McCawley (1981) for further discussion). The reason is that there are propositions denoted by sentences in natural languages that are true (or false) up to a point, i.e. neither completely true, nor completely false. For instance, in the sentence *Mary is about 20 years old*, the expression *about 20 years old* has an uncertain meaning boundary. Accordingly, its truth value could be a matter of degree. This cannot be handled by any conventional theories. Although both three-valued and four-valued logics seemingly have an undefined value to represent the degree of truth, all they do is just conflate any value which is not all-or-none type into a general category without any further structured exploration.

As an illustration, according to conventional truth conditional semantics, the sentence *Mary is about 20 years old* is true if and only if there is an individual called Mary and she is about 20 years old. This allows a biconditional statement—*u* satisfies the formula *about $n(x)$* iff *u* is in the set of *about n* . It leaves *about n* as an unanalysed primitive, and does little to capture the meaning of it. As Klein (1980) points out, the treatment utilizes a semantic metalanguage in which fuzzy expressions occur, and presupposes the notion of fuzziness. This kind of treatment is not particularly informative or useful. We need an account which can define expressions like *about n* adequately. FST (Fuzzy Set Theory, Zadeh 1965) can do a better job in that it captures degrees of truth, which will be demonstrated in Chapters 5, 6 and 7 below.

The study of FQs here concentrates on semantics, with little account of syntax and phonology. The reason is that the very issue of FQs' semantics is so complex that it is more than enough to deal with for a thesis of limited length. It would be wiser to tackle one aspect of FQs adequately than to touch on several aspects superficially. However, a further study of FQs interfacing phonology, semantics and syntax would be of benefit.

In terms of organisation of the thesis, the presentation is in four parts. In the first part (Chapters 1 & 2) I shall give a general discussion of the issues involved such as terminology, approach and methodology, the distinction between ambiguity, vagueness and fuzziness, causal factors of fuzziness, and a review of previous work. Following that, the second part (Chapters 3 & 4) is a discussion of the semantics of FQs. Chapter 3 is an empirical study based on the data in Channell (1983) and Zhang (myself). Chapter 4 is a discussion of pragmatic effects and compositionality of FQs. In the third part (Chapters 5, 6 & 7), a formal semantic account of FQs is proposed in which FQs are treated as fuzzy operators. The fourth part (Chapter 8) contains interdisciplinary implications and conclusions. The four parts are logically arranged. Part one provides background knowledge. Part two discusses the semantics of FQs from different aspects. Part three provides a formal framework. Finally, implications and conclusions are drawn up.

1.1 Survey of the fundamental notions

Two notions need to be clarified at the outset: *fuzzy* and *FQ*.

1.1.1 Fuzzy

How can the membership of the set of *twentyish* be defined (putting aside contextual factors: the membership of *twentyish* for a person's age and for a medical measurement could be different)? In other words, how old does one have to be to be *twentyish*? 15? 16? 17? 18? 19? 20? 21? 22? 23? 24? 25? Obviously, any attempt to fix a single answer will be impossible. Expressions like this are considered as *fuzzy* ones to which an application of a particular referent or state of affairs is not a clear-cut case.

Furthermore, such a sentence as *Mary is twentyish* may very often be neither true, nor false, nor nonsensical; but only true to a degree. For instance, if Mary is exactly 20 years old, the sentence might be 100% true. If Mary is

15 years old, the sentence might be 60% true. If Mary is 40 years old, then the sentence might be totally false. The reason is that the membership of *twentyish* is fuzzy, i.e. some elements in the domain neither definitely belong to the set nor definitely do not belong to it.

It is imperative that the concept *fuzzy* used in this discussion has nothing to do with the negative part of its literal meaning, like *misuse*, *mistaken*, or *not well defined*. In fact, the term *fuzzy* is a technical term, and has a precise definition throughout the discussion. However, it appears that the term *fuzzy* sometimes confuses people. Accordingly, it has been suggested that the term *continuous* could replace the term *fuzzy*. For example, *fuzzy logic* could be called *continuous logic*. It seems that whatever it is called makes little difference, as long as we keep it well defined (see Section 1.2 for further discussion on the distinction between fuzziness, ambiguity and vagueness).

Fuzziness occurs in two layers with respect to two types of language users (individual and group). The individual type means that an FQ is defined in correspondence with an individual's view. Fuzziness occurs on an individual level, when an individual, Mary for instance, is unsure about the boundary of a *twentyish man*. The group type, on the other hand, means that an FQ is defined corresponding to the views of a group of people. An FQ may be viewed as non-fuzzy at the individual level, but not necessarily at a group level. For instance, Mary may say that an interval for a *twentyish man* is from 15 to 25; John might insist that it is from 16 to 24. What follows is that at an individual level the expression is not fuzzy at all, because Mary and John are individually certain about it. If we examine it at a group level the expression could be viewed as a fuzzy one, since Mary's and John's responses are different and they may not reach a unified decision.

It is empirically proven that fuzziness at an individual level is less than that at a group level. For example, Wallsten *et al* (1986a) conclude that the membership function of fuzzy terms is useful and reliable for individuals, rather than the group (see Section 2.2 for the details). Our data in Chapter 3 also showed that there was indeed a discrepancy among the subjects in terms of intervals they designated for FQs.

1.1.2 Fuzzy quantifier

The quantifiers² that are most often referred to are *all*, *a*, *the*, *any*, and *some*. These are called *logical quantifiers*, because their meanings are not context-dependent. What I intend to explore here is another kind of quantifier, known as *FQs*, which is context-driven. The generic term FQ covers a collection of quantifiers whose representative elements are: *several*, *many*, *few*, *a few*, *about 10*, *approximately 5*, *nearly 10*, *10 or so*, *10-odd* and *3 or 5*.

An FQ is defined here as a quantifier which has no clear-cut meaning boundary in terms of what precisely the number should be. What is special about an FQ is that it generates, in an approximate fashion, a set of numbers. For instance, when John says that Mary is *20 or so*, he means to say that Mary's age is within a permissible latitude (i.e. a possible interval which is appropriate for an FQ) allowed by *20 or so*. We may say that *20 or so* is certainly less than *2,000 or so*, or *There are many students in the hall* logically entails *There are several students in the hall*. However, we are less sure about the precise boundaries of *many* and *20 or so*.

FQs have the property of being context-dependent. For instance, *many* may be interpreted differently in the following two sentences: *Many people are in my room* and *Many people are in Tian An'men Square*. Due to the different sizes of *my room* and *Tian An'men Square*, *many* would be interpreted as having a narrower interval in the former than in the latter (see Section 4.1.2 for further discussion). We term quantifiers like *many* as non-standard quantifiers or non-logical quantifiers in the sense that they are context-dependent, as opposed to so-called logical quantifiers (e.g. *all*, *a*, *the*). They are also called natural language quantifiers (e.g. Moxey and Sanford, 1993b). It should be noted that FQs are typical natural language quantifiers rather than atypical, since there are many more FQs in natural languages than so-called logic quantifiers.

²In this thesis, the term *quantifier* is used as defined in Barwise and Cooper (1981), where a quantifier is a noun phrase: determiner + noun. However, for the convenience of exposition, I may leave the noun out from time to time.

1.2 Fuzziness—vagueness—ambiguity

In research, keeping terms clearly defined is important in preventing confusion among readers. It appears that fuzziness, vagueness, and ambiguity represent different linguistic phenomena; hence there is a need to identify these differences.

There is a clearer distinction between ambiguity and the other two than between vagueness and fuzziness. In fact, some researchers do use *fuzzy* and *vague* interchangeably. For instance, at the beginning of this century Peirce (1902: 748) gave his definition of vagueness: "A proposition is vague when there are possible states of things concerning which it is intrinsically uncertain whether, had they been contemplated by the speaker, he would have regarded them as excluded or allowed by the proposition." This definition of *vagueness* fits the characteristic of *fuzziness* in my terms. The differentiation between *fuzzy* and *vague* may be insignificant in other studies and using one or the other is purely a matter of preference, but in some linguistic areas the two terms are defined in a different sense, such as in Kempson (1977) (see Section 1.2.2 for further discussion). This is a point which should be addressed.

1.2.1 Fuzziness

The term *fuzzy* is defined in Crystal (1991: 148) as:

"A term derived from mathematics and used by some LINGUISTS to refer to the INDETERMINACY involved in the analysis of a linguistic UNIT OR PATTERN. For example, several LEXICAL ITEMS, it is argued, are best regarded as representing a SEMANTIC CATEGORY which has an INVARIANT core with a variable (or 'fuzzy') boundary, this allowing for flexibility of APPLICATION to a wide range of entities, given the appropriate CONTEXT. The difficulty of defining the boundaries of *cup*

and *glass* has been a well-studied example of this indeterminacy. Other items which lend 'fuzziness' to language include *sort of*, *rather*, *quite*, etc."

It is interesting that Crystal states that *fuzzy* is derived from fuzzy mathematics. It indeed appears that researchers who have a science background of some sort tend to use *fuzzy*, perhaps because the term *fuzzy* comes from the term *fuzzy set* proposed by Zadeh (1965), and was originally used in fuzzy mathematics. Similarly, if one talks about something related to FST one may prefer to use the term *fuzzy*. That is one reason I chose to use the term *fuzzy* in this work, rather than *vague*; the other being that, as mentioned, some linguists (e.g. Ullmann (1962) & Kempson (1977)) have already defined *vague* linguistically in a different sense (see next section for a detailed discussion). It is therefore wiser and clearer to use the term *fuzzy* with a fresh application.

Crystal does not list the term *vague* in his *A Dictionary of Linguistics and Phonetics*. He might consider the term *fuzzy* more technical-oriented than *vague*, and it has a distinguishing feature in terms of its connection with *fuzzy mathematics*.

An influential work on fuzziness is Zadeh's (1965) FST. He suggests that fuzziness can be formally handled in terms of a *fuzzy set*, a class of entities with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each entity a grade of membership ranging between zero and one, notated as $[0, 1]$ (see Section 5.2 for more discussion).

Lakoff (1973a) applies FST to the study of meaning. He points out that there is a certain degree of *fuzziness* around componential boundaries. To consider *bird-likeness*, it appears that *robin* is a central member, as it belongs to *bird-likeness* completely. *Bat* is a peripheral member as it hardly belongs to *bird-likeness*. Thus, a better way of representing the meaning of *bird-likeness*, especially the referential meaning of it, is to rank relevant members as to the degree of their *bird-likeness*—the degree to which they match the core member of *bird-likeness*. Here is a *bird-likeness* hierarchy, reproduced

from Lakoff(1973a):

robins
eagles
chickens, ducks, geese
penguins, pelicans
bats

(1.1)

The hierarchy is not a bad approximation. Also, some experiments carried out by Heider (1971) have shown a distinction between central members of a category and peripheral members of the category. She surmises that if subjects have to respond *true* or *false* to sentences of the form *A (member) is a (category)*—for example, *A chicken is a bird*—the response time would be faster if a member is a central member (a good example of the category) than if it is a peripheral member (a poor example of the category). Some of the examples of central and peripheral category members that emerged from her study are listed in Table 1.1:

TABLE 1.1: Heider's examples

Category	Central Members	Peripheral Members
toy	ball, doll	swing, skates
bird	robin, sparrow	chicken, duck
sickness	cancer, measles	rheumatism, rickets
metal	copper, aluminum	magnesium, platinum
sport	baseball, basketball	fishing, diving
vehicle	car, bus	tank, carriage
body part	arm, leg	lips, skin

Heider's work shows clearly that category membership is not simply a yes-or-no question, but rather, a matter of degree. Different individuals may have different category-rankings depending on their experiences, their world knowledge and their beliefs.

Supposing that instead of asking about category membership we ask about the truth values of propositions that assert the category membership. The degree of truth corresponding to the ranking of category membership in (1.1) is listed in (1.2), adapted from Lakoff (1973a):

- | | | | |
|----|----------------------|---|-------|
| a. | A robin is a bird. | (<i>true</i>) | |
| b. | An eagle is a bird. | (less true than <i>a</i>) | |
| c. | A chicken is a bird. | (less true than <i>b</i>) | |
| d. | A penguin is a bird. | (less true than <i>c</i>) | (1.2) |
| e. | A bat is a bird. | (false, or at least very far from true) | |
| f. | A cow is a bird. | (absolutely false) | |

We have to make it clear that the examples given here have to be understood in terms of ordinary language. Scientists might make (a), (b), (c) and (d) absolutely true, and (e) and (f) absolutely false. Based on the scientific conception, the fact that a penguin is not a typical bird does not make it less true than that a penguin is a bird. For more discussion of FST, see Section 5.2.

1.2.2 Vagueness

Kempson (1977: 124-128) defines four types of vagueness:

1. Referential vagueness, where the meaning of a lexical item is in principle clear enough, but it may be hard to decide whether or not the item can be applied to certain objects;
2. Indeterminacy of meaning, where the meaning of an item itself seems indeterminate;
3. Lack of specification in the meaning of an item, where the meaning is clear but is only generally specified;

4. Disjunction in the specification of an item's meaning, where the meaning involves an either-or statement with different interpretation possibilities.

What happens in (1), referential vagueness, is that we do not have clear-cut criteria to distinguish the extensional meaning of items like *city* or *town*; *mountain* or *hill*; *forest* or *wood*; and *house* or *cottage*. For example, the relationship between the expression *city* and a place called *Perth* in Scotland is not absolutely clear, i.e. it is not certain if *Perth* in Scotland can be called a city.

Let us look at Kempson's example, *John's sheets*, to illustrate (2): indeterminacy of meaning. It may be used to describe not only the sheets John owns, made or designed, but also the sheets which go on the bed in which he is going to sleep. There is indeterminacy of meaning when *John's sheets* stands in isolation, because there are several possible interpretations of it. This example is used by Kempson to consider the phenomenon of one general term (e.g. *John's sheets*) having different possible meanings.

In fact, it seems that *John's sheets* could also be seen as an example of (4) disjunction. It involves a few interpretation possibilities as shown in (2a) in Fig. 1.1 below. However, it is not a case of fuzziness as far as my definition of fuzziness is concerned, because if we talk about fuzziness of *John's sheets*, we would look into whether the extensional meaning of *John's sheets* is clear-cut or not. If the expression is fuzzy, then we cannot be certain whether or not some of those possible interpretations are in the denotation of the expression. This can be represented in Fig. 1.1:

FIGURE 1.1: John's sheets

Fig. 1.1 shows that in (2a) the relations between *John's sheets* and those possible meanings (i.e. M1, M2, ...) are certain, i.e. they definitely belong to *John's sheets*. This is what Kempson's example exhibits. On the other hand, (2b) reveals the uncertainty between *John's sheets* and its possible meanings. This is what a fuzzy expression would depict.

In addition, indeterminacy of meaning of *John's sheets* in (2a) could be resolved in context. We may be able to pick up one of those interpretations which fits in a certain context. On the other hand, for (2b) context may not resolve fuzziness (see page 19 for further discussion on contextual effects). So, *John's sheets* is not fuzzy in my terms.

Then, in terms of type (3), lack of specification, Kempson says: "The simplest example of lack of specification is an item like *neighbour* which is unspecified for sex, or for that matter, race, or age, etc. It can be applied to people as disparate as a tiny, five-foot Welshman studying Philosophy, and a six-foot Ghanaian girl who has seven children and who only did four years' schooling". However, this appears to be a type of generality. There is uncertainty over whether Perth is a city or not, but there may be no dispute about both the Welshman and the Ghanaian girl being neighbours. The expression *neighbour* is a general term, i.e. unspecified in Kempson's terms.

There is a distinction between *unspecified* and *fuzzy*. The concept of *unspecification* concerns an expression not constituting or falling into a specifiable category, whereas the concept of *fuzziness* concerns an expression having an uncertain extensional denotation. For instance, Kempson calls the expression

neighbour unspecified in terms of sex, age or race. We say that the expression is fuzzy, because we do not know whether or not a person living one mile away is a neighbour. What fuzziness concerns is if an entity is denoted by an expression, but not the nature of the entity. As far as the example of *neighbour* is concerned it can be both unspecified (as Kempson implies) and fuzzy (as I imply).

In terms of type (4), Kempson discusses *or* in the sentence *The applicants for the job either had a first-class degree or some teaching experience*. The implication that *or* contributes to the sentence is that one of the two conjuncts is true, or possibly both are true. That is to say *or* in this instance may or may not be used in an inclusive sense: an applicant could have a first-class degree, or some teaching experience, or both. Then, the sentence given would be either true or false totally. This is not a case of fuzziness, because a fuzzy sentence such as *About 200 students left*, would have a degree of truth.

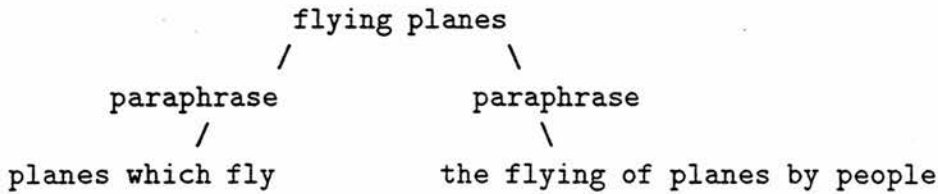
To conclude, of Kempson's four types of vagueness only type (1) presents a case of fuzziness in my terms, characterized by having no precise extension for an expression.

1.2.3 Ambiguity

Ambiguity is defined as: expressions which have more than one unrelated meaning. An expression is ambiguous if it has several paraphrases which are not paraphrases of each other. One example often quoted is:

Flying planes can be dangerous. (1.3)

This sentence is ambiguous, since the expression *flying planes* itself has two unrelated meanings: *planes which fly* and *the flying of planes by people*. This can be illustrated in Fig. 1.2:

FIGURE 1.2: Ambiguity example

In Fig. 1.2, *flying planes* has two paraphrases which are not paraphrases of each other. In what follows, the distinction between ambiguity, vagueness and fuzziness is to be explored.

1.2.4 Distinction between the three concepts

First, let us look into the distinction between fuzziness, ambiguity and generality. It appears that fuzziness deals with uncertain extensions, ambiguity with more than one unrelated meaning, and generality with lack of content. Fine (1975) explores this by providing some hypothetical examples. Suppose that the meanings of predicates, $nice_1$, $nice_2$, $nice_3$, are given by the following clauses:

- $$\begin{array}{ll}
 (1) & (a) \quad n \text{ is } nice_1 \text{ iff } n > 15, \\
 & \quad (b) \quad n \text{ is not } nice_1 \text{ iff } n < 13; \\
 (2) & (a) \quad n \text{ is } nice_2 \text{ iff } n > 15, \\
 & \quad (b) \quad n \text{ is } nice_2 \text{ iff } n > 14; \\
 (3) & \quad n \text{ is } nice_3 \text{ iff } n > 15.
 \end{array}
 \tag{1.4}$$

Predicate $nice_1$ is fuzzy, because its meaning is under-determined. As shown in (1), n is $nice_1$ iff $n > 15$ and n is not $nice_1$ iff $n < 13$. That is, the range from 13 to 15 is an under-determined area; we do not know if the area should or should not go to $nice_1$. To give an example, *The students' number is about 200*, we might say that 199 is definitely *about 200* and 500 is definitely not

about 200, but we are less certain whether or not 290 is *about 200*. So, *about 200* is fuzzy in the sense that its meaning is under-determined.

On the other hand, *nice*₂ is ambiguous, because its meaning is over-determined. Namely, *n* is *nice*₂ iff *n* > 15 and *n* is *nice*₂ iff *n* > 14, then *nice*₂ could have two values simultaneously, i.e. 14 and 15. A term that has two values at the same time is over-determined. For example, *bank* is ambiguous because it has two readings: the rising ground bordering a lake or river; and a financial institute. Finally, *nice*₃ is highly general or un-specific, because *n* is *nice*₃ iff *n* > 15. That is to say, any number above 15 is *nice*₃, which is a non-specific meaning. For instance, the meaning of *item* is considered to be general in the sense that it does not specify the nature of the things it denotes.

Sorensen (1990) also discusses the differences between ambiguity, generality and fuzziness (vagueness in Sorensen's terms). Sorensen says that the distinction is crucial for avoiding confusions emanating from the locution "Word *w* means either *x* or *y* or *z*". Under the generality reading, this "meaning fork" says all utterances of the word mean a disjunction of subclasses, as in "Child' means boy or girl". Let "*M*_{*wr*}" read "*w* means *r*", and suppose that *w* ranges over utterances of a word (or statement) and *r*₁, *r*₂, ..., *r*_{*n*} are alternative readings. Then, a general term always means: either *x* or *y* or *z*, i.e.

$$(w)M_w(r_1 \vee r_2 \vee \dots \vee r_n). \quad (1.5)$$

Under the ambiguity reading the utterances have a disjunction of meanings (or senses) as in "Bank' either means a financial institution or the side of a river". An ambiguous term always either means *x* or means *y* or means *z*, i.e.

$$(w)(M_{wr_1} \vee M_{wr_2} \vee \dots \vee M_{wr_n}). \quad (1.6)$$

Under the fuzziness reading the utterances all mean the same thing but we do not know which precise meaning that is. Each alternative is a definition designed to hold for all utterances of the word, as *noonish* either always

means within one minute of noon or always means within two minutes or A fuzzy term either always means x or always means y or always means z , i.e.

$$(w)M_{wr_1} \vee (w)M_{wr_2} \vee \dots \vee (w)M_{wr_n}. \quad (1.7)$$

Next, a discussion about homonymy and polysemy will make the distinction between ambiguity, vagueness, and fuzziness clearer.

Homonymy and polysemy

In terms of the concepts of *word* and *lexical item*, I consider that the set of lexical items is a subset of the superset of words. For instance, there is one word *tap*, but two lexical items: *to give someone a tap on the shoulder*, and a *water tap*. The Chinese word *hui* 'ability/a meeting'³ has at least two lexical items: *a meeting* and *ability*. Moreover, each lexical item can be divided into different senses. For example, the lexical item *hui* 'ability', has at least two senses: *can* and *understand*.

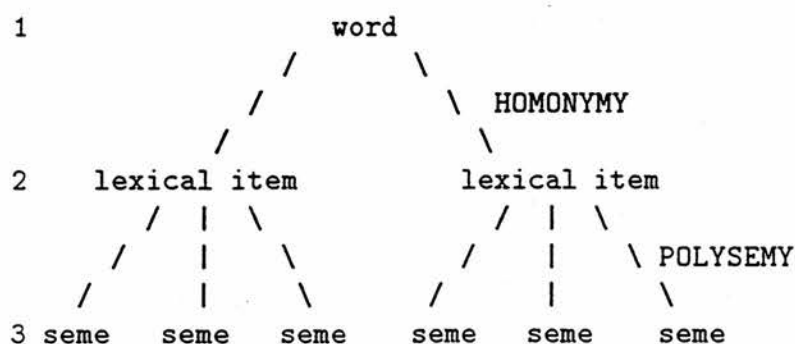
At the level of words, if a word has more than one unrelated meaning, then we term it homonymy. An example of homonymy is the word *tap*, since its two lexical items, as mentioned above, are not semantically related. At the same level, if senses derived from the same lexical item are semantically related to each other, then we call it *polysemy*. Consider the Chinese word *hui* in the following sentences:

³From hereon, FQs in Chinese are represented by *pinyin*—a common phonetic system used to symbolize Chinese characters. The English translation is in single quotes.

- a. *Wo* *hui* *shuo* *Yingwen.*
 'I can speak English.'
- b. *Wo* *hui* *yi* *de* *diandiantou.*
 I understand meaning particle nod
 'I nodded understandingly.'
- (1.8)

The two semes—*can* and *understand*—in (a) and (b) represent two different but related meanings in that presumably if one can deal with something then one would understand what one is doing.

The taxonomy in Fig 1.3 illustrates homonymy and polysemy:



Example:

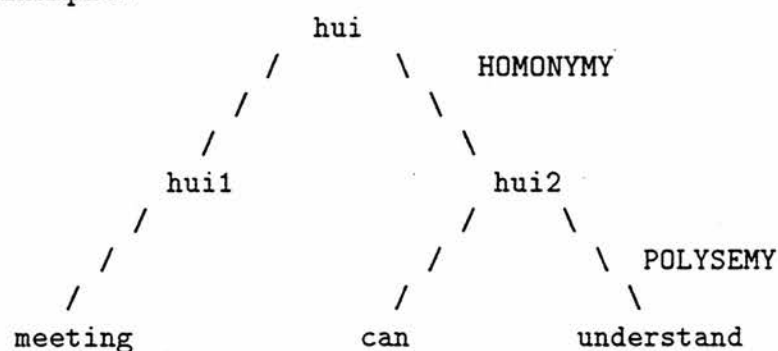
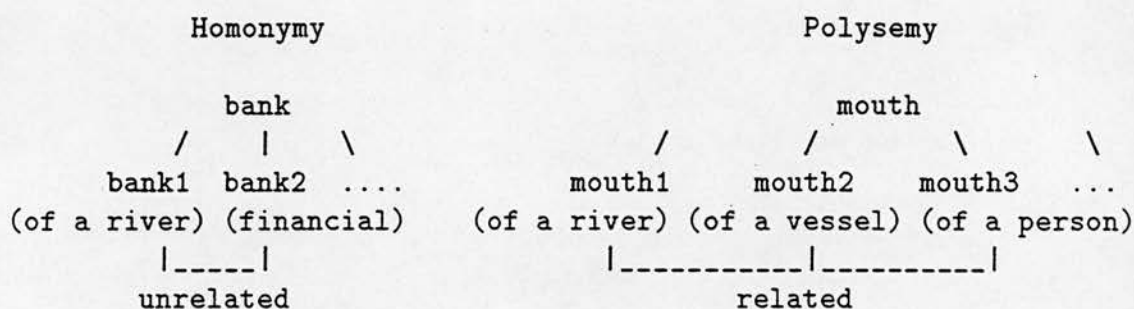


FIGURE 1.3: Chinese example of homonymy and polysemy

It is shown that there are three levels: the word level, the lexical item level, and finally the seme level. Correspondingly, there are two relations: homonymy and polysemy. Homonymy is a property of a word represented by the relation between its lexical items. Polysemy is a property of a lexical item represented by the relation between its semes. The question is how these three levels and two relations are connected to the distinction between ambiguity, vagueness and fuzziness. Let us examine Fig 1.4:

FIGURE 1.4: English examples of homonymy and polysemy

It appears that ambiguity is associated with the word level and is connected with homonymy only, e.g. the word *bank* illustrated in Fig. 1.4. Vagueness, on the other hand, is represented by polysemy, as *mouth* in Fig. 1.4, as is fuzziness. Fuzziness and vagueness are not connected with *homonymy*. For instance, fuzziness between *vermilion* and *pale red* exists in terms of polysemy under the lexical item *red*. In terms of levels, fuzziness and vagueness are associated with all three levels, the word, the lexical item and the seme.

Furthermore, a fuzzy expression is defined as an expression which has no clear-cut boundary. However, a vague expression is defined as an expression which has more than one related meaning, and the question of whether these meanings have clear-cut meaning boundaries is irrelevant. For instance, the meaning boundary of *about 200* is not determinate, which concerns fuzziness. On the other hand, vagueness concerns more than one related meaning: for example, *John's book* has the book John owns, the book John wrote, the book he has been reading, the book he has been told to read, the book he was carrying when he came into the room, etc.. Whether or not these meanings have a clear-cut boundary does not concern vagueness. What fuzziness concerns in this case would be whether or not the denotation of each possible meaning of *John's book* is determinate. For instance, *the book John wrote* is fuzzy, because authorship is fuzzy—how much of the book would John have to write to become one of its authors?

Next, a distinction between the three phenomena can be made by looking at their connections with context.

Contextual factors

It is known that elimination of ambiguity can be carried out if an ambiguous word is associated with a given context. For example, *punch* is ambiguous. It means *to hit* in *I was punched by him*; or *a drink* in *I made gallons of rum punch for the party*.

This may also be the case for examples of vagueness, i.e. vagueness may also be removed by referring to context. For instance:

- a. I read John's book, which was written by his father.
- b. John and Mary have both written books and I have just managed to get John's book.

(1.9)

From the context we know that in (a) *John's book* means the book he owns, not the one he wrote; whereas in (b) *John's book* means the book he wrote, not the one he bought. When *John's book* stands as an isolated lexical item, it is vague; but the vagueness could be removed once it is associated with context.

However, in the case of fuzziness even an adequate context is irrelevant to defuzzification. For instance, *Mary is about 20 years old* is fuzzy; because we cannot reach an agreement of the exact numerical value of *about 20 years old*, the sentence remains an approximation in whatever context. This reveals that fuzziness is inherent, but vagueness and ambiguity are contextually resolvable.

Finally, it appears that ambiguity has two forms: syntactic form and lexical form. To take an example of syntactic ambiguity—the sentence *Young men and women came to my party yesterday* has at least two readings. However, it

seems that vagueness and fuzziness are primarily involved with the meaning per se, i.e. different syntactic structures do not produce vague/fuzzy meanings. This further differentiates ambiguity from fuzziness and vagueness.

An overall profile of the three concepts corresponding to various parameters discussed above, is summarized in Table 1.2:

TABLE 1.2: Ambiguity—vagueness—fuzziness

	homo/poly	Clear-cut boundary	More than one meaning	Context resolved	Syntactic or semantic
Ambiguity	homonymy	yes/no	yes	yes	syn/seman
Vagueness	polysemy	yes/no	yes	yes	semantic
Fuzziness	polysemy	no	no	no	semantic

The table shows that the three concepts differ in several aspects. They are examined at the three levels (word, lexical item, and seme) and connected with the two relations (homonymy and polysemy). It is my contention that ambiguity is associated with the level of words, is related to homonymy only, and is represented by both syntactic and semantic forms. In contrast, vagueness and fuzziness of an expression do not emerge from syntactic structure. Vagueness and fuzziness can be examined at all three levels of word, lexical item and seme, but are primarily related to polysemy. In general, vagueness and fuzziness are under-determined, and ambiguity is over-determined. In terms of whether or not ambiguous and vague expressions have a clear-cut boundary, the answer is yes and no. For example, *John's sheets* is vague in that it has more than one related interpretation, such as *the sheets John bought* or *the sheets John made*, etc. Whether or not the meaning boundary for these interpretations is fuzzy is not an issue as far as vagueness is concerned, as is ambiguity, i.e. it may be fuzzy or it may not. Finally, an ambiguous or vague term has more than one meaning (e.g. *bank* or *John's book*), but not a fuzzy term. A fuzzy term may not have a determinate meaning boundary, but it has only one meaning. For example, *about 20* means a number approximate to 20. We say that a fuzzy term has no more than one

meaning in the sense that although a fuzzy term may have more than one extension, it only has one intension, which differs from a vague or an ambiguous term (see Section 7.2 for more discussion on intension and extension).

In conclusion, the discussion in this section verifies that the term *fuzziness* has nothing to do with *misuse*, and is indeed a technical term. Fuzziness differs from ambiguity and vagueness in that it is not simply a result of a one-to-many relationship between a term and its subsets. (e.g. *John's book*), nor a list of alternative meanings of an expression (e.g. *bank*). Fuzziness is defined in terms of the denotation of an expression. Fuzziness is inherent in the sense that it is not resolvable even with resort to context. On the other hand, vagueness and ambiguity may be contextually resolved, i.e. some readings can be eliminated by incompatibility with a given context⁴.

1.3 Types of FQs

The types of FQs are examined in terms of the way they are constructed. There are three types of FQs which will be discussed in this work.

1.3.1 The three types

TABLE 1.3: Types

Type I:	few, a few, many, a lot
Type II:	about n , n or m , n -ish, nearly n , n or so, n -odd
Type III (semi-FQs):	fewer than n , more than n , at least n .

(n and m indicate numerals.)

⁴It may be argued that the claim that fuzziness is not resolvable by context is a theoretical assumption; it is not empirically testable because there is no way, at least at the moment, we can extract context from a linguistic discourse.

In terms of Type I FQs, more can be listed: *not quite all, nearly all, most, very many, an awful lot, a majority, a comfortable majority, quite a lot, quite a few, several, not a lot, not many, only a few, hardly any, very few*. We will only investigate a proportion of these FQs listed. FQs in Type I do not have numerals like n and m which occur in Types II and III. For example, in *about 20*, 20 is a number required for FQs in Types II and III. The number could be a single number like 20 in *about 20*, or two numbers like 20 and 30 in *20 or 30*.

In terms of the semantics of the three types, most of the FQs in Type I are proportional, except FQs like *several* and *a few* (at least, the two have less sense of proportion). FQs in the other two types are cardinal. Generally speaking, proportional FQs are semantically more complex than cardinal FQs. That is because a cardinal FQ decides whether a set has certain properties based on the number of entities in the set. However, a proportional FQ also has to consider the relation between the sets involved. For instance, we may interpret *about 20 students left* by just checking how many students left. With *Many students left*, we also need to decide whether or not the set of students who left, compared to some norm in a given context, is a significant number. The interpretation of proportional FQs requires more than just checking some straightforward numbers in a relevant set. We will elaborate on this throughout the following discussion.

In terms of construction, some FQs of Type I have only one element (besides the common noun), like *many*. On the other hand, FQs in Types II and III must have more than one element: approximator (e.g. *about*) + numeral (e.g. 200) (see Section 1.3.2 for a definition of *approximator*). Approximators have to appear with numerals to result in approximations. It is clear that this constraint applies equally to all the approximators that fall into Types II and III, such as *nearly, or so, odd* and *more than*. However, FQs in Type I do not have the form of approximator + numeral, a form that FQs in Types II and III must have.

A difference between Type III and the rest is that the former may, in a mathematical sense, have a clear-cut boundary. For instance, we may say that *more than 20* has a precise interval: from 20 to *positive infinity*. However, in ordinary language *more than 20* would be understood as an FQ having a

fuzzy interval, just like any other FQs. That is the reason I consider FQs of Type III as semi-FQs, which will be discussed fully in Section 3.2.3.

1.3.2 Adaptor and fuzzifier

Approximators⁵ are of two types: adaptors and fuzzifiers. An adaptor operates on an item to modify it to a certain extent, and is exemplified by *sort of*⁶ and *very*. On the other hand, a fuzzifier operates on an item resulting in a fuzzy expression, and is exemplified by *about* and *or so*.

What differentiates an adaptor from a fuzzifier is that an adaptor operates on an item which is a fuzzy expression. All it does is alter the meaning of the fuzzy expression to a certain degree. For example, the adaptor *very* in *very many* may alter the original fuzzy meaning of *many* to a certain degree, i.e. intensifying it. A fuzzifier, however, operates on an item which is a non-fuzzy expression. What a fuzzifier does is create a fuzzy meaning. Thus, the fuzzifier *about* combines with an exact number *200* to result in an FQ denoted by *about 200*, i.e. *about* makes the meaning boundary of *about 200* fuzzy. In other words, adaptors can only make a quantity change while fuzzifiers can make a quality change. Most approximators, such as *very*, *about*, are function words—words without contentive elements. They do not denote entities, i.e. they do not denote individuals (e.g. the word *table* denotes a class of individuals) or events (e.g. the word *festival* denotes a class of events).

Usually, an approximator has only one function, either as an adaptor or a fuzzifier. However, there is at least one exceptional approximator: *-ish*, which has a dual function. For instance, if *-ish* combines with a non-fuzzy

⁵Approximators are also termed as *approximatives* in Moxey and Sanford (1993b) and Wierzbicka (1986). For instance, for *about 20*, *20* is the numeral, and *about* is the approximative.

⁶I would draw attention to a subtle difference between the two readings of *sort of*: *sort of* (*kind of*) and *sort of* (*somewhat or to a modest extent*). For instance, *a sort of bird* means a kind of bird, whereas *sort of a bird* means somewhat a bird. In this paper, *sort of* is used consistently to mean *somewhat or to a modest extent*.

expression like in *twentyish*, it acts as a fuzzifier; whereas it could also modify a fuzzy expression like *reddish*, in which it acts as an adaptor.

Fuzzifiers tend to appear with numbers. Anywhere that a number occurs, a fuzzifier can be added to the number and result in an FQ, except where *exactly* or something like that is used. For instance, *I have been in Britain for about four years* can still be acceptable and meaningful without the fuzzifier *about*. It is in this sense that I term *about* and the like as fuzzifiers which bring a fuzzy reading to expressions and propositions which would otherwise usually be precise⁷.

Adaptors rarely combine with an FQ formed by a fuzzifier. For example, we do not say *sort of about 200*, where *sort of* is an adaptor and *about* is a fuzzifier. One thing both adaptors and fuzzifiers have in common is their effect on the truth values of the propositions associated with them. For instance, the truth value of the proposition *198 is 200* would be false; *198 is about 200* may be true. Also, the truth value of *Steam is a gas* would be different from the truth value of *Steam is sort of gas*. Jim Miller (personal communication) points out that the latter assertion places the steam on the periphery of the set of gas, whereas the former assertion leaves it open how central steam is in the set. As Prince *et al* (1980) suggest, the addition of an approximator to a proposition P_i results in the formation of a proposition P_j , where $i \neq j$.

My definitions here are in the same vein as in Prince *et al* (1980). They define an approximator as affecting the propositional content, either by *adapting* a term to a non-prototypical situation (e.g. *sort of a bird*) or by indicating that some term is a *rounded-off* representation of some figure (e.g. *about 20*). In their terms approximators differ from what they called shields, which affect the degree and type of speaker-commitment that is inferred by implicating that the speaker is uncertain because she/he speaks from knowledge or beliefs acquired via *plausible reasoning* or that she/he has no direct knowledge but

⁷It is noted that even exact numbers may be understood as fuzzy. For example, often the sentence *I have been in Britain for 4 years* may well mean no more than *I have been in Britain for about 4 years*, i.e. the fuzzifier is unstated in the former sentence but implied by the context or understood by people.

is *attributing* the belief to a particular other. The taxonomy is illustrated in Fig 1.5, reproduced from Prince *et al*:

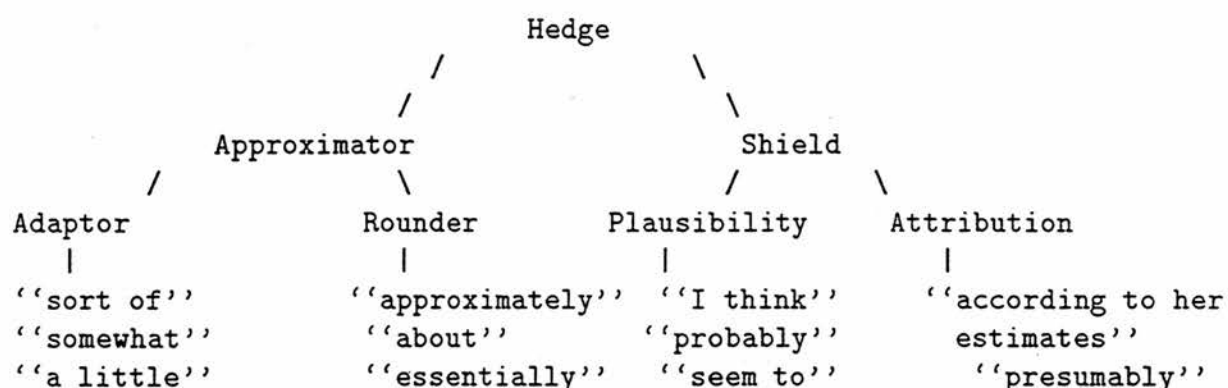


FIGURE 1.5: Types of Hedges

It is claimed by Prince *et al* that approximators affect the truth condition of the propositions associated with them. Adaptors and rounders all implicate non-prototypicalness. In contrast, shields do not affect the truth condition of the propositions associated with them. Thus, the truth value of *A robin is a bird* and *I guess/according to Mary a robin is a bird* shall be the same. The only effect of the latter is that the speaker implicates that she/he is not fully or personally committed in the usual or *unmarked* way to the belief that the relevant state of affairs actually pertains. Shields are not my interest here—we will concentrate on approximators.

1.4 Causal factors for fuzziness

In this section the matter of where fuzziness comes from will be discussed. Through the discussion the nature of the semantic imprecision of FQs will be revealed. It is expected that clarification of this will deepen our discussion of the semantics of FQs.

Ullmann (1962: 118) delineates the following causal factors of fuzziness:

1. Generic character of words;
2. Meaning is never homogeneous (i.e. it is context-bound);
3. Lack of clear-cut boundaries in the non-linguistic world;
4. Lack of familiarity with what the words stand for.

He explains (1) as words referring to “not single items but classes of things or events bound together by some common elements”. For instance, the word *bird* generates a set of objects where “to have feathers” could be considered as a common element. But, the objects in the set differ in some other aspects, and we do not have a precise concept to represent them. This is seen as a conflict between language, thought and the world, which will be discussed in Section 1.4.1.

Factor (2) shows that interpretation of meaning is context-dependent; Ullmann’s implication is, as Channell (1983) points out, that ultimately exact interpretations will appear. In fact, this might not necessarily be the case for fuzziness. For instance, it is suspected that the meaning of *There are about 10 people in the classroom* may not be precise no matter what kind of context it is put into.

Factor (3) says that fuzziness is caused by the real world. Contrary to this, it is claimed in Section 1.4.1 below that fuzziness is not a characteristic of reality. Factor (4) says that the reason for the existence of fuzziness is that we are unfamiliar with what the words stand for. A word could be unfamiliar to a child, a foreigner, or even a native adult speaker. If we do not know what a word stands for then we do not know its meaning. However, this is not a causal factor for fuzziness defined in this work. We define *fuzzy* as a technical term; it means an expression has no clear-cut meaning boundary. For example, we may be absolutely sure about the meaning (definitional meaning, to be precise) of *city*, but we are less sure whether or not a particular place qualifies as a city. This kind of fuzziness differs from a situation where

a particular person happens to not know what the word *city* means at all. So, factor (4) does not seem to be a causal factor, as far as the definition of fuzziness in my terms is concerned. Next, we shall discuss a number of causal factors for fuzziness.

1.4.1 A language concept

This section explores the issue of fuzziness as a language concept rather than a property of reality. It is argued that fuzziness exists because thought, language, and the world do not allow definitive mappings, since they consist of entities of different types.

Locke—the 17th century philosopher of language and mind, author of the *Essay Concerning Human Understanding*—did not believe that language is divine and natural. He claimed that language is created by people, who sort and dominate objects by the sensible qualities which they find in them. Lack of a universal understanding of the world leads to a lack of accurate and sharp criteria for defining the denotation of expressions (e.g. *twentyish*). That is, fuzziness seems to emanate from people's minds. More specifically, fuzziness is caused by human indecisiveness in applying a linguistic item to some element of a domain.

Locke devised in Book Three of the *Essay* an account of meaning which he called *semantic individualism*. That is, the interpretation and understanding of meaning depend on each individual. Moreover, meaning varies in different situations. Locke claimed that each individual understands words in his own particular way, owing to the impenetrable subjectivity of ideas to which words are tied. Fuzziness is indeed associated with subjectivity, since comprehension of the same expression depends greatly upon our backgrounds, world knowledge and experience. For example, answers to the question *Is Mary beautiful?* may differ according to people's varying aesthetic criteria; our differing tastes enhance different aspects of a meal.

Taking Locke's semantic individualism as a basis, it appears that because each individual has his own understanding of meaning, semantic imprecision

inevitably occurs. Furthermore, even with one individual there might also be inconsistencies in interpretations as a result of having no clear-cut criteria in defining fuzzy expressions.

Goocher (1965) provides empirical evidence which suggests that people's inferential activities indeed cause fuzziness in terms of interpretation of expressions like *often* and *seldom*. Take *going dancing* as an example; those who disliked and had little experience of the activity tended to give higher frequency quantifiers than those who enjoyed and had more experience of it. For the former, going dancing once a week might be *often*, but it might be *seldom* as far as the latter is concerned⁸.

However, there is a view claiming that fuzziness is a characteristic of reality. For instance, Ullmann (1962) says that one of the four causal factors for fuzziness is "lack of clear-cut boundaries in the non-linguistic world." An example is *sea water*. Generally speaking, *sea water* means the salt water in the sea. However, the sea water in the natural world is usually mixed up with rain water and water of other kinds. Thus, the distinction of sea water is difficult to make in practice. Objective fuzziness is even adopted as the basis of an ancient fable. Once, Aesop's drunken master made an unrealistic promise that he would drink up all the sea water. When he sobered up, he turned to Aesop for help. The suggestion given by Aesop was to quibble that what the master had promised was to drink unadulterated sea water, but the water in the sea was actually not pure. Aesop took advantage of the fuzziness of *sea water* to save his master from embarrassment by confronting people who were waiting by the seashore.

I do not agree with the assumption that the fuzziness of *sea water* comes from the fact that sea water itself is fuzzy. Sea water itself, like anything else in the world, is just physical stuff. It is neither fuzzy nor precise, it just exists in the world. The fuzziness of the expression is caused by the fact that people are not capable of defining it a precise denotation, at least in ordinary

⁸Newstead (1988: 59) comments that it is possible that this finding is a special case of an effect of the expected frequency, since those who like the activity or indulge in it regularly might have a higher expected frequency for such an event. For more discussion on the role of expectation of language users, see Section 2.4 and Section 4.1.2.

language. That is, the fuzziness of *sea water* is caused by providing no sharp criterion for defining the concept. Similarly, we can say that the expression of *a tall man* is fuzzy, but not that the man himself is fuzzy just because we are not sure whether he is tall or not.

Opposing the view that there are fuzzy objects in the world is the view that the world is precise. Tye (1990) disagrees with the view that the world is precise, as do I. However, I do not incline to believe his view that there are fuzzy concrete objects in the world. Tye is in favour of Ullmann's idea mentioned above. The example Tye gives is Mount Everest. He thinks that there is no line which sharply divides the matter composing Everest from the matter outside it. Therefore, Everest's boundary, in Tye's view, is fuzzy, i.e. Mount Everest is a fuzzy object. He says: "Let us hold that something x is a borderline of F just in case x is such that there is no determinate fact of the matter about whether x is an F ". (1990: 535-536)

How could we know that a concrete object has borderline parts? Tye does not elaborate on this. Is it somehow that the object itself illustrates it? We have little evidence to say so. A reasonable explanation seems to be that only human beings can set up some criteria by which those concrete objects are classified and accommodated. Consider Mount Everest again. The reason for its physical boundaries not being precise is, as Tye says, that there is no line which sharply divides the matter composing Everest from the matter outside it. Then, Tye draws his conclusion that Mount Everest itself is fuzzy. However, it is obvious that Mount Everest itself cannot possibly draw any line—precise or fuzzy. It is we, as human beings, who give criteria to the concept of *Mount Everest*. Mount Everest is fuzzy because we cannot reach an agreement on its denotation, but Mount Everest itself is just something which unconsciously exists in the world.

Another example is *rainbow*. Although using a kind of non-ordinary language, physicists may somehow be able to identify the denotation of *rainbow*; in ordinary language, it is fuzzy. In terms of the visual colour spectrum we cannot decide precisely where the edge of a rainbow is, i.e. there are bands, but no bounds. It again appears that the object in the world being called *rainbow* is neither fuzzy nor precise, just something in the sky. When people try to identify the edge of it, they may not come to an agreement. So, it is

the expression of *rainbow* that is fuzzy, rather than the object itself. My view is shared by Russell (1923), Black (1937), Fodor (1977), Fine (1975), Danell (1978), and Schmidt (1974). They all agree that fuzziness is a phenomenon of language, not of reality. Moreover, even if our perception of something (e.g. *rainbow*) in the world were absolutely accurate, language communication of such information could not be (see the next section for the details).

To sum up, it is claimed that fuzziness is a property of language and human inferential activity, but not a property of concrete objects in the real world. Fuzziness is due to the fact that there is no precise correspondence, i.e. a mismatch, among human thought, language and the real world; they are different in various aspects. As far as linguistics is concerned we say that the denotation of an expression is fuzzy, and is a matter of linguistic categorisation. Cognitive property may be significant in the way the world is structured by humans, but fuzziness (in denotation) is essentially a linguistic concept.

1.4.2 Lack of need for preciseness

In this section, we discuss the lack of need for preciseness in language. Consider the sentence, *Mary is twentyish*, which has a fuzzy meaning. In communication, there may be no need to make the sentence precise after all. That is, this fuzzy sentence may just serve our communicational purpose well; a precise sentence is simply not required. Another example is Mount Everest. It is true that there is no sharp criterion to decide where to draw a line which divides the matter composing Mount Everest from the matter outside it. Nevertheless, the fact is that there is usually no need to do so in day-to-day conversation anyway.

This factor is compatible with Grice's (1975) conversational principles. He outlines *cooperative principles* which see certain *maxims* followed in conversational situations:

Maxim of Relevance: Do not say something that is not relevant.
Maxim of Quantity: Do not say more than you need to say. (1.10)

In the case of fuzziness, the two maxims play a key role; in particular, the Maxim of Quantity, i.e. people should not say more than they need to say.

Then, the question is how people communicate with each other appropriately if Locke's account of *semantic individuality* (see Section 1.4.1 for more details) is the case. In other words, how can an individual elicit a common meaning which can be understood by other people? On this point, Locke presented an account of *secret reference*. What he meant by this is that there is a presupposition of meaning among people, by which they can understand each other up to a point. Locke thought that each individual has a private language that might make absolutely adequate communication difficult, but it does not mean that people cannot communicate at all. Language users have a common *secret reference* that makes communication in an approximate fashion possible.

Empirically, Locke's *secret reference* account is verified in our tests. It was found that there was a high agreement on typical members of an FQ. For instance, in Channell's (1983) test (see Appendix 1, Table 2.2), most subjects agreed that 15 was in the meaning boundary of *about 15 people*, i.e. 15 was considered as a prototype. The test results showed that the fuzziness tended to emerge in a peripheral area. Using Locke's account of *secret reference*, subjects had a kind of common understanding about the typical member of *about 15 people*, which is also called *public meaning* in Locke's terms. It is speculated that semantic individualism, by and large, appears in the area of the peripheral members of an expression. Because language users have a common understanding of the best examples of expressions in a given context, it should be no problem for them to communicate with one another.

There is another explanation about how people communicate with fuzzy language. Although the extension of a fuzzy word is interpreted differently, its intensional denotation may be consistent among users. For instance, *many* may well have very different numerical values corresponding to various

given contexts. Its intension (*a significant number compared to a norm*), however, could remain consistent over contexts and among language users. Successful communication among language users may be due to a common understanding of the intension of fuzzy words, rather than the extension (see Section 7.2 for more discussion of the matter of intension and extension).

Lyons says: "We communicate with one another for the most part successfully, as far as the application of words like 'dog' are concerned, only because we do not usually find ourselves operating in the fuzzy or indeterminate areas of a word's meaning" (1986: 70-71). If by this he means people use fuzzy words to communicate without consciously being aware of the fact that they are fuzzy, then it seems only partially true. It might be the case that people do know that words like *dog* are fuzzy, but they are able to deal with it as it is and do not think that the word needs to be precise. Fodor (1977: 212) holds a similar view. She states: "A realistic view of natural language surely must recognise that ordinary people often do use the word *bird* without intending to include penguins, or use the word *fish* intending to include whales. To what extent do we really care, in our everyday conversation, whether the words we use carve nature at its joints?"

To sum up, although a fuzzy term is imprecise, very often there is no need to specify a precise boundary for the term. People may communicate well enough, no matter knowing or not that they are using fuzzy language. There might be some situations in which a precise numerical value for an FQ is simply not needed. Furthermore, an intensional meaning, such as *a significant number* for *many*, may just serve communicative purposes well (see Moxey and Sanford (1993b) for further discussion of this point).

1.4.3 Need for fuzziness

It is claimed in the last section that in some situations, there is simply no need for precision. In this section, we discuss the need for fuzziness, and its important role in communication. Communicative need is indeed another causal factor for fuzziness.

As argued in Section 1.4.1, it could be the case that human perception and cognition are not fuzzy, but the concepts and language required for communication are. Linda Moxey (personal communication) argues correctly that in order to convey a distance or a quantity we have to have an appropriate set of linguistic labels which can be scaled in a similar way by different language users. These labels only approximately describe the actual quantity to be described. Thus even if our perception/cognition were 100% accurate, our communication of such information could not be. For example, given a particular scene in the sky, people might easily perceive exactly the same (or very similar) shape/range of colours etc. The range of words we have to describe such scenes, however, is much more severely limited than our perception, and the label chosen by different people will be different. In fact, the label chosen by one individual may differ from one occasion to the next because of the function which the scene is playing in the communication. That is why we have to use fuzzy expressions.

We need to have a means of communication which is flexible enough to cope with the indefinite variety of our thoughts and experiences. In languages, fuzzy expressions refer to categories of individuals or events, not just single items, so they can function as such a means. For instance, *about 200* could be understood as denoting a set of numbers, where some numbers in the set, say 190, are seen as more typical or better members than others, say 180. Moreover, FQs can be used to generalize over several instances. For example, a person is asked how many friends he has: he may answer *a few* because the number may be consistent with this description, but varies from time to time. It is not the reality which is fuzzy, nor our perception, but the requirement of communication needs fuzziness.

We may all experience times when we feel like fuzzy communicating or feel it is better to communicate in this way. Fuzzy language can make our language communication more adequate and efficient. It appears that fuzziness is needed as much as precision, which is stated in Moxey and Sanford (1993b: epilogue):

“Every day, in many situations, we use expressions which seem to provide us with only vague information. The weather forecaster

tells us that 'some showers are likely in Northern regions during the night', a statement which is vague with respect to number of showers, location and time. Yet such messages are informative, and often it is not possible for the producer of the message to be more precise. A tutor tells his students that 'only a few students fail their exams outright'. This does not give a precise incidence. Yet it might be equally misleading to do so. For example, to say that 12% failed outright last year says nothing about other years, while to say an average of 8% over the last five years says nothing about variability. We argue that a precise, numerical statement can be sometimes more misleading in reality than a vague statement."

The usefulness and necessity of fuzzy language in communication has been ignored for a long time by philosophers and linguists. As Lyons (1981: 203) says, their attitude to fuzzy language is "a highly prejudiced and unbalanced view" and

"Not only is it frequently, and erroneously, associated with the view that all sentences have precise and determinate meanings; it is based on the equally erroneous assumption that clarity and the avoidance of vagueness and equivocation are always desirable, regardless of what language game we are playing."

In fact, fuzziness plays a very important role in language communication. Channell (1994: Ch.8) gives a list of purposes and situations for using fuzzy language. Here are some items on the list, with examples provided by me.

(1) Giving the right amount of information. In Grice's (1975) view of the Maxim of Quantity, we should not say more than we need to. Fuzzy terms are needed for conforming to this rule of conversation, by giving the right amount of information appropriate for a given situation, not too little and not too much. For instance, when John's daughter came back from a school fair, he asked her: "How many people were there?". She answered: "A lot."

A lot, as an FQ, might give just about the right amount of information John wanted to know. He didn't expect to be given a precise number for people who went there.

(2) Deliberately withholding information. People tend to use fuzzy language on purpose, in order to withhold information. For example, a middle-aged lady is asked about her age, she might say *over 30 years old*, when in fact she is 39 years old.

(3) Lacking specific information. Naturally, if one does not know precise information about something, one has to give as close an approximation as possible. For instance, if one does not know exactly how many people in a room, but has a rough idea that there might be about 20 people. Then, one has to use *about 20 people* to answer the question "*How many people are in the room?*"

(4) Self-protection: If one wants to safeguard against later being shown to be wrong, even though one knows the exact information, one might like to use fuzzy language. For instance, a lecturer knew exactly 200 students pre-enrolled for his course, but when being asked, he may prefer to say *about 200*. The reason is that there might be some changes later on, so he uses the FQ as a safeguard.

Channell also demonstrates some other purposes and situations of using fuzzy language: using language persuasively; lexical gaps; displacement; power and politeness; informality and atmosphere; and women's language. It is interesting to note that Channell says that some respondents who took part in her test told her that women use more fuzzy expressions than men do. Moreover, in applied linguistics, Brown (1979) from his observations of foreign speakers of English, makes a point that fuzziness is needed in terms of language appropriateness.

The causal factors for fuzziness discussed in this section can complement each other. Sometimes we may be able to define an expression precisely, but we still leave it fuzzy, because there is no need to specify. For instance, one may know Mary's precise age, but one still says *Mary is twentyish* because there is no need to be precise. At other times, there is indeed a need to specify

an expression precisely, but the mismatch between language, thought and the world prevents us from doing so. Let us consider the example *Mount Everest*. If for some reason we really need to know precisely where the line is which sharply divides the matter composing Everest from the matter outside it, we may find ourselves unable to do so. Also, there could be a situation in which we are able to specify a fuzzy expression, but it would not be socially appropriate, e.g. to specify a lady's age precisely. Certainly, there would be cases where several factors are involved simultaneously. To consider the sentence *Mary is tall*, we may have different interpretations in terms of whether she is tall or not; meanwhile there could also be no need to specify her precise height.

Finally, it is worth pointing out that there are two kinds of fuzziness. One could eventually be made precise, such as the kind of fuzziness emerging from lack of knowledge. This kind of fuzziness can be dealt within the framework of a theory called the supervaluation theory (see Chierchia & McConnell-Ginet (1990) for a discussion of the theory). The other kind of fuzziness is caused by other factors rather than lack of knowledge. This kind of fuzzy term cannot be made precise, which may be dealt by FST (see Chapters 5, 6, and 7 for more discussion).

In conclusion, on the one hand the mismatch between language, thought, and the world creates fuzziness. On the other hand, often it is not precise meaning we want, and fuzzy terms play a significant part in language and communication. Our discussion shows that language is fuzzy because it cannot be precise, need not be precise, and communication is sometimes improved by not being precise. Fuzziness is viewed here as a characteristic of language, but not of reality. Thus, a need for studying fuzziness in languages is justified.

1.5 Approach and methodology

Although fuzziness is an important aspect of language, it has not yet received enough attention in linguistics. There is also no well-established linguistic

approach to it. A general approach used in my work is fuzzy semantic. It explores FQs in terms of their denotational meanings represented by membership functions.

1.5.1 Denotational approach

In the same vein as in Cann (1993), a denotation is defined here as consisting of two things: intension and extension (see Section 7.2 for further discussion of this). The reasons for using a denotational approach here are fourfold.

Firstly, the denotational meaning of linguistic expressions is a primary concern in semantics. Semantics differs from syntax, logic and psychology in that semantics deals predominantly with the denotational meaning of expressions. One of the criteria of an adequate semantic theory is this: A semantic theory must provide an account of the relation between linguistic expressions and what might be called things in the world. Also, one of the principles of semantics states that core meaning is determined by the relation between linguistic expressions and the entities that they refer to. In other words, denotation forms part of the basis of semantic theory. Especially in truth-conditional semantics, the denotational meaning is used to judge whether or not a proposition is true. That is, from the point of view of truth-conditional semantics, we must have some denotational meaning in mind to be able to assess the truth or falsity of a proposition. For example, the truth value of *About 200 students left* depends on whether or not the number of students who left was approximate to 200. So, as far as semantics is concerned, denotational meaning of expressions requires investigation. The denotational theory has been well established and has proved extremely fruitful in the study of semantics, although there is still some debate on it. It is thus expected that a denotational approach will be useful for the study of FQs here.

Secondly, there is a need for the extensional denotation of FQs in communication. In some situations, we do need to know the numerical value of an FQ, although in an approximate way. As an illustration,

Husband: There will be many people coming tonight for our party,
we'd better prepare enough food.

Wife: How many roughly?

(1.11)

In this anecdote, it appears reasonable for the wife to ask for an approximate number to prepare food for the party.

Thirdly, researchers have reported that subjects experienced little difficulty coming up with numerical values in one form or another (e.g. Mosteller and Youtz 1990). This claim is also supported by a large percentage of subjects who were able to give numerical intervals in my Chinese test (see Chapter 3 for the details). In the Chinese data (see Appendix 2, Table 2.2), out of 24 items studied, 100% of subjects marked numerical intervals for 13 items: 99.9% — 97.78% for 9 items; and 81.48% for two items. In Channell's (1983) English data (see Appendix 1, Table 1.2), out of 32 items tested 100% of subjects marked numerical intervals for 15 items; 99.9% - 90% for 12 items; only 5 items were below 90%. This empirical evidence provides a foundation for a denotational approach in the sense that the denotational meaning of FQs can be elicited.

Finally, the study of the denotational meaning has an important role in emerging technologies, such as Expert Systems. For instance, a simulation processing has to somehow represent a degree of uncertainty or confidence, in order to simulate human expertise or experience. FQs are often used by experts. For instance, one may say that *Give a few minutes to cool the soup down, then stir it for another 5 or so minutes*. This piece of information is fuzzy. The question then is how to make machines to process these FQs. To solve this kind of problem, a denotational study is needed.

However, it could be said that there is no need for specific numerical values in understanding of FQs. For example, *many* in *There were many people in my room* may simply imply that there were more than a normal number in the room. Also, *few* in *There were few people in my room* could just suggest that there were less than a normal number in the room. What matters in

these two sentences is the inference, or some other information conveyed by them, rather than numerical values. This may be reasonable from the point of view of other fields, but not from a semantic point of view which has been explained above. In particular, the idea that the numerical value is a dismissible matter is less obvious with, for example, Type II FQs. In the sentence *There were about 50 people in my room*, the numerical value of *about 50*, however crude, appears to be a main concern, as far as semantics is concerned.

Indeed, sometimes we may only use intensional denotation of an FQ in communication, rather than its extensional denotation. In other words, it could be that in non-experimental situations, *a few* often means *a small number* and does not get mapped to a more fine-grained scale at all. Although in some situations *a few* in *A few students went to the party* may mean *a small number* and its numerical value may not be a concern, we cannot then claim that *a few* does not have an extensional denotation at all, because it is equally true that we do use the extension of *a few*, say 4 in a given context, in communication. Although pragmatically we may not always need to know the extensional denotation of *a few*, in a semantic theory we must have some provision for explaining its intension, extension, and the relation between them. What concerns semantics is the issues of how the extension of an FQ is determined by its intension, and extensional meaning, i.e. numerical meaning in the case of FQs is part of the meaning for most, if not all, expressions.

I am aware that in some situations, the numerical values between FQs, e.g. *a few students* and *few students*, may not be differentiable. However, it does not mean that the two are extensionally equivalent or they have no extensions at all, since *a few* and *few* may differ extensionally in some other situations⁹. Another example: in some situations, *few* and *very few* may not be different in numbers. However, in the sentence *There were few customers in the shop yesterday; there were very few customers today*, *few* and *very few* may not convey the same numerical value. *Very few* probably means a smaller number

⁹Also, taking the morpho-syntax seriously, *few students* and *a few students* are not equivalent. Indeed, the use of the indefinite signals that a non-numerical meaning is intended, but there is still a relation between the cardinality of *a few* and that of *few* with respect to some norm as denoted by *students*.

compared to that of *few*. The point is that we cannot claim that *a few* and *few*, similarly *few* and *very few*, are the same in numbers, since *a few* differs in meaning from *few* if there is some situation in which the former is the case but not the latter, and vice versa.

It must be made clear that the denotational approach promoted in my work is different from an approach called the *uni-dimensional* approach (see Moxey and Sanford (1993b) for further discussion of the problems of this approach). The general idea of the uni-dimensional approach is that the meaning of FQs can be and should be captured by scale-values adequately. One of its ultimate goals is to provide some kind of codification for regulating the meaning and usage of fuzzy terms (see Mosteller and Youtz (1990) for a discussion of codification). However, my work here attempts to capture general semantic patterns of FQs in terms of their denotational meanings, not codification of any sort. It is not my interest to map each FQ into a fine-grained scale to get some kind of precise numerical value.

As argued above, as far as the semantics is concerned, denotational meaning of FQs is important to explore. However, it is not assumed that FQs cannot and should not be explored from other perspectives. For example, FQs may also be examined from the point of view of pragmatics or psychology, where how context affects the meaning or how people understand FQs would be in focus. Moxey and Sanford (e.g. 1993b) have done some important work on FQs by using a multi-dimensional approach. They have proved on empirical grounds that other aspects of meaning differ between quantifiers, such as attentional focus¹⁰ and inferential pattern (see Section 2.4 for a review of their work). I do not assume that FQs only differ numerically; in fact I agree that they may differ in some other ways, as argued by Moxey and Sanford. However, my interest in this work is the denotational meaning, so non-numerical aspects of FQs will not be my focus here.

¹⁰An attentional theory on natural language quantifiers has been developed by Moxey and Sanford (1993b) (see Section 8.4 for the details).

1.5.2 Membership function

Denotational meanings of FQs here will be represented by membership functions as defined in FST (see Section 5.2 for the details). FQs in my work are defined as fuzzy semantic units by assigning them a grade of membership and a degree of truth for sentences containing such units. In the case of FQs, conventional semantic theory is regarded not as an empirical hypothesis, but as a default truth. It tends to make a rigid distinction between semantic units and to represent an over-simplified form that does not capture the richness of FQs in natural language. For instance, there is no absolute boundary for FQs like *about 200 people*, but the all-or-none principle of the conventional theory cannot represent the fuzzy semantics. It appears that fuzziness can be reasonably handled within the framework of fuzzy semantics led by FST. This will be demonstrated throughout the following discussion.

The method used in this work is primarily theoretical, i.e. my main focus is the theoretical semantics of FQs, formal semantics in particular. Some sample empirical data from Chinese and English will be discussed for the purpose of illustrating the possible semantic trends and how my formal semantics would work with more appropriate membership functions. Although it would be surprising if every language turned out to be identical, there should be some common patterns to be observed. This kind of pattern will be explored and formalized in my work. It is important to make clear about the capabilities of my formal system and its relationship to the statistical results. Because my data here is a sample, it will not be considered as a justification of my formal treatments. All the membership functions used in my formal semantic framework are hypothetical.

Finally, the focus in my study here is the semantics of FQs with respect to ordinary language, not scientific language. That is, I am interested in the FQs in the context of natural language. This is an important point to be borne in mind since an expression may be interpreted differently depending on whether it is considered in ordinary language or scientific language. For instance, colour terms are fuzzy in day-to-day language, but there are precise scientific definitions for each colour term.

1.6 Conclusion

A *fuzzy quantifier*, as a technical term, is a quantifier with a “hazy” meaning boundary, such as *about 20* and *many*. Being used here as a technical term, it is not fuzzy at all in the sense that it is appropriately defined. Approximators, such as *about* and *very*, have a function to fuzzify or modify. Given that language is relatively efficient and the functional vocabulary more so, the fuzzification and modification determined by an approximator must be different from those defined by all others. What are these membership functions? How do they vary? How can we define FQs’ semantics using them? The following chapters will discuss these questions.

FQs are indeed an important linguistic phenomenon to be studied in linguistics. Because of their unique characteristic of having a *hazy* boundary, the non-standard theory, FST, is employed in my work here. I am aware of the fact that FQs can be explored through some other perspectives, but in my semantic work here it is essential to explore the denotational meaning of FQs represented by membership functions.

2

Previous work on fuzziness

In this chapter, we will review the following works: Black (1937) concerning consistency, Wallsten *et al* (1986a) concerning membership functions, Mosteller and Youtz (1990) concerning codification, Moxey and Sanford (1993b) concerning pragmatic effects, and Wachtel (1981) concerning a formal treatment for FQs.

2.1 Black's work

As early as 1937, Black proposed a quantitative description of fuzzy items, called a *consistency profile*. He defines the consistency of application of a term T to an element s , $s \in S$, as:

$$C(T, s) = \lim_{\substack{M \rightarrow \infty \\ N \rightarrow \infty}} \frac{M}{N} \quad (2.1)$$

where M stands for the number of positive judgments that T applies to s ,

N the number of negative ones, and *lim* stands for a limit of M or N . The consistency profile is then defined as the function $C(T, s)$ over the domain of applicability S . This can be represented in Fig. 2.1, reproduced from Black (1937):

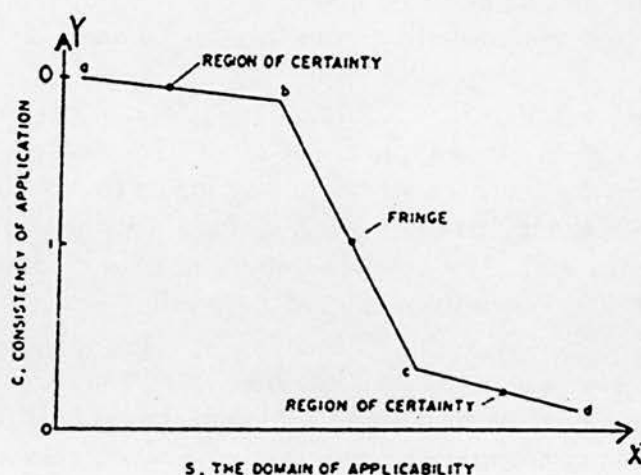


FIGURE 2.1: Hypothetical consistency profile

As Hersh and Caramazza (1976) point out, Fig. 2.1 exhibits a typical example in which the most doubtful cases correspond to $C(T, s) = 1$, i.e. 50% yes and 50% no. This profile, then, is used to define the fuzziness of a term by taking the slope of the curve from point b to point c , shown in Fig. 2.1, as an index of fuzziness.

The quantitative description proposed by Black assumes the consistency of fuzzy items in terms of their denotational variations. This differs slightly from Zadeh's (1965) FST in that Zadeh makes a quantitative description of a fuzzy item by focusing on its membership function (see Section 5.2 for further discussion of this point), rather than its consistency of application. However, there is a similarity between them in that they all consider the application of fuzzy item as a matter of degree.

Hempel (1939) develops (2.1) further, to restrict the range of C to a closed interval 0 to 1. His definition is:

$$C(T, s) = \lim_{\substack{M \rightarrow \infty \\ N \rightarrow \infty}} \frac{M}{M+N} \quad (2.2)$$

where the consistency of application of a term T , to an object $s \in S$ will have a range in $[0, 1]$, and the borderline cases take on values of about 0.5.

Importantly, Black assumes that fuzziness must be characterized with consistency, i.e. the variability of applications of an item has to be predictable and regular, otherwise there would be no way for us to make some kind of distinction between them. His assumption is empirically justified, in part, by the data in Wallsten *et al* (1986a) where the membership function is reliable on an individual basis (see Section 2.2 for the details).

What I intend to study in my work is compositionality which shares Black's idea in that there must be something common among FQs, otherwise semantic theories would not make much sense (see Section 4.2 for a discussion on compositionality). One of my ultimate goals in this thesis is to pull out common properties of FQs, and formalize them.

2.2 The work of Wallsten *et al*

Wallsten and his colleagues (Wallsten, Budescu, Rapoport, Zwick and Forsyth 1986a, henceforth Wallsten *et al*; Rapoport, Wallsten and Cox 1987) investigate ways by which the meaning of probability terms, in specific contexts and to individuals, can be represented adequately. They employ a technique similar to the membership function defined in FST, i.e. employing $[1,0]$ to indicate a degree of membership. There is little empirical work done on membership functions per se, although there is considerable theoretic literature on FST. Wallsten *et al*'s work claims, on empirical grounds, that a membership function is useful for representing the meaning of fuzzy terms, like *toss-up* and *improbable*. Their empirical data elicited from sophisticated and technically elegant experiments enhance the theoretical idea of the notion

of membership function in FST. They established and assessed membership functions by using a modified pair-comparison procedure in two experiments.

In the first experiment, ten probability terms were tested: they were *almost certain*, *probable*, *likely*, *good chance*, *possible*, *toss-up*, *unlikely*, *improbable*, *doubtful*, and *almost impossible*. Twenty graduate students were recruited as subjects. There were three sessions, and three parts in each session. Session one was for practice only and the other two were for collecting data. For each session, part one elicited an interval for a given term. The results of this part were then used to determine the unique probabilities to be used in Parts 2 and 3 for each subject. The second part of the session was to present one probability term with a pair of spinners which were radially divided with segments of opposing colours denoting different probabilities. as shown in Fig. 2.2, reproduced from Wallsten *et al* (1986a: 350).

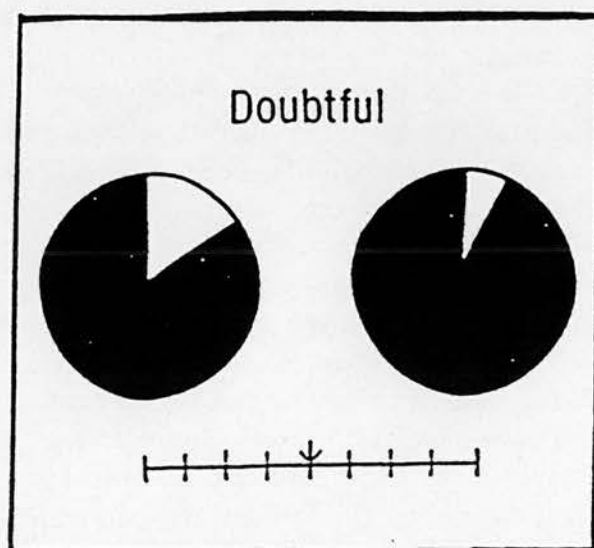


FIGURE 2.2: Sample experimental scenario

Subjects estimated the degree to which one probability rather than another was better described by a probability term. For instance, subjects had to judge, as shown in Fig. 2.2, that the term *doubtful* was better described by the probability shown in the left-hand spinner or the probability shown in the right-hand spinner. By moving the arrow along the line underneath the two spinners, subjects indicated to what extent one probability was better than the other. If the left spinner was considered absolutely better described, the subject would move the arrow to the far left. On the other hand, if the right spinner was absolutely better described, then one would move the arrow to the far right. Leaving the arrow in the middle indicated that the term was equally well described by the two spinners. The probabilities on the two spinners were changed from trial to trial, and varied factorially for each term. Based on various scaling models, membership functions were plotted for each term against probability values which were derived by calculating the degree of displacement on the line using a conjoint-measurement theory (Norwich and Turksen, 1982, 1984; Sjöberg, 1980).

Part 3 reversed the procedure, conducted to validate the results of Part 2. That is, on trial a spinner representing a particular probability was written at the top of the screen, and two probability terms were presented below it. In the same manner as Part 2, the subject moved the arrow on the line segment to indicate the degree to which one term rather than another better described a particular probability. For instance, the arrow position on the line indicated to what extent a probability 0.3 was better described by *doubtful* than *likely*.

The results showed that individual subjects were reasonably stable over sessions, despite the fact that there was no probability term for which all subjects' membership functions had exactly the same shape. For Parts 2 and 3, data were analyzed at the individual level. Membership functions were represented by the shape of curves plotted against probability values given by each individual. It is concluded by Wallsten *et al* that subjects can compare degrees of membership in such a way that leads to a consistent, meaningful, and interpretable scaling of fuzzy meanings, according to either a ratio model or a different model.

It turned out that 67% of membership functions observed were interpretable

as meaningful test results, 30% of these were monotonic, and 31% were single-peaked. Thirty-three percent were not interpretable because they were multi-peaked, which was explained by Wallsten *et al* as measurement error. After the first experiment, Wallsten and his colleagues realized some problems with sampling probabilities over ranges specified by individuals, i.e. individualizing the stimuli for each subject. All the trials in Parts 2 and 3 depended on a single determination of an upper and lower probability for each term obtained in Part 1. Then, if a subject made an error in Part 1, that error would be carried through all the subsequent results, making the comparison between subjects more difficult. To provide more complete and reliable membership functions for each subject, a second experiment was conducted.

In the second experiment, eight subjects were selected who had given relatively consistent and more reliable membership functions in the first experiment than the other 12 subjects. This time, the stimuli were reduced to six probability terms—*almost certain*, *almost impossible*, *unlikely*, and *possible* were dropped out. There was no Part 1, instead probabilities were selected on the basis of results from Experiment 1 and the same values were used for all subjects. The subjects' performances were similar to those described earlier, in spite of some minor response procedure change.

The results for the second experiment were more interpretable, with 56% of membership functions as monotonic and 44% as single-peaked. Again there was little within-subject variability over sessions, but there was considerable difference between subjects, except for *toss-up*. *Toss-up* was given similar single-peaked functions by all eight subjects, whereas none of the other five terms was given precisely the same function by any two subjects. This can be illustrated as in Fig. 2.3, reproduced from Wallsten *et al* (1986a: 360-361).

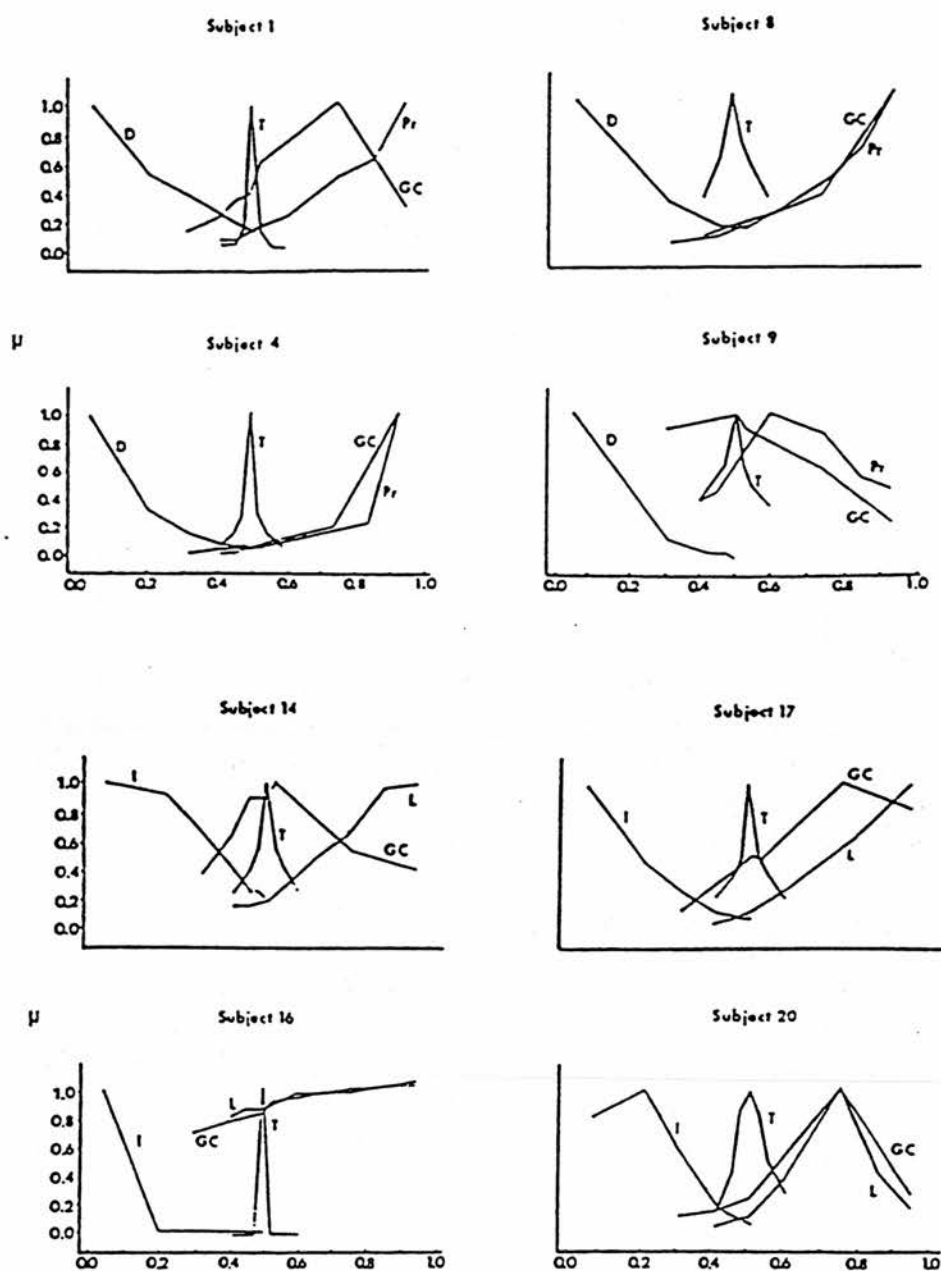


FIGURE 2.3: Probability

The functions are coded as follows: D = doubtful; GC = good chance; Pr = probable; T = toss-up; I = improbable; L = likely.

As shown, for those monotonic functions, the best described probability is at the end of the range, whereas for those single-peaked functions, it is at the centre of the range. Wallsten and his colleagues conclude that the more extreme probability terms tend to have monotonic functions, and more central terms tend to have single-peaked functions. They also believe that the judgments obtained were highly reliable, and satisfied a set of necessary conditions required for measurement of membership. The results, they claim, were fitted well with the scale values extracted by appropriate algebraic models (Budescu & Wallsten, 1987). It is also concluded that theoretical issues regarding the use and understanding of probability phrases are more appropriately investigated at an individual level rather than a group level.

The construct of the membership was further validated by the work in Rapoport, Wallsten and Cox (1987). In their work, a comparison between a direct (magnitude estimation) and an indirect (graded pair-comparison) procedure was carried out through establishing membership functions for some probability terms. It is concluded that there is consistency in terms of membership functions obtained from the modified pair-comparison, direct estimation, and ranking procedures. Indirect procedure is slightly more accurate than the direct procedure, but the direct estimation technique may obtain sufficiently good results in general.

It is empirically shown by Wallsten *et al* (1988: 42) that the interpretation of a probability term appears to depend on context. The data in Fillenbaum *et al* (1987) yield that the function representing a term for a given individual is located more centrally and covers a broader interval when the phrase is received from, rather than selected for communication to, another person. Moreover, base rate (Wallsten, Fillenbaum & Cox 1986b) and desirability of the events (Cohen 1986) also have an impact on the function of a probability term.

The work done by Wallsten *et al* lays an empirical foundation for my claim that the meaning of FQs can be represented by membership functions. Also, their findings on context effects provide empirical evidence for my discussion on the relation between FQs and context (see Chapter 4 for more discussion).

Wallsten *et al* make an assumption that fuzzy terms investigated in their

work are compositional in that the shape of curve generated by a term would be the same. On the other hand, Moxey and Sanford (1993b) do not agree with this on the grounds that the shape may change. My work will show that compositionality is not about the shape of curves, otherwise it would not sustain at least in the case of FQs; I agree with Moxey and Sanford that context may alter the shapes (see Chapter 4 for further discussion on this).

2.3 Mosteller and Youtz's work

Mosteller and Youtz (1990) explore the possibility of codification for probability and frequency items. They gathered empirical data from 20 different studies in which 52 expressions were tested. A comparison shows that the variation of numerical averages of opinions on quantitative meaning of 52 items was modest, although they were tested in a variety of populations (e.g. students, physicians, other medical workers, and science writers), formats of questions, instructions, and contexts. The consistency is claimed to suggest that the test results may be useful for codification.

Mosteller and Youtz conclude that it is possible to offer codifications by determining what these terms mean to the people who use them, and see how satisfactory people find them. For instance, they found that *likely* was judged to represent an average proportion of 0.71 of the time with an interquartile range of about 0.15. That is to say, if one says something is likely, then one means that it occurs about 71% of the time plus or minus 7.5%. This kind of average is codified in their work for a number of probability and frequency expressions. It is claimed that codification achieves a better communication in that expressions codified can be used and interpreted consistently.

The codification idea is also promoted in the work done by Kong *et al* (1986). Their study, conducted through a nationwide interactive computer network based at the Massachusetts General Hospital, was carried out to test the meaning of commonly used probability terms. For 12 expressions used in the test, there was little variation in the median values assigned to them among the three groups: physicians, medical students, and other professionals. Also,



the same order remained in terms of means for seven of the 12 expressions compared to those found in an earlier study by other researchers, although numerical values assigned to the seven items were not the same. It is then concluded that consistency among groups of professionals is encouraging for the future prospect of codifying the meaning of such expressions. The investigation of codifying is needed to standardize the variation in the use of probability terms. The average numerical values presented in the work could well be used to enhance communication among medical professionals and the like.

The effort put into codification shows that there is a desire to have a relatively consistent assignment to a fuzzy expression, and it is indeed not difficult to get an average from different contexts and different individuals. The question is whether or not this kind of codification is acceptable and adequate. In discussing Mosteller and Youtz's work, H. Clark (1990) says that the meaning of *tall* means "greater than some norm in height", which should not change over contexts. What changes is the normal height we infer in order to fill out the meaning of *tall* with respect to a certain context. We are likely to infer different heights for *tall* when it is used in *tall grass* vs *tall tree*¹. Clark states that fuzzy items discussed in Mosteller and Youtz have the same property as *tall*, i.e. they are relative adjectives or adverbs. For instance, *often* really means "often relative to C"; it varies from context to context (see H. Clark (1990) for a detailed discussion). Any codifications by averaging over contexts would not be adequate in the sense that we cannot possibly exhaust all the contexts.

While Wallsten and Budescu (1990) applaud the work done by Mosteller and Youtz (1990), they also state that regularity should not be taken to suggest that a major codification of the language of probability is a goal to be pursued, for at least four reasons: consistent individual differences; personally vague meanings; context effects; and the necessity of fuzziness. It is argued that probability phrases should not be made to appear more precise than they really are. There are situations in which the nature of

¹What Clark talks about may also be explained as a difference between intension and extension of a word (see Section 7.2 for the details); that is, the intension of a word is relatively independent of context, but not its extension.

fuzzy judgment is useful information in and of itself. Communication about uncertainty should always be as precise as possible, but never more precise than warranted by the data.

It appears to me that Mosteller and Youtz's work for codification could be useful for some situations where a specific value for a fuzzy term is required. For example, in the case of weather broadcasting, we may need to know a numerical value for some fuzzy items, say *likely*. Then, codification can be considered as one of those alternatives to achieve this goal. However, as Wallsten and Budescu (1990) point out, in ordinary language fuzzy terms are not precise, and there is no need to make them precise (see Section 1.4 for more discussion). Also, codification is not universally applicable, especially to those complex fuzzy terms. For example, codification might be much easier for *about 200 people* than for *many*, while 200 can act as a norm, but it is hard to get a unitary norm for *many* in isolation. It could be 10, or 1,000, very much depending on context.

It is concluded that codification can be done if required for a certain purpose, but it would be mistaken to overestimate its usefulness. It appears to me that as far as the semantics of fuzzy expressions is concerned, it is the semantic structure that is important, not those specific values. This point will be addressed throughout the following discussion.

2.4 Moxey *et al*'s work

Moxey *et al* (e.g. 1986, 1987, 1990, 1991, 1992, 1993, 1994 & 1995) have done extensive research on natural language quantifiers in terms of how they are understood, reasoned and used. They aim at:

1. Investigating what differentiates quantifiers from one another in terms of conditions of use.

2. Examining how the understanding of quantified statements might fit into more general accounts of language understanding, and of reasoning. (Moxey and Sanford, 1993b: 111)

Since my work here concentrates on the semantics of FQs, Moxey and Sanford's work of the effect of context on the interpretation of FQs will be reviewed in detail.

One of their experiments was carried out for testing the proportions denoted by FQs like *many* in three different contexts. The subjects were 450 university students who were presented with one of the following, where QUANT represents a quantifier:

"The residents association's Xmas party was held last night in the town hall. QUANT of those who attended the party enjoyed what might be called the social event of the year." (Residents Association condition)

"At yesterday's party conference, Mr Cameron spoke about the effects of education cuts on British universities. QUANT of his audience were convinced by his conclusions." (Party Conference condition)

"A survey has recently been carried out to find out whether or not female students prefer to be examined by female doctors. QUANT of the local doctors are female." (Survey condition)

Subjects were divided into 30 independent groups of 15. One third of those groups (150 subjects) were presented with the Residents Association text, another third with the Party Conference text, and the remainder with the Survey text.

Within each of the above texts, QUANT was replaced with one of the following ten quantifiers: *few*, *a few*, *very few*, *only a few*, *quite a few*, *not many*, *many*, *very many*, *quite a lot* and *a lot*. One tenth of the subjects (15 subjects) in each topic condition were presented with one of the ten quantifiers. On a separate sheet of paper, each subject was then posed a question. One third of the subjects in each of the above conditions were then asked the same

questions. For instance, subjects presented with the Residents Association text were asked "What percentage of the residents do you think enjoyed the Xmas party?"

One of the findings was that the higher the expectation manipulated in the three contexts above, the higher the percentage assigned to quantifiers. Thus, our prior expectations about the proportion of an FQ play a role in interpreting FQs, i.e. the interpretation depends in part on one's prior expectation about what the proportion being described might be. Initially, this effect was discovered only in the category of high-ranking quantifiers. In a later experiment (Sanford, Moxey & Grant, submitted) where ten quantifiers were tested in ten contexts rather than three contexts as in the former test, it was found that a good linear relation existed between the interpretation given to each quantifier and the base rate of expectation associated with each context, however much weaker. It appears that contextual effects do exist among lower-ranking quantifiers, if sufficient situations are manipulated.

Some empirical study on levels of interpretation of FQs has also been carried out by Moxey (1986) and Moxey and Sanford (1993a), the only work of this kind. They set three basic conditions which created three different base rate frequency expectations. A total of 192 subjects were divided into three groups. They were all university students. One group was tested for level 1 meaning (sentence meaning): what percentage the quantifier denoted literally. A second group was tested for level 2 meaning (speaker's meaning): what proportion they thought the speaker had expected before the speaker discovered the facts. A third group was tested for level 3 meaning (listener's meaning): what proportion the speaker thought the listener might have expected before the listener heard the quantified statement.

The findings showed that the quantifiers affected levels 2 and 3, and proportions estimated for some quantifiers differed between levels. For instance, *quite a few* was given a low value in both levels 2 and 3, which means that less had been expected than what really happened. Quantifiers, such as *very few*, *few*, *not many* and *only a few* were given a higher value in level 2, but *a few* was given a low value in the same level. Similarly, those negative quantifiers obtained a higher value in level 3, except *few*. So, it indeed appears that the interpretation of natural language quantifiers involves one's prior

expectations about what the proportion being described might be. There are other contextual factors that can affect the interpretation of FQs, which will be elaborated in Chapter 4.

Apart from their contributions to the issue of contextual effects on FQs, Moxey and Sanford also investigate quantifiers from a non-numerical perspective using an interdisciplinary approach—psychology integrating with linguistics and logic. In Moxey and Sanford's view, a multi-dimensional investigation is needed for adequately studying natural language quantifiers (see Section 8.4 for further discussion). One of the major contributions Moxey and Sanford have made is to argue that other aspects of meaning differ between quantifiers, in addition to numerical meaning. For example, it is argued that when combining with a quantifier, *very* may not intensify numerical value, instead *very* in *very few* enhances the strength of claim. It is also argued that FQs in communication may not be mapped into a numerical value in a fine-grained scale. People do not necessarily need to compute a fine-grained scale for each single FQ they use (see Moxey and Sanford (1993b) for further discussion).

In terms of language understanding and language use, Moxey and Sanford's points on FQs are plausible. I incline to their view that FQs can be explored in other aspects, apart from their numerical values. For example, we may explore the cognitive meaning of FQs by investigating a wider range of issues, like the intention of speakers, attentional focus (see Section 8.4 for further discussion), etc. We may look at a relation between meanings and their functions, a kind of communicative/psycho-linguistic relation. Moxey and Sanford's work puts forward plausible insights from a different perspective, which is certainly worthwhile pursuing. While I agree that FQs do differ in other aspects of meaning, as far as semantics is concerned denotational meaning of FQs needs exploration (see Section 1.5.1 for my arguments).

So far, we have reviewed a few empirical works. Next, we will review a formal account on FQs. Two more formal theories associated with FQs, FST and GQT (Generalized Quantifier Theory, Barwise and Cooper 1981), will be discussed in detail in Chapter 5.

2.5 Wachtel's work

Wachtel's (1980, 1981) treatment for a number approximation is to designate an interval in which an *appropriate round number* is a kind of prototype while other numbers in the interval are supposed to have certain membership with regard to the round number. The membership of one number n , situated on one side of the round number, equates to another number m 's membership, situated on the other side with the same distance away from the round number n . Wachtel proposes that an approximation containing *approximately* is true for an interval of numbers, whose central point is the prototype. The length of an interval in any given case is determined by a function from C (a set of contexts) into F (a set of rounding functions). This function draws from a domain of numbers into a set of numbers to select an appropriate approximating number or numbers. The notion of *round number* is used in a technical sense defined as in (a) in (2.3) below, where R is a function from a set of contexts C into a set of rounding functions F , and N is a set of numbers. F is a set of functions from N to N , and is defined as in (b), where $|n|$ is the absolute value of n . The expression *about* is treated as denoting a function from numbers and contexts into sets of numbers. This function is represented as *about'*, and is defined as in (c). For all $n \in N$ and all $c \in C$,

- a. For all $n, m \in N$ and all $c \in C$, n is an appropriate round number for m in c iff $R(c)(m) = n$;
 - b. $F = \{f: (x, y) [f(x) = y \rightarrow (z)[|z - y| \leq |x - y| \rightarrow f(z) = y]]\}$;
 - c. $\text{about}'(n, c) = \{x: R(c)(x) = n\}$.
- (2.3)

R selects the rounding function that is operative in the context of utterance. A proposition, such as *John has about 20 apples*, is true in context c if and only if 20 is an appropriate round number in c for the number of apples that John has—i.e. if and only if the number of apples that John has is a member of $\text{about}'(20, c)$.

To show how (b) in (2.3) works, let us examine the following relation:

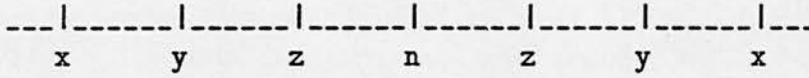


FIGURE 2.4: An illustration

The function in (b) assumes that the function of “if x , then y and z ” is valid if and only if taking n as a prototype, z is nearer to n than y and x are, y in turn is nearer to n than x is.

Wachtel (1981) expands the treatment in (2.3) in order to reply to Channell’s (1980) objections to the analysis of approximations outlined in Wachtel (1980). He claims that different approximators can be handled within the same framework he proposes in (2.3), with no radical changes. The formula that begins by taking into account the symmetrical distribution of a number approximation exemplified by *approximately* n , also applies to skewing number approximations exemplified by *n or so*. He says that although the function F given in (2.3b) appears to induce a symmetrical distribution, skewing approximators can also be handled by F . The simplified version of the expanded formula is given as follows (see Wachtel (1981) for details):

Let $A = \{i, j, k, l\}$ where i and j are non-skewing approximators, which only differ in their permissible latitudes; and k and l are skewing approximators which also only differ in their permissible latitudes. $H = \{h_1, h_2, h_3, \dots\}$, which is a function from number to number, indicating non-skewing or skewing approximations. $G = \{g_1, g_2, g_3, \dots\}$, which is a function from context to context, indicating different lengths of permissible latitudes. Then, for all $n \in N$ and all $c \in C$,

$$\begin{aligned}
 a. \quad i'(n, c) &= \{x : R(g_1(c))(x) = h_1(n)\}; \\
 b. \quad j'(n, c) &= \{x : R(g_2(c))(x) = h_1(n)\}; \\
 c. \quad k'(n, c) &= \{x : R(g_1(c))(x) = h_2(n)\}; \\
 d. \quad l'(n, c) &= \{x : R(g_3(c))(x) = h_3(n)\}.
 \end{aligned} \tag{2.4}$$

It is shown that the only difference for i and j is g_1 for i and g_2 for j , i.e. they

differ in lengths of permissible latitudes in various contexts. The similarity between i and j though is that they are both non-skewing approximators, having the same h_1 . Now, k differs from i in having a different h , meaning k is a skewing approximation whereas i is a non-skewing one. Finally, l differs from i , j and k in both g and h . It appears that the above treatment does cover both skewing and non-skewing FQs.

Comparing (2.3) and (2.4), the general framework involving F and R remains unchanged, although in the latter some parameters are added. A general formula is shown in (2.5), where A is the set of approximators. For all $a \in A$, there is some pair $\{g, h\} \in G \times H$ such that, for all $n \in N$ and all $c \in C$,

$$a'(n, c) = \{x : R(g(c))(x) = h(n)\}. \quad (2.5)$$

That is, R and F remain unchanged, and the introduction of G and H is for the specification of the semantics of each individual approximator.

Channell (1983: 208) states that the problem with Wachtel's account is its extreme generality which leads to counter-intuitive predictions. She says "For the number approximation, because it treats all numbers as equally available for use in approximations, equivalence is predicted, such as:

- a. This is approximately one inch long.
 - \equiv
 - b. This is approximately 2.067 cms long.
- (2.6)

(1 inch = 2.067 cms)²

which is clearly wrong, given all we know about differences between round and non-round numbers."

In fact, in my view it is a misunderstanding to assume that Wachtel's account predicts (a) and (b) in (2.6) are exactly equivalent. According to Wachtel, the mapping function itself should be the same for the two sentences above,

²However, usually 1 inch = 2.54 cms, the current author notes.

but this does not predict the equivalence of their actual values. On the contrary, the problem of round and non-round numbers could be sorted out by the function of G , defined in (2.4), since different lengths of an interval could be made by G . That is, $g(a)$ and $g(b)$ could make the intervals of (a) and (b) in (2.6) different.

Wachtel takes contextual factors into account in his formal treatment of number approximations. There is ample evidence to indicate that context is an important factor in specifying an FQ's meaning boundary. For instance, Hörmann (1982) reports that people typically think of *a few mountains* as being four-five mountains, while *a few crumbs* means eight crumbs (see Chapter 4 for more examples). In my treatment of FQs here, I shall also take contextual factors into account (see Chapter 6). Wachtel's work conveys the idea of prototypability well, and it is mainly on the shape of curves (symmetrical and skewing). However, in his framework, there is no provision for membership function and degree of truth. This makes his work limited in dealing with FQs. In my formal treatment, membership function and degree of truth are primary issues to be investigated (see Chapters 5, 6, and 7 for further discussion).

2.6 Conclusion

The review in this chapter shows that there may have regularities in the semantics of FQs, and it is proven empirically that membership function is indeed reliable on an individual basis. Moreover, contextual factors can affect the interpretation of FQs. It is then concluded that for any adequate semantic theory of FQs, the role of context must not be neglected.

It appears that there is little work which explores FQs in terms of semantic patterns and membership function. In particular, it is not yet clear whether or not FQs are compositional, and if so on what grounds. There is a need to work on these issues; my work here is an attempt to meet the need.

What follows is an empirical study of FQs in both Chinese and English, and

what is aimed for is common semantic properties of FQs. There will be a comparison between English and Chinese FQs, in order to explore similarities (and differences) between them.

3

An empirical study of the semantics of FQs

In this chapter, we will analyze the semantics of FQs based on empirical data from Channell (1983) and Zhang (the current author). The study of FQs in this chapter is intended to observe semantic trends statistically. Although the data here is a sample and the data analysis is preliminary, they serve my purposes. The data is meant to give a starting point for the theoretical analysis later¹.

3.1 The data

In both Chinese and English data, a numerical experimental method was used to test a possible interval of a given FQ. Channell did an elicitation test mainly on English FQs in Type II, such as *about n*, *n or so* and *n or m*. My questionnaire data covered Chinese FQs in Types I, II and III.

¹On the other hand, the restricted usefulness of such sample data for a theoretical treatment is borne in mind (see page 41 for further explanation on this).

3.1.1 Zhang's questionnaire for Chinese FQs

A pre-test was carried out on the questionnaire. The aim was to test the hypothesis that subjects were able to designate a numerical interval for FQs. This was a prerequisite of the main study. FQs were chosen from standard Mandarin Chinese, some being the equivalents of FQs in English tested in Channell's (1983) work, to enable a comparison between the two sets of data. A total of 60 FQs were tested, and each subject was presented with all 60 FQs. The questionnaire, presented in standard Mandarin Chinese, is enclosed in Appendix 3 and its English translation is enclosed in Appendix 2.

A total of five subjects were tested, all of them native speakers of Mandarin and postgraduates at the University of Edinburgh. The results showed that all five subjects were able to give a numerical interval for FQs tested.

Main Test:

The ultimate objective was to investigate semantic patterns of FQs in Chinese.

Method:

Data was gathered through a questionnaire. The response rate was 97%. The percentage of subjects who were able to give an interval is listed in Appendix 2, under the title "%subjects marking intervals" in column 2 of each table. This column indicates how well subjects judge FQs in intervals.

Subjects: A total of 135 Mandarin-speaking adults, most of whom were university students in either the UK or China, completed their questionnaires. To elicit serious responses, subjects were told that the questionnaire was for academic research.

Materials:

The questionnaire was the one used in the pre-test. For discussion in this thesis I chose 24 out of the 60 FQs because they roughly matched those tested in Channell (1983). In doing this I intended to compare the English data with the Chinese.

These FQs all have imprecise meaning boundaries, i.e. they are non-logical quantifiers. They are commonly used natural language quantifiers. Most FQs were tested in isolation. I anticipated that some FQs, like *many*, could hardly be designated an interval without a given discourse. So subjects were invited to mark intervals for *many* and the like out of 100; that is, subjects were expected to tell *How many out of 100 is many*. They were also told to give an interval in percentage for some of the FQs.

I also wished to examine how contextual factors might affect the designation of FQ intervals. Therefore I presented two FQs embodied in a sentence to compare them with the same FQs tested in isolation. The results will be discussed later.

Design and Procedure:

The questionnaires were distributed in person to those subjects in Edinburgh and the rest were sent through the post to China. Once the subjects completed the questionnaire they either handed or sent them back.

Every subject saw all 60 FQs presented and gave one numerical interval for each of the 60 FQs, i.e. each subject estimated 60 intervals in total. They were told to make just one choice for each FQ and to come to a decision independently. They were also invited to make comments if they wished to.

One hundred and thirty-five subjects means 135 intervals for each FQ tested. To obtain a histogram for each FQ, every given interval was divided into n equally spaced numerical values, using the AWK (Aho *et al.*, 1988) programme. For instance, a given interval 180-220 was divided into a range of numbers—180, 185, 190, 195, 200, 205, 210, 215, 220. A histogram was then made up by taking all 135 such ranges of numbers into account, in 95%

confidence. For the calculation of test results I used the statistics package BMDP (see Appendix 2 for the details). As an illustration, for *2,000 ren zuoyou* 'about 2,000 people' the histogram was given as below.

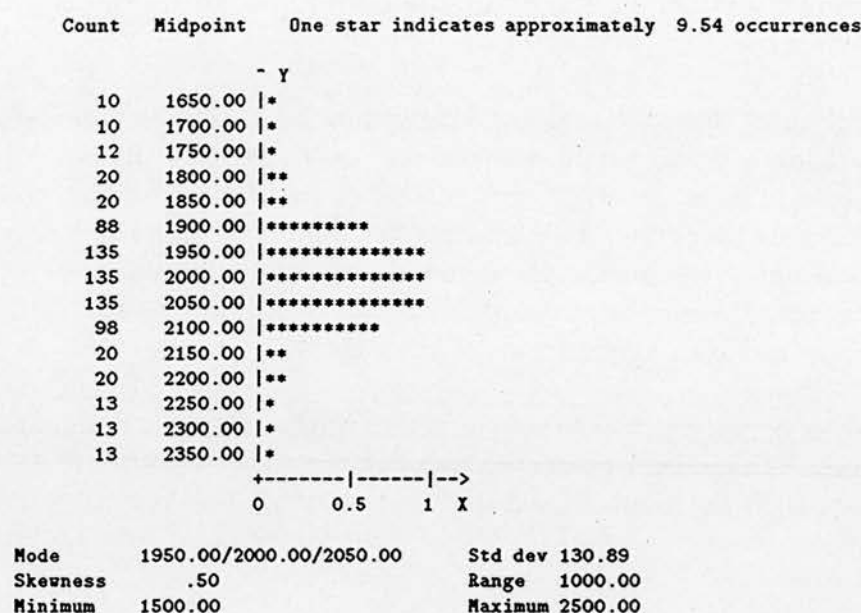


FIGURE 3.1: *2,000 ren zuoyou* 'about 2,000 people'

Notes:

1. The mode number underneath the histogram indicates the most frequent occurrences. There are some cases where for an FQ the mode number is a set of numbers rather than a single number. For instance, in Fig. 3.1 a set of numbers from 1,950 to 2,050 are all mode numbers.

2. In my Chinese test, there were only five FQs tested which had 100% agreed mode numbers (see Appendix 2, Figs. 2.10, 2.13, 2.16, 2.17, and 2.20). 79% of FQs tested had no 100% agreed mode numbers, i.e. the mode number was given by less than the total of 135 subjects. For instance, in Appendix 2, Fig. 2.15, the count for the mode number is 129 rather than 135. This indicates that although the FQs are all fuzzy, we could perhaps say that some are fuzzier than others, because there is a lower agreement on their mode

numbers.

3. Each number on the Y-axis corresponds to a score on the X-axis. The more stars a number gets the higher grade of membership it gets and the more typical a member it becomes. For instance, in Fig. 3.1 the number 2,000 gets the most stars, which means it is more typical of *2,000 ren zuoyou* 'about 2,000 people' than, for example, the number 2,350.

4. For the formulas of computing *standard deviation*, *kurtosis* and *skewness*, refer to (10.1), (10.2) and (10.3) in Appendix 2.

Here the FQ is represented by a membership function over the [1.0] interval. In the histogram, and throughout the following, the ordinate denotes two things, the count in the first column and the midpoint in the second. The count indicates how many subjects estimated the particular number corresponding to the second column as a member of *about 2,000 people*. For instance, 100% of subjects (i.e. 135 counts shown in the first column) thought that the number 1,950 shown in the second column of the same row was in the boundary of the FQ, whereas only 20 subjects thought 2,200 would be.

The abscissa denotes membership values representing the degree to which an individual number on the ordinate (second column) is a member of the FQ. The grade of membership function was derived by dividing the count in the first column by 135 (total number of the subjects). This is a ratio representing a particular count against the best estimate. For instance, the membership of 2,000 is 1, obtained by dividing 135 by 135; and 2,100 has a membership value of 0.73, obtained by dividing 98 by 135. As mentioned above, most FQs in my test were not given 100% agreed mode. That means that they could not have a full membership 1. For example, for *jiangjin 200 ren* 'nearly 200 people', the mode was 111, divided by 135; then its maximum grade of membership was 0.82, as shown in Appendix 2, Fig. 2.9.

Results:

The results, displayed in detail in Appendix 2, show the following trends:

1. People were able to estimate a numerical interval;

2. There was variance between subjects, which represented fuzzy meaning of FQs;
3. The same kind of FQ had the same kind of semantic behaviours.

To test the effect of context I deliberately set two FQs which were tested both with and without a given linguistic discourse respectively. When I tested them in isolation subjects were asked to designate an interval in percentage for *daban ren* 'majority of people', and the same for *shaoban ren* 'minority of people'. In testing these two in context of utterance I presented them combined in a sentence, as listed in (3.1).

- | | | | | |
|----|---------------------|--------------------|---------------|------------------|
| a. | <i>Juchangzhong</i> | <i>dabanren</i> | <i>zai</i> | <i>guzhang.</i> |
| | in the theatre | majority of people | in process of | applaud |
| | 'A majority of | people in the | theatre were | applauding.' |
| b. | <i>Juchangzhong</i> | <i>shaobanren</i> | <i>zai</i> | <i>shuijiao.</i> |
| | in the theatre | minority of people | in process of | sleep |
| | 'A minority of | people in the | theatre were | sleeping.' |
- (3.1)

The test results are shown in Table 3.1 (see Appendix 2. Table 2.2. Group 2).

TABLE 3.1: Effect of context

No	Item title	%subjects designating interval	Mean	Mode (modal interval)
1	daban ren 'majority of people'	81.48	68.75	70.00 (60-80)
2	shaoban ren 'minority of people'	81.48	29.37	30.00 (30-40)
3	daban ren 'majority of people' (in the context)	97.78	66.58	70.00 (60-80)
4	shaoban ren 'minority of people' (in the context)	97.78	26.52	30.00 (20-30)

Note: The number here denotes percentage.

Items 1 and 2 are context independent, whereas items 3 and 4 are context dependent. The scores in column 3 (%subjects designating interval) show that subjects felt more confident in designating FQs in the context than out of it. The percentage of subjects designating intervals in isolation was 81.48%, which is lower than 97.78% for the FQs which were judged in the two sentences given in (3.1).

Furthermore, in column 5 the modal interval (20-30) given to item 4 is lower than that of item 2 (30-40). The mean in column 4 also varies corresponding to the item with or without a contextual setting. Conversely, there are some comparative similarities. For instance, the mode number in column 5 is the same for each item with or without context.

In general, there is no significant difference in Table 3.1. One of the reasons for this may be that the two FQs tested, *majority of people* and *minority of people*, are not known as typical FQs (e.g. *many*). If some typical FQs, like *many people*, are tested, the test result may be different. What can be

concluded from the test results here is that some FQs vary over context less than others.

Further discussion of the test results and a comparison with Channell's results will be carried out in Section 3.2 below.

3.1.2 Channell's elicitation test for English FQs

Channell (1983) discusses two types of expressions, number approximations (e.g. *about ten*) and fuzzy category identifiers (e.g. *a film or something like that*). The former is my interest here. Her test had two main objectives:

1. To test the hypothesis that number approximations designate intervals of numbers;
2. To find out the length of intervals which different approximations designate, and the placing of the intervals relative to the exemplar number(s) present.

Channell used 26 first-year University of York students as subjects for her elicitation test, which consisted of 32 examples of putative number approximations. Some were attested examples from her data and the rest were invented sentences. The test items contained four different approximators (*about n*, *n or so*, *n or m* and *not less than n*) and a selection of exemplar numbers (see Appendix 1 for the details).

Before seeing the test materials the subjects heard a short taped extract of a conversation in which an approximation was used. They then read a transcript of it, in which the approximation was underlined². Then, subjects

²Channell explains that one of the purposes for doing this was to "encourage them to believe that every test item they read was an attested example from my corpus of data. For this reason also, each item was written between quotation marks. I wanted to encourage them to act, as far as possible, as *hearers* of the test stimuli: that is, rather than ask themselves 'Do I say this and what do I mean when I do?', to ask 'If I heard this, what would I understand?'"

were given test papers to designate intervals of various approximations that appeared in 32 examples.

For detailed test results, refer to Appendix 1. Table 1.2 in that appendix displays general results for *about/(a)round n*, *n or m*, *n or so* and *not less than n*. Table 1.3 gives special results for *n or m* approximations.

3.1.3 Remarks on the written instructions

I am aware that the way in which FQs are tested can influence how they are interpreted. For instance, the wording of a question as well as the way it is presented may affect the test results. To minimize bias, written instructions similar to those employed by Channell were used in my Chinese test. Channell gave a sequence of numbers and the subjects were expected to mark their intervals on it. The written instructions each subject saw in her test are reproduced below:

Example:

"You find you get *five or six articles* and they're all very much the same."

Someone who thought this could ordinarily refer to anywhere between three and eight articles (inclusive) would mark their answer as:

1 2 (3 4 5 6 7 8) 9 10

IF YOU FIND THE NUMBERS GIVEN NOT EXACT ENOUGH PLEASE WRITE IN ANY ADDITIONAL NUMBER YOU NEED. PLEASE WORK ALONE.

Compare this with my written instructions:

Example: *jī (ge)* 'several' (no English equivalent for the measure noun *(ge)*)³

³From hereon, for presenting a Chinese FQ, an item bracketed is a measure noun or

Someone who thought its interval could be numbers between 2 and 8 (inclusive) would give their answer as 2-8.

As shown, instead of giving a sequence of numbers I simply invited subjects themselves to provide an interval, the reason being that it might somehow mislead subjects if any kind of sequence of numbers is provided. For instance, Channell presented a sequence of numbers 1 2 3 4 5 6 7 8 9 10 for *five or six articles*, indicating that she presupposed the permissible latitude allowed by *five or six articles* would be somewhere within the range of 1-10. Therefore, subjects would hesitate to go beyond the given range, possibly bringing about biased responses. Furthermore, if a subject is asked to mark *about 20,000 people* with a given sequence of, say 15,000 16,000 17,000 18,000 19,000 20,000 21,000 22,000 23,000 24,000 25,000, and then wants to designate an interval of, say 18,500-21,500, the numerals are not shown in the given sequence. The subject may feel uneasy about changing the given sequence, suspecting the new sequence he gives to be inconsistent with that provided. Having considered these potential problems, no sequence at all was provided in my Chinese questionnaire.

numeral classifier. It is a word used with a noun which shows the sub-class to which the noun belongs. Measure nouns play a very important role in Chinese; there are over 50 of them. Chinese has an extensive system of measure nouns. For instance, *wu shi (tou) niu* 'fifty head of cattle' is an obvious enough measure noun to the speaker of English. Less obvious are measure nouns like (*ben*), a volume, as in *Ta you yi (ben) shu* 'She has one (volume of a) book', or the all-purpose measure noun (*ge*) as in *Ta you wu (ge) pingguo* 'She has five (of them) apples'. For this (*ge*) there is no English equivalent. In Chinese, it is always preferable to put (*ge*) after numerals than to use no measure noun at all. Therefore, in my Chinese test I put (*ge*) in FQs like *liang, san (ge)* '2 or 3', *san, wu (ge)* '3 or 5' and *shi'er, san (ge)* '12 or 13'. In addition, for Chinese *n* or *m* approximations, a Chinese comma is often needed. It should note that the shape of a English comma used here is slightly different from a Chinese comma. For the convenience of word-processing, I use the English comma here.

3.2 A semantic analysis

In our data discussed above, each respondent gave a lower bound and an upper bound for an FQ that estimated the range of acceptability. To give membership functions, histograms were computed as shown in Fig. 3.1 above. They depict the semantic tendency of an FQ which will be investigated in this section.

The following semantic analysis is primarily based on the empirical data, but to discuss FQs in a wider range our analysis will not be limited to the range that our data covers.

3.2.1 FQs in Type I

FQs of this type, such as *few*, *many* and *most*, are generally regarded as identifying proportions. For instance, *few* signals a low approximate percentage, say fewer than 50%; on the other hand, *most* indicates a high approximate percentage, say more than 50%.

Then, FQs of Type I can be divided into two sub-types: FQs denoting a larger proportion and FQs denoting a smaller proportion⁴. Some Type I FQs may be arranged in a sequence, e.g. *few—many—almost all*. *Few* in the left most denotes a smaller proportion and *almost all* in the right most denotes a larger proportion. The proportions denoted by the FQs in the sequence increase gradually from the left end to the right end.

⁴Here, the concepts of *small* or *large* are of course defined in a relative way. For instance, it could be said that a *few* denotes a smaller proportion when compared to *many*, whereas a *few* represents a larger proportion when compared to *few*.

Less variable

In my Chinese test the modal interval estimated by subjects, for *ji (ge)* 'several' was 3-5; for *yixie ren* 'quite a few people' was 20-30; for *xuduo ren* 'many people' it was 70-80. It appears that *several* represents a smaller interval than *quite a few* and *many*.

The variability of degree of fuzziness can be represented or measured by *standard deviation*. Standard deviation is the most frequently used measure of variability or dispersion of a distribution, i.e. the degree to which scores vary from the mean. It is *standard* in the sense that it looks at the average variability of all the scores around the mean, and all the scores are taken into account. The figures for standard deviation (see Appendix 2, Table 2.2, Group 1) consistently increase from *ji (ge)* 'several' to *xuduo ren* 'many people': 2.13 for *ji (ge)* 'several', 12.88 for *yixie ren* 'quite a few people' and 19.75 for *xuduo ren* 'many people'. *Several* is less variable than the other two.

This claim is also supported by other empirical data. Let us examine Fig 3.2, reproduced from Moxey and Sanford (1993b).

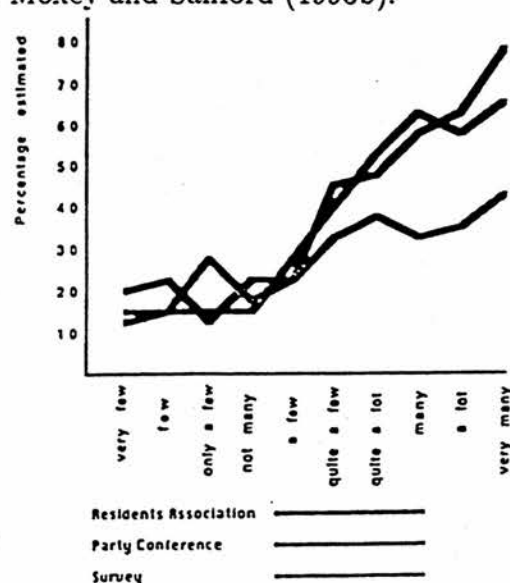


FIGURE 3.2: FQs in Moxey and Sanford

In Fig. 3.2, from *a few* to the left, small-denoting FQs change little with context. On the other hand, from *a few* to the right, FQs are more variable with context.

One of the explanations for this phenomenon, as stated in Moxey and Sanford (1993b), is that FQs which denote larger proportions are unmarked in the sense that an FQ of this sort is the name for a continuum as well as for a proportion on the continuum. For instance, in English *many* can be the name of a continuum (e.g. *How many people are there?*), as well as naming a proportion on the continuum (e.g. *There are many people*). It appears that *many* in the interrogative sentence does not denote a proportion at all; whereas it does denote a proportion in the declarative sentence. On the other hand, its antonym *few* is *marked*, because the question *How few people are there* does not name a continuum, but limits a proportion that is only part of the continuum. In other words, *How few people are there* indicates that the presupposition for the question is that there are few people. Because FQs denoting smaller proportions are marked (e.g. *few*), they are less changeable.

Another explanation can be given in terms of logical entailment in the material sense. FQs denoting larger proportions logically entail FQs denoting smaller proportions. For instance, *many* entails *a few*. However, the reverse does not hold, i.e. *a few* does not entail *many*. As Moxey and Sanford (1991b) state: "In this sense smaller denoting quantifiers are dependent on the interpretations given to larger denoting quantifiers, and this gives them less room for manoeuvre." For instance, *very few* is near to zero, and *few* is not far from zero either. Similarly, we do not normally say *about 1 person* and *about 1 cent*, since there is nothing much to approximate from zero to 1. So, FQs denoting smaller proportions can only vary within a limited scope, because they are marked and logically dependent on FQs denoting larger proportions.

What can be added is that some Type I FQs often denote an interval rather than a proportion. For example, in Chinese *ji (ge)* 'several' may be viewed as denoting an interval say 2-8, i.e. the variance of *ji (ge)* 'several' may be limited within the interval. However, it is not the case for *xuduo* 'many', which could be 5 in one context, 2,000 in another. Thus, FQs that may denote an interval appear less variable. If this is so then FQs in Types II

and III are less fuzzy than most FQs in Type I because they all denote an interval, such as *about 20 students*.

Furthermore, it appears that FQs on the both ends are less variable than FQs in the middle of a sequence. Supposing there is a sequence of Type I FQs: (*none*), *few*, *many*, *almost all*, (*all*). Then, items on two ends of the sequence appear less variable than items towards the central. *Few* normally would not mean more than 50%, and *almost all* would not mean fewer than 50%. However, *many* may mean either more than 50% or fewer than 50%. Basically, the proportion denoted by *many* is supposed to be larger than a norm with respect to a given context, but it is not necessarily more than 50%. For instance, a sentence *Many Chinese are restaurant owners* might describe a situation in which the number of owners who are Chinese is proportionately greater than the number of owners of non-Chinese nationality. As an illustration, suppose in Edinburgh there are 2,000 restaurants and the percentage of Chinese owners is, say, 10%. Then, the proportion expressed by the sentence *Many Chinese are restaurant owners* is significant, despite the fact that the number of Chinese owners may be nowhere near half of the total number of restaurant owners in Edinburgh.

The claim that some FQs in Type II are less fuzzy is also applicable to other types of FQs. For example, in my Chinese data the standard deviation of *liang, san (ge)* '2 or 3' is 0.93, but 1.33 for *san, wu (ge)* '3 or 5' (see Appendix 2, Figs. 2.18 and 2.19). There may be two reasons for this. One is that *liang, san (ge)* '2 or 3' has its prototypical numbers adjacent, so its histogram was more pointed. The other is the floor effect, i.e. *liang, san (ge)* '2 or 3' is nearer to zero (see Section 4.1.1 for more discussion of the floor effect), so it is less variable and fuzzy.

Different shapes

It appears plausible to say that there are two kinds of curves that FQs in Type I may generate: single-peaked and monotonic, although we do not have empirical data on Type I FQs with a monotonic curve. A general trend is that FQs on the two ends tend to be monotonic, and FQs in the middle tend

to be single-peaked. This can be represented in Fig. 3.3.

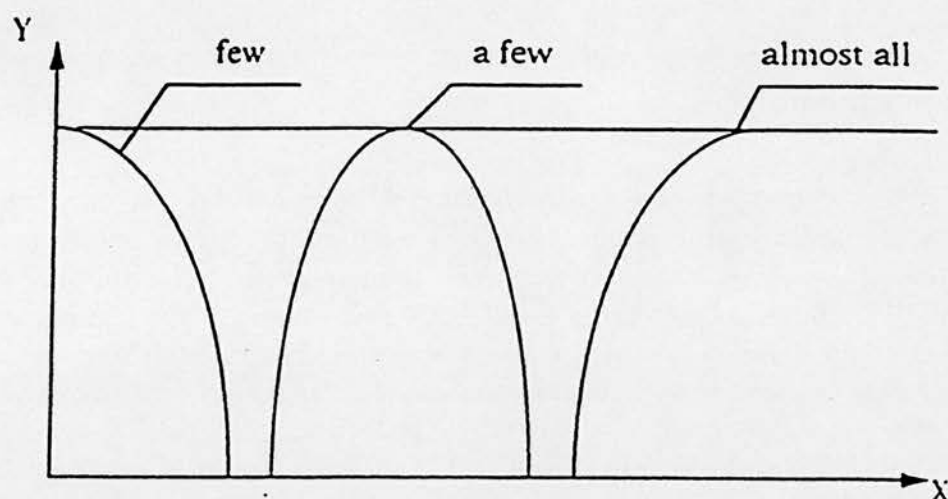


FIGURE 3.3: Different shapes

As we can see from Fig. 3.3, *few* and *almost all* have a monotonic curve, while *a few* has a single-peaked one. It would be counter-intuitive, for example, to give a monotonic curve to *a few* in both English and Chinese. My data also showed a single-peaked curve for *ji (ge)* 'several' (see Appendix 2, Fig. 2.1).

The trend for Type I FQs here parallels with the findings in Wallsten *et al* (1986a), where probability and frequency terms were studied. The data showed that *doubtful* and *good chance* on the near ends were mostly given a monotonic curve, but *toss-up* in the middle was unanimously given a single-peaked one. Refer to Section 2.2 for more discussion.

3.2.2 FQs in Type II

Approximators in Type II FQs approximate the prototypical number to make an approximation. FQs in Type II are exemplified by *nearly 200*, *about 200*, *200 or so* and *200-odd*. This type of FQ differs from the FQs in Type I in that the former has a salient number like *200* in *about 200*, whereas FQs, such

as *many* and *several*, in Type I do not have this kind of numeral involved. However, FQs in both types have one thing in common: imprecise meaning.

N zuoyou 'about *n*'

It is expected that *about n*, *approximately n* assign an interval most likely with *n* as a prototypical number, by which a set of appropriate numbers are selected. Take *about 200* as an example; its interval would be approximate to 200. In Chinese, there are a few FQs which have similar meaning as *n zuoyou* 'about *n*', such as *dayue n* and *n shangxia*. In English, there are also similar FQs as *about n*, such as *around n* and *approximately n*. It is expected that *about n*, *approximately n* have single-peaked curves. Take *about 2,000 people* as an example; as illustrated in Fig. 3.4, the curve is single-peaked (see Appendix 2, Fig. 2.16).

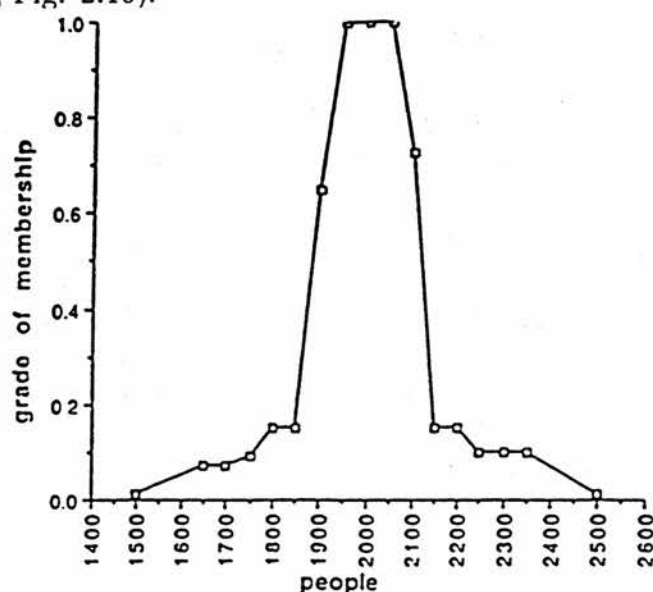


FIGURE 3.4: *2,000 ren zuoyou* 'about 2,000 people'

According to my Chinese data shown in Fig 3.4, any number which falls outside the interval of 1,500–2,500 would not be considered a member of *2,000 ren zuoyou* 'about 2,000 people'. In fact, all *about n* approximations in our data showed a similar shape of distribution, as shown in Fig 3.5 and

Fig 3.6, reproduced from Chinese and English data (see Appendix 1, Figs. 1.1, 1.2 & 1.3 and Appendix 2, Figs. 2.15, 2.16 & 2.17).

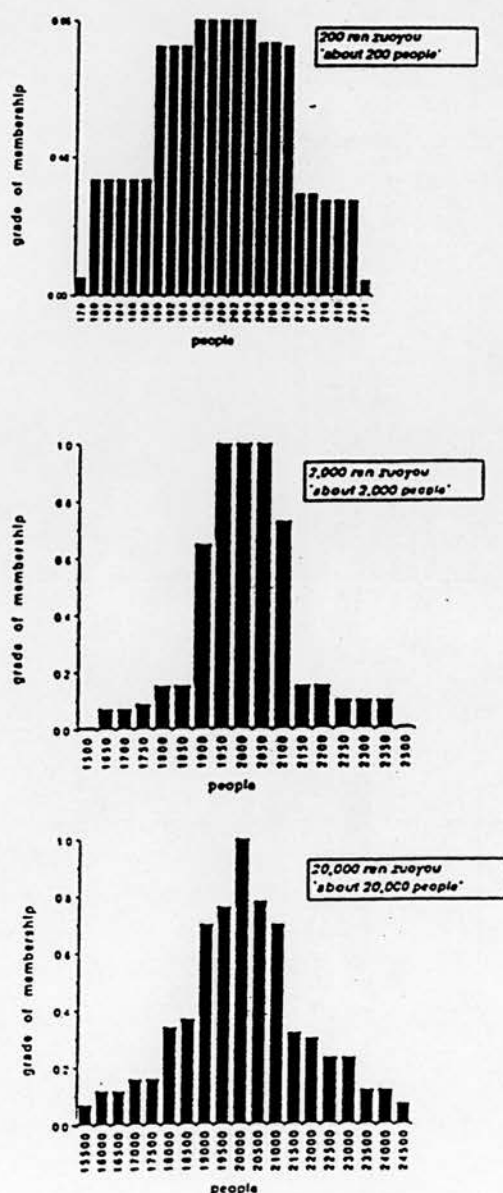
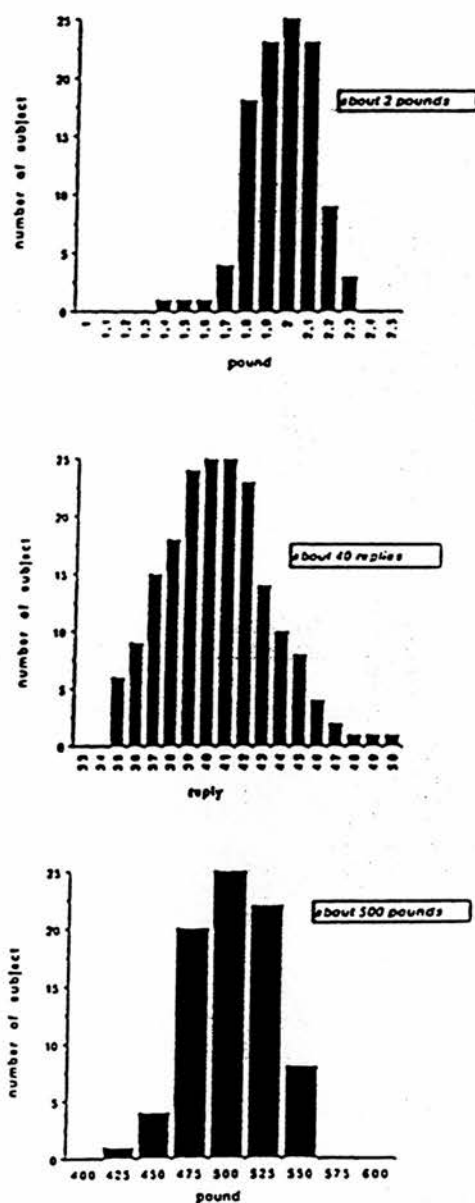


FIGURE 3.5: *About n* in Chinese data



It appears evident that these functions of *about n* displayed in the two sets of figures are similar for *about* in English and *zuoyou* 'about' in Chinese, i.e. they are translational equivalents.

Finally, with *-ish* approximation, it is expected that *about twenty people* would have a similar curve as *twentyish people* would have. In other words, *n-ish* approximation is similar to *about n* in generating a single-peaked curve around *n*.

N (,/zhi) m 'n or m'

N or m approximation differs from those considered above in that it has two numbers—*n* and *m*. In Chinese, an *n or m* approximation can be made either by putting a comma or *zhi* 'reach' between *n* and *m* (e.g. *liang, san (ge)* '2 or 3' versus *shi zhi ershi* '10 or 20').

The detailed data for *n or m* illustrated in Table 3.2 and Table 3.3 are reproduced from Appendix 1, Table 1.2 and Appendix 2, Table 2.2, Group 6.

TABLE 3.2: *N or m in English data*

No	Item title	Mode
1	3 or 4 metres	3.50
2	4 or 5 regions	4/5
3	6 or 7 hours	7
4	8 or 9 feet	9
5	8 or 10 students	8
6	15 or 20 people	16/17/18
7	70 or 80 people	75
8	3 or 4 hundred pounds	350/375
9	2 or 3 thousand people	2250
10	2 or 3 thousand gallons	2500

TABLE 3.3: *N or m in Chinese data*

No	Item title	Mode
11	liang, san (ge) '2 or 3'	3
12	san, wu (ge) '3 or 5'	4
13	shi'er, san (ge) '12 or 13'	13

Of 13 items from Tables 3.2 and 3.3, only five of them (items 1, 7, 8, 10, 12) were given a mode number which was the mean of n and m . Interestingly, 11 out of 13 items were given a mode which was the mean or above the mean of n and m . Only four out of 13 items had a mode that was below the mean. Several items have more than one mode number. Five out of 13 items take the m in n or m as their mode numbers (items 2, 3, 4, 11 and 13). For

instance, item 3, *6 or 7 hours*, has 7 as its mode number⁵.

N or *m* has some features that need to be explored in greater depth. Firstly, the two numbers (*n* and *m*) given are relevant to the interval designated. This is displayed in Table 3.4 and Table 3.5 (see Appendix 1, Table 1.3 and Appendix 2, Table 2.3):

TABLE 3.4: *N* or *m* in English data

No	Items	%subjects specifying interval cont. both <i>n</i> and <i>m</i>	%subjects specifying int. bounded by <i>n</i> and <i>m</i>	%subjects specifying nos below <i>n</i>	%subjects specifying nos above <i>m</i>
1	3 or 4 metres	57.7	76.9	30.8	73.1
2	4 or 5 regions	100.0	57.6	50.0	46.1
3	6 or 7 hours	69.4	88.5	42.3	80.8
4	8 or 9 feet	88.5	73.1	46.2	65.4
5	8 or 10 students	96.2	77.0	61.5	57.7
6	15 or 20 people	92.3	77.0	65.0	69.2
7	70 or 80 people	80.8	88.5	57.7	65.4
8	3 or 4 hundred pounds	69.2	61.5	30.8	53.8
9	2 or 3 thousand people	69.2	76.9	41.1	65.4
10	2 or 3 thousand gallons	73.1	76.9	57.7	57.7
	Mean over 10 items:	79.6	75.4	48.3	63.5

⁵One may argue that this could be just a chance effect, i.e. given two numbers, the mode must be one or the other. In fact, this is not necessarily the case. For instance, subjects did give 3.5 as the mode number of *3 or 4 metres* in Channell's test shown in Table 3.2. Perhaps, the effect of the denotation of the head noun (i.e. whether it is count or mass) on the interpretation of the FQ also plays a role here. That is, because *metre* is a measure plural, subjects were able to give 3.5 as the mode number. For more discussion of the effect of a head noun, see Section 4.1.2.

TABLE 3.5: *N or m* in Chinese data

No	Items	%subjects specifying interval cont. both <i>n</i> and <i>m</i>	%subjects specifying int. bounded by <i>n</i> and <i>m</i>	%subjects specifying nos below <i>n</i>	%subjects specifying nos above <i>m</i>
11	liang, san (ge) '2 or 3'	100	50.36	0.01	49.63
12	san, wu (ge) '3 or 5'	99.99	42.96	0.01	55.56
13	shi'er, san (ge) '12 or 13'	99.99	30.37	34.81	63.7
	Mean over 3 items:	99.99	41.23	11.61	56.3

Note: Column 3 indicates the percentage of subjects who specified the interval of an *n or m* approximation as the one containing both *n* and *m*. Column 4 indicates the percentage of subjects who specified the interval of an *n or m* approximation as the one being bounded precisely by *n* and *m*. Column 5 indicates the percentage of subjects who specified the interval of an *n or m* approximation below *n*, and the last column indicates those above *m*.

As we can see from Table 3.4 and Table 3.5, of 13 items tested the mean percentage of subjects marking both *n* and *m* as part of the interval was 79.6 for English, 99.99 for Chinese. It appears that the *n* and *m* are relevant to the designation of a permissible latitude for *n or m* approximation, although they are not necessarily the mode number.

Secondly, the interval for *n or m* does not necessarily take *n* and *m* as the two outermost numbers. Our data showed that the percentage of subjects marking the interval bounded precisely by the two numbers was 75.4 for English and only 41.23 for Chinese.

Channell (1983) claims that one of the constraints on *n or m* approximation

is that n must be smaller than m . It may be the case in English, but not in Chinese. For instance, in Chinese it is perfectly appropriate to say *san, liang (ge)* '3 or 2' in which n is bigger than m . However, Channell points out correctly that *or* can also be associated with non-approximate expressions. For instance:

Zhang has *two or three* apples. (3.2)

Sentence (3.2) has at least two readings: one means that the number of apples Zhang has involves a possible approximation, e.g. from 1 to 7 as observed in my Chinese data (see Fig. 3.7); the other reading is that Zhang has either exactly two or exactly three apples. In the second reading, *or* does not act as a fuzzifier.

In terms of a proposition containing an n or m approximation, *or* is neither used as *inclusive* (e.g. in the sentence *You must be working very hard or be ill, because you look pale*) nor *exclusive* (e.g. in the sentence *He told me that he did not know what to do: stay or leave*), because the interval of n or m is supposed to be an interval that could also be designated beyond both n and m . In other words, an n or m approximation represents not only one possibility (exclusive) or two possibilities (inclusive), but many alternatives (a set of appropriate numbers). This special feature distinguishes *or* in an n or m approximation from any other conventional *or*. In other words, although *or*, both in the case of an n or m approximation and in the case of conventional usage, represents alternatives, an n or m approximation should not be considered to be equivalent to other conventional *or* cases.

It is shown in our data that n or m tends to have a single-peaked curve, the same as *about* n (see Appendix 2, Figs. 2.18, 2.19 and 2.20). However, hypothetically speaking n or m could be bimodal, depending very much on the value of n and m in n or m approximations. Let us examine Fig. 3.7 (reproduced from Appendix 2, Fig. 2.18) and Fig. 3.8.

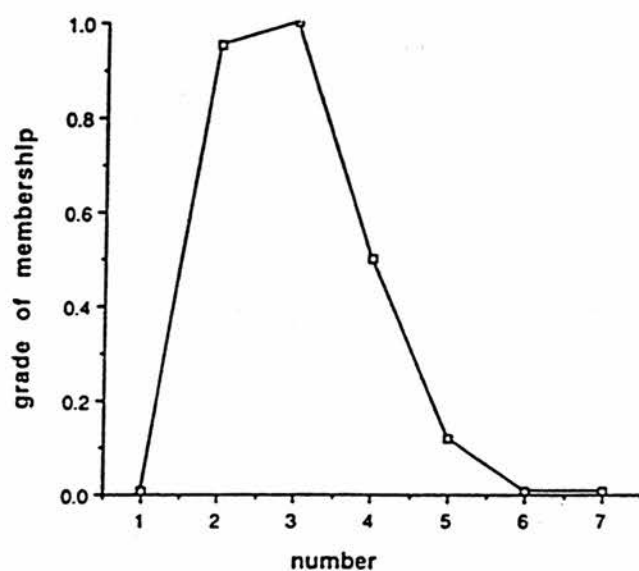


FIGURE 3.7: liang, san (ge) '2 or 3'

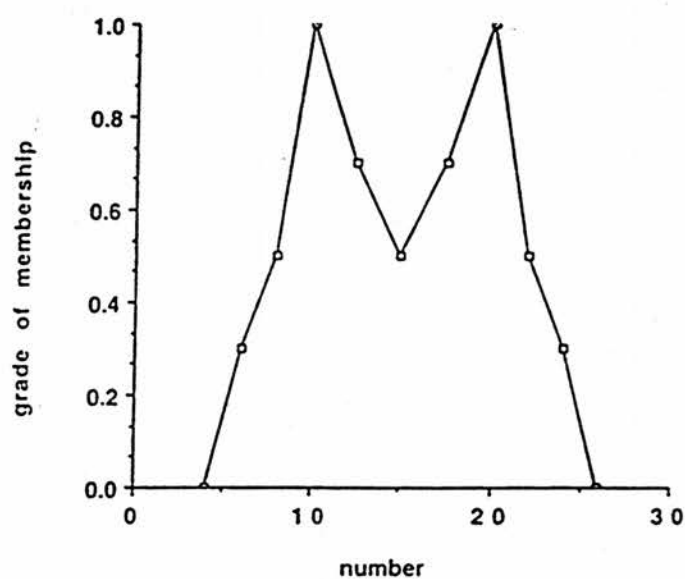


FIGURE 3.8: shi zhi ershi '10 or 20'

As my Chinese data show, if the two prototypical numbers in *n or m* are adjacent (or close to each other like *san, wu (gc)* '3 or 5'), then it most likely generates a single-peaked curve as shown in Fig. 3.7. On the other hand, it is suspected that *shi zhi ershi* '10 or 20' in Fig. 3.8, where 10 and 20 are not adjacent, may have a bimodal curve. This appears to be the case in English as well. So, it appears that the meaning of an *n or m* approximation may practically depend on the semantic properties of the head noun, as well as the difference between *n* and *m*.

N duo 'n-odd' and *n or so*

In Chinese, *200 duo* '200-odd' means *a little bit more than 200*. It differs from *n or so* in English in that it would most likely take *n* as a lower bound. *n or so* would not, i.e. *200 or so* may very often include some number below 200. Our data show that *200 duo ren* '200-odd people' has a distribution which expands upwards from 200, as shown in Fig. 3.9 (see Appendix 2, Fig. 2.12).

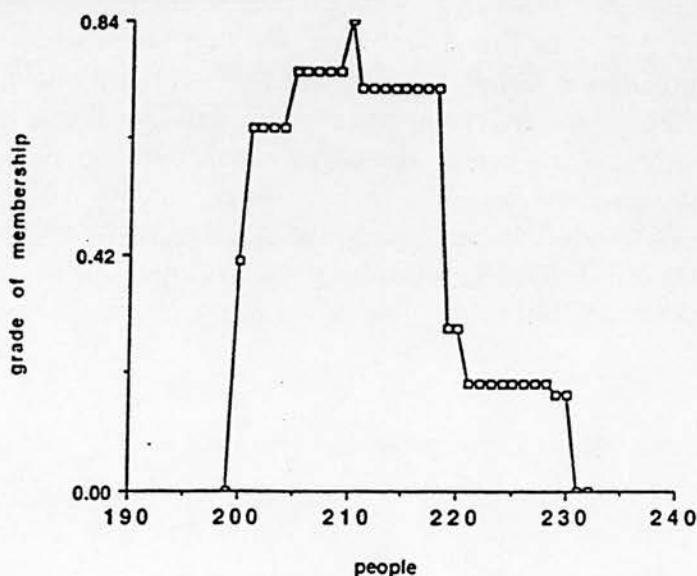


FIGURE 3.9: *200 duo ren* '200-odd people'

We can see from Fig. 3.9 that the distribution is expanded upwards. Let us

then examine Channell's data for *500 pounds or so* in Fig. 3.10 (see Appendix 1, Fig. 1.5):

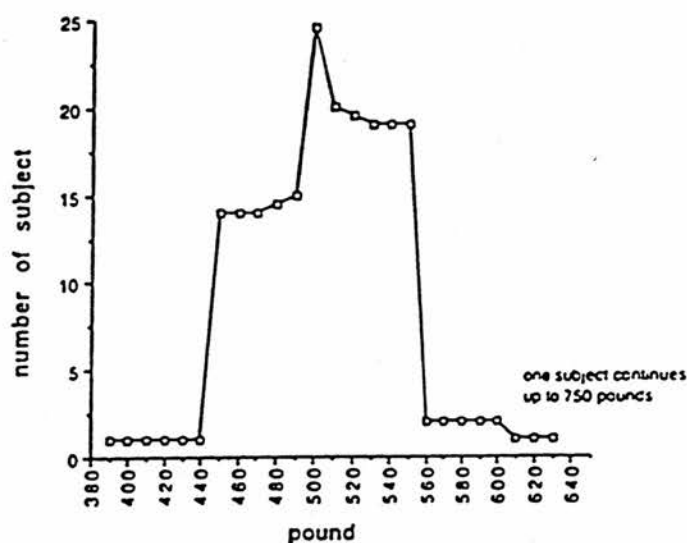


FIGURE 3.10: *500 pounds or so*

Comparing Fig. 3.10 with Fig. 3.9, we can see that the *n or so* distribution in Fig. 3.10 spreads out below the *n* much further than *n-odd* in Fig. 3.9, although both of them tend to expand upwards. As a result, the mode number for *n-odd* in my Chinese data was consistently given as a number greater than the numeral *n*, while in *n or so* of English only 3 out of 10 had been given a mode number which was greater than *n*. This is shown in the Table 3.6 and Table 3.7, reproduced from Appendix 1, Table 1.2 and Appendix 2, Table 2.2. Group 4.

TABLE 3.6: *N or so* in English

Item title	Mode
a bottle or so	1
2 spoonfuls or so	2.5
6 or so books	6
10 lbs or so	10
10 or so litres	10
200 or so people	200/210
500 pounds or so	500
2,000 pounds or so	2,200
3,000 or so students	3,000
30,000 or so visitors	30,000

TABLE 3.7: *N-odd* in Chinese

Item title	Mode
200 duo ren '200-odd people'	210
2,000 duo ren '2,000-odd people'	2,100
20,000 duo ren '20,000-odd people'	20,500

This is due to the fact that in Chinese, *200-odd* means *a bit more than 200*, i.e. the approximation is supposed to be above *n*. But, in English it would not be unusual if one judges 195 to be in the interval of *200 or so*.

With respect to similarities between *n or m* and *n or so*, Wachtel (1981) says that if one treats *so* as a pro-numeral, both types would in fact fall into a single category of *n or m* approximation, where *m* is either a specific numeral as in *n or m*, or a pro-numeral as in *n or so*. In addition, *so* being understood

as a proform is also consistent with its many other uses as a proform, such as in *I hoped that I could have gone to visit my parents during this summer vacation, but I did not do so.*

Jiangjin n 'nearly n'

Nearly n has not received as much attention as *about n* and *n or so*. In fact, the existence of this type of FQ acts as a kind of balance with regard to *n-odd* in that the distribution of the *nearly n* approximation tends to expand downwards as opposed to *n-odd*. Let us examine Table 3.8. reproduced from Appendix 2, Table 2.2. Group 3.

TABLE 3.8: *Nearly n* approximations

Item title	Mode
jiangjin 200 ren 'nearly 200 people'	195
jiangjin 2,000 ren 'nearly 2,000 people'	1,990
jiangjin 20,000 ren 'nearly 20,000 people'	19,500

It is shown in Table 3.8 that the three items were all given a mode which was not *n*, but a number less than it.

The similarity between *n-odd* and *nearly n* is that none of them have *n* as their mode number. However, only 28% of responses counted the number 200 as a member of the interval for *jiangjin 200 ren* 'nearly 200 people'; whereas 47.4% of responses counted 200 as a member of the interval for *200 duo ren* '200-odd people'.

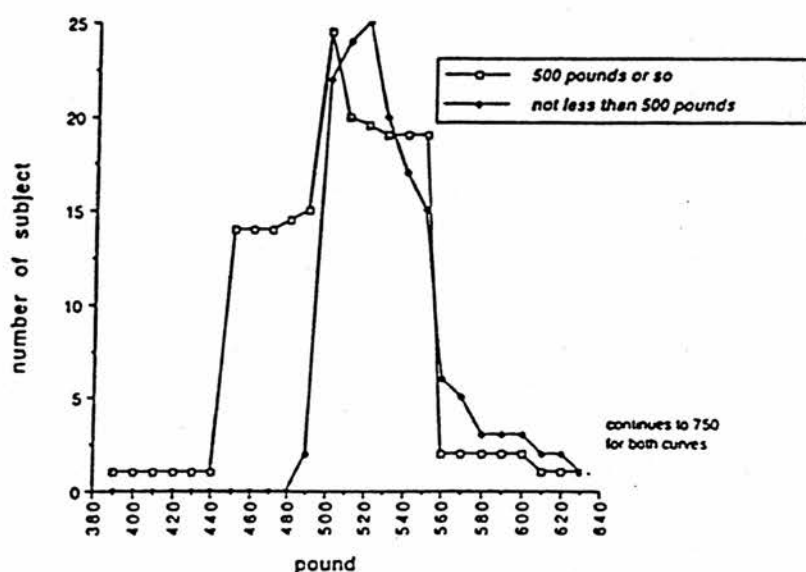
3.2.3 FQs in Type III (semi-FQs)

FQs in Type III are called semi-FQs in the sense that they have some special characteristics that ordinary FQs do not have (see Section 1.3.1 for further discussion of this point). In Chinese, they are exemplified by *n yishang* 'more than *n*' and *n yixia* 'fewer than *n*'.

Semantically speaking, *more than n* may have a curve with an open-ended upper limit, and *fewer than n* may have a curve with an open-ended lower limit. On the other hand, pragmatically speaking, what is commonplace for *more than n* or *fewer than n* is a single-peaked curve (see Chapter 4 for an explanation in terms of the relationship between semantics and pragmatics). For example, we would not interpret *more than 20 people* in *More than 20 people went to the party* as a positive infinity in ordinary language. It would be something like from 20 to 30. If it is over 30 people, according to Grice's (1975) cooperative principles (i.e. "One should not say more than one needs to say" in particular) we should say *more than 30* rather than *more than 20*. Thus, semi-FQs indeed have fuzzy meaning boundaries, as do other types of FQs, but they can be relatively less fuzzy in a semantic sense.

In Wachtel's (1981) terms semi-FQs are called *partial specifiers*. It is predicted that they shall specify upper (e.g. *at most n*) or lower (e.g. *at least n*) limits. For instance, *not less than n* would designate intervals which do not go below *n*. However, in Channell's data, 27 and 24 percent of subjects respectively gave intervals below *n* for the two *not less than n* approximations tested (see Appendix 1, Table 1.2). Based on her data, Channell (1983: 97-101) claims that *not less than n* and the like are actually used to approximate in the same way as other FQs, such as *n or so*.

However, it appears that Channell's claim is not correct. First, *not less than n* may indeed have a monotonic curve in a semantic sense, but *n or so* may not. Even in Channell's English data, the test results for *n or so* and *not less than n* were not quite the same. The following in Fig. 3.11 is a comparison between *not less than n* and *n or so*, reproduced from Channell's data (see Appendix 1, Figs. 1.5 & 1.7).

FIGURE 3.11: A comparison

As shown in Fig. 3.11, both curves have an upper bound of 750. However, an obvious difference between them is that the distribution of *500 pounds or so* is below the number 500, much further than *not less than 500 pounds*. A majority of subjects felt that the lower bound of *not less than 500 pounds* should not go below 500. Thus, it appears that in English, *n or so* and *not less than n* are not equivalent.

Let us then examine *more than n* and *n-odd* in Chinese data. To an extent my Chinese data support Wachtel's idea that *more than n*, as well as *fewer than n*, is a partial specifier. None of my subjects gave intervals above *n* for *fewer than n*, and below *n* for *more than n* (see Appendix 2, Tables 2.4 & 2.5). *More than n* is illustrated here in Table 3.9, reproduced from Appendix 2, Table 2.4:

TABLE 3.9: Chinese data: semi-FQs

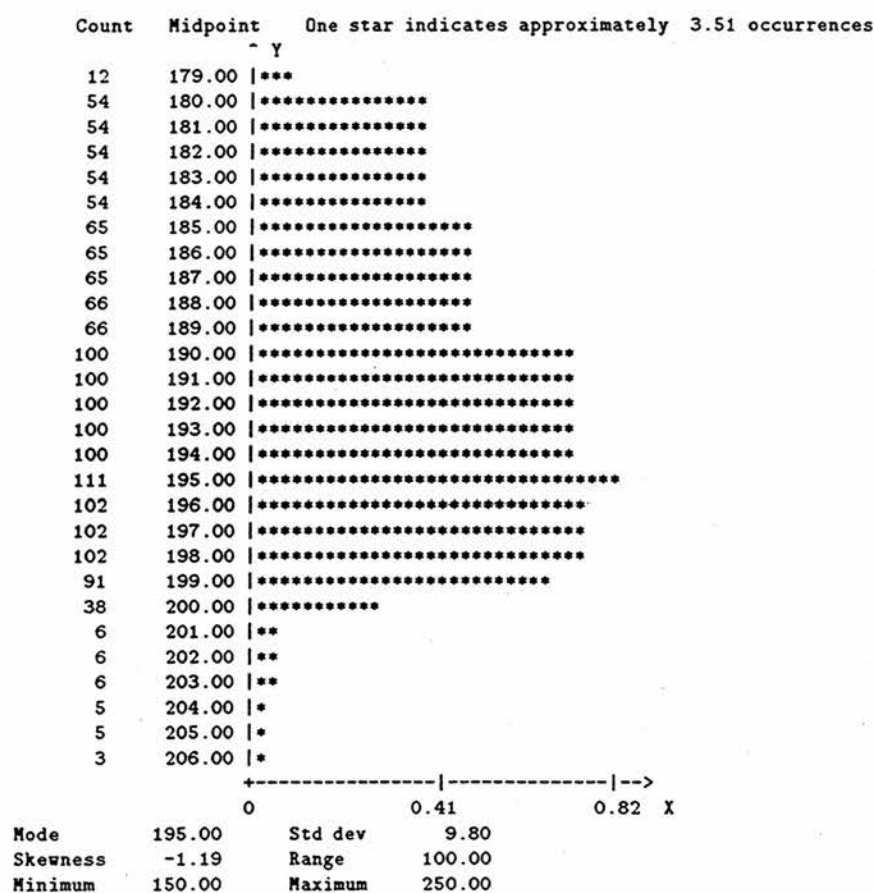
Item	% subjects specifying u-bound as positive infinite	% subjects specifying l-bound as n	% subjects specifying l-bound below n	% subjects specifying l-bound above n
200 yishang 'more than 200'	28.36	70.15	0	29.85
200 ren yishang 'more than 200 people'	18.7	58.06	0	41.94
Mean over 2 items:	23.53	64.11	0	35.9

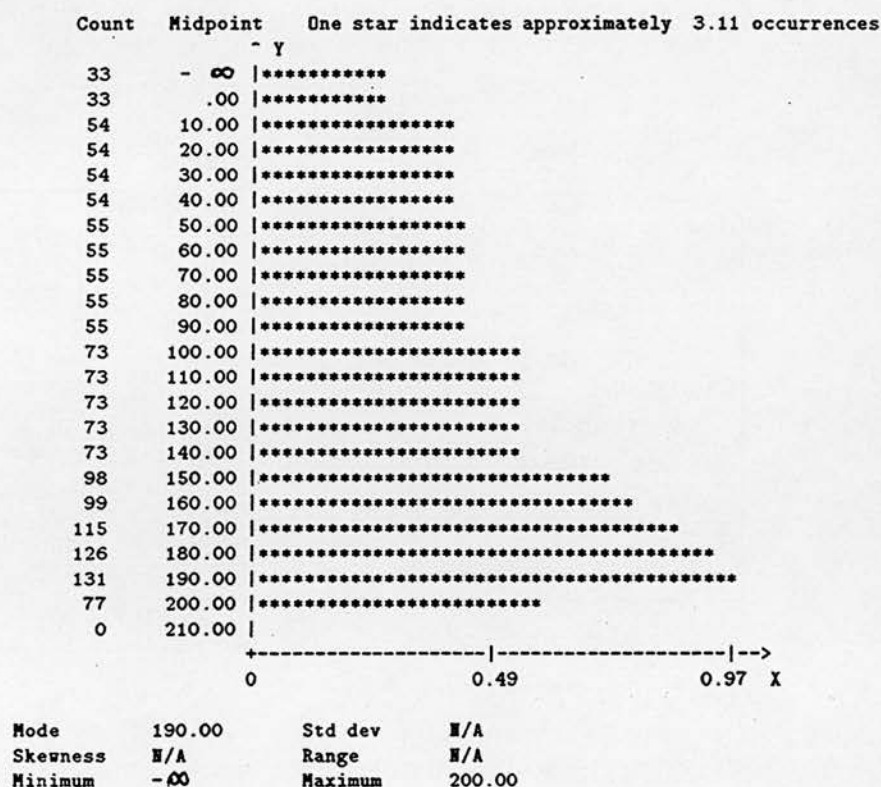
As shown in Table 3.9, 23.53% of subjects marked the upper bound of *more than n* as positive infinity⁶, whereas none of the subjects marked the upper bound of the three tested items of *n -odd* as positive infinity (see Appendix 2, Table 2.2, Group 4). This implies that in Chinese, *more than n* and *n -odd* are semantically differentiated.

Furthermore, as shown in Table 3.9, 64.11% of subjects marked n as a lower bound of *more than n* , and no one marked the lower bound below 200. On the other hand, there were a few subjects who did think that the lower bound of *n -odd* could be below n (see Appendix 2, Figs. 2.12, 2.13 & 2.14). This is compatible with the English data shown in Fig. 3.11 in that a small number of subjects thought that the lower bound of *not less than n* could go below n . In addition, as we can see in Table 3.9, more subjects (64.11%) thought the lower bound of *more than n* should be exactly n , rather than some numbers above it (35.9%).

Likewise, by comparing *nearly n* and *fewer than n* , a similar result is found. Let us examine Fig. 3.12 and Fig. 3.13, reproduced from the Chinese data (see Appendix 2, Figs. 2.9 & 2.22).

⁶I suspect that to be able to give positive infinity represents a high level of knowledge that my subjects had. Most of them were university students who were doing a science degree. It would be interesting to examine the performance of other groups of people.



FIGURE 3.13: 200 *yiria* ‘fewer than 200’

Comparing the two figures, it can be seen that the number 200 plays a precise boundary role in *fewer than n*, but not in *nearly n*. No subjects marked a number above 200 in *fewer than 200*. On the other hand, some subjects did mark above 200 for *nearly 200* shown in Fig. 3.12. So in Chinese data, the numerals in semi-FQs—*more than n* and *fewer than n*—may be taken as the outer limits of FQs. This is, however, not the case for other FQs, like *n-odd* and *nearly n*.

Interestingly, within the Chinese data there were contrasting test results between *200 yishang* ‘more than 200’ and *200 ren yishang* ‘more than 200 people’. For the former, the modal interval was $200-+\infty$; for the latter it

was 200–300. This also applies to *200 yixia* ‘fewer than 200’ and *200 ren yixia* ‘fewer than 200 people’. Consider Table 3.10 (see Appendix 2, Table 2.2, Group 7):

TABLE 3.10: *More than n and fewer than n*

Item	Modal interval
200 yishang ‘more than 200’	200- $+\infty$
200 yixia ‘fewer than 200’	$-\infty$ -200
200 ren yishang ‘more than 200 people’	200-300
200 ren yixia ‘fewer than 200 people’	100-200

Why has this happened? It seems that *ren* ‘people’ has some impact. The subjects tended to designate an interval for *more than n* (without *people*) or *fewer than n* (without *people*) in a more mathematical sense. This was represented by giving a modal interval 200- $+\infty$ to *200 yishang* ‘more than 200’ and $-\infty$ -200 to *200 yixia* ‘fewer than 200’. While considering FQs combined with *ren* ‘people’, the subjects then gave 200–300 as the modal interval for *200 ren yishang* ‘more than 200 people’ and 100–200 for *200 ren yixia* ‘fewer than 200 people’, as shown in Table 3.10. So, *ren* ‘people’ indeed had a significant effect on the results.

In Channell’s data (see Appendix 1, Figs. 1.6 & 1.7), subjects did not think that positive infinity could be given as the upper bound of *not less than n pounds*. This is because in her test *not less than 500 pounds* was embedded in the sentence *The repair bill certainly would not be less than 500 pounds*. With reference to *the repair bill*, it would sound odd to give positive ∞ for the upper bound of *not less than 500 pounds*. Also, the other tested FQ, *not less than 150 pounds*, was set in the sentence *She was wearing a dress costing not less than 150 pounds*, it is again quite understandable why subjects did

not mark an interval with positive infinity as its upper bound. It would be unusual if one says the dress she was wearing cost an infinite amount, i.e. pragmatically the situation described is not plausible. In other words, comparing test results of *more than n* in Chinese and *not less than n* in English, although both can mean a number between *n* to positive infinity, none of the subjects in the English test gave a positive infinity. It is suspected that pragmatic factors play a role here, because *not less than n* was tested in a sentence. On the contrary, *more than n* in Chinese was tested in isolation (see Section 4.1.1 for further discussion).

For more examples, let us look at the *fewer than n* approximation in Table 3.11 (see Appendix 2. Table 2.5).

TABLE 3.11: *Fewer than n* in Chinese data

Item	% subjects specifying l-bound as negative infinite	% subjects specifying u-bound as <i>n</i>	% subjects specifying u-bound below <i>n</i>	% subjects specifying u-bound above <i>n</i>	% subjects specifying l-bound as 0/1
200 yixia 'fewer than 200'	23.88	54	46	0	16.42
200 ren yixia 'fewer than 200 people'	0	53.63	46.37	0	24.81
Mean over 2 items:	23.88	53.8	46.2	0	20.62

On specifying the number 200 as the upper bound, there is no big difference between the two items listed in Table 3.11, 54.48% vs 53.63%. Also, there was no one who specified the upper bound above 200, as no one specified the lower bound below 200 for 200 yishang 'more than 200' and 200 ren yishang 'more than 200 people' shown in Table 3.9. However, the percentage (46.2%) specifying the upper bound of *fewer than n* as a number below *n* is higher than the percentage (35.9%, shown in Table 3.9) specifying the lower bound of *more than n* above *n*.

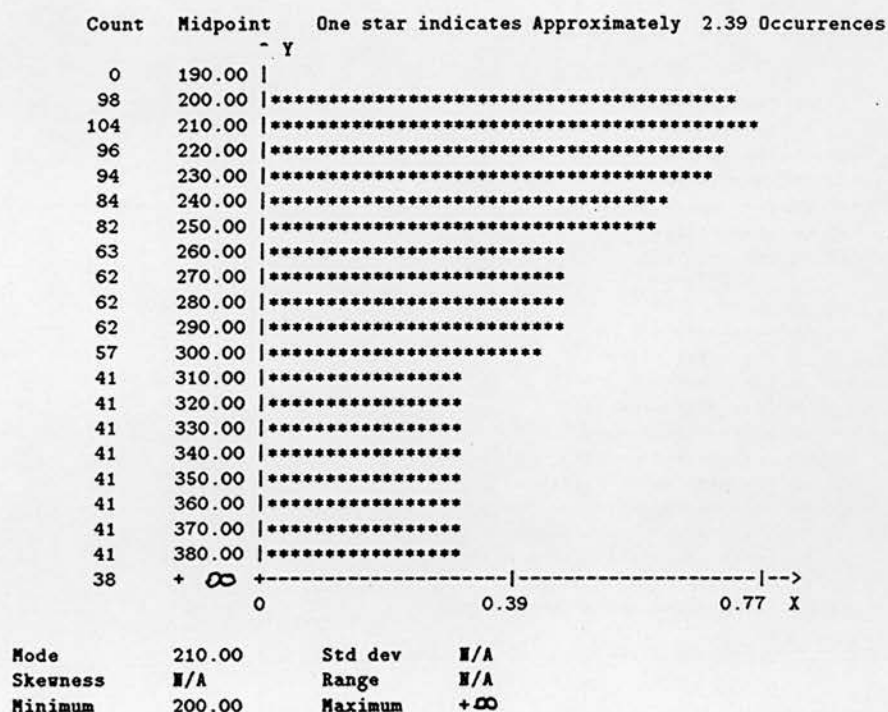
There were 23.88% of subjects who thought the lower bound of *200 yixia* 'fewer than 200' should be negative ∞ . However, no subject gave negative ∞ for *200 ren yixia* 'fewer than 200 people'. Apart from mathematical or non-mathematical considerations, it is common sense that there can never be negative people. It may be because of this that no single-subject gave negative infinity as a lower bound for *200 ren yixia* 'fewer than 200 people'. On the other hand, for *200 ren yishang* 'more than 200 people', although the percentage giving positive ∞ as its upper bound was lower than that of *200 yishang* 'more than 200', there were still 18.7% of subjects who thought that the upper bound of *200 ren yishang* 'more than 200 people' was positive ∞ . However, this is not at all the case for *200 ren yixia* 'fewer than 200 people'. Moreover, there were more subjects (24.81%) marked 0/1 as the lower bound of *200 ren yixia* 'fewer than 200 people', compared to 16.42% for *200 yixia* 'fewer than 200'.

Incidentally, no subject gave a fractional number to both *200 ren yishang* 'more than 200 people' and *200 ren yixia* 'fewer than 200 people'. The simple reason is that people exist in wholes, unless one's view of the world permits person-partials—say a person who lives in a half-alive and half-dead condition and who is considered to be 50% person and 50% non-person.

It can then be concluded that the significant interval difference between *more than 200* vs *more than 200 people* and *fewer than 200* vs *fewer than 200 people* in Chinese data supports the definition of a quantifier in GQT, i.e. determiner + noun. It is illustrated that in some cases the nature of the head noun does influence the interpretation of an FQ, i.e. the meanings of a determiner and a combination of the determiner + noun are not the same. That is one of the reasons why I also define a quantifier as a combination of determiner and noun in this thesis.

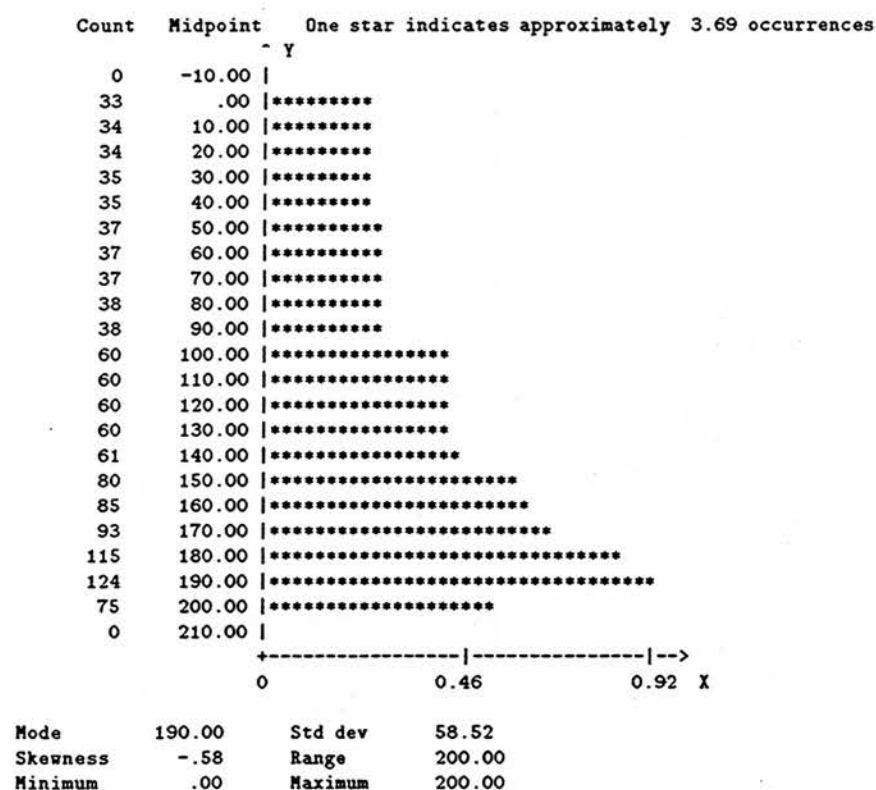
Finally, in terms of their distributions, semi-FQs can again be divided into two subtypes: upward skewing and downward skewing.

Upward skewing: Semi-FQs of this type are exemplified by *above n*, *more than n*, *not less than n*, etc.. The distribution of *200 yishang* 'more than 200' is represented graphically in Fig. 3.14 (see Appendix 2, Fig. 2.21):

FIGURE 3.14: *200 yishang* 'more than 200'

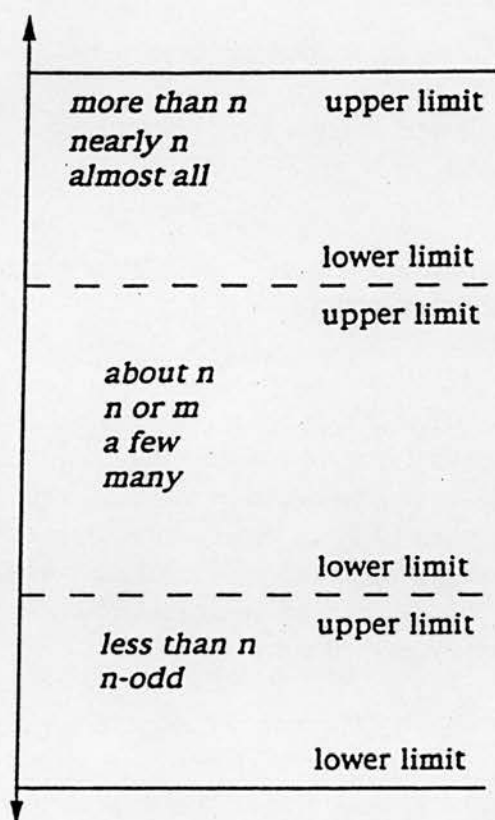
In Fig. 3.14 the distribution is indeed skewed upwards.

Downward skewing: The distributions of semi-FQs of this type, such as *fewer than n* and *under n*, tend to skew downwards in contrast to FQs with upward skewing distributions. The test results in Chinese for *fewer than n* are given in Fig. 3.13 above, and Fig. 3.15 below reproduced from Appendix 2, Fig. 2.24.

FIGURE 3.15: *200 ren yixia* 'fewer than 200 people'

It is clearly displayed in Fig. 3.13 and Fig. 3.15 that their distributions are skewed in the opposite direction to that of *more than n*.

Now, all three types of FQs have been discussed. What follows is a figure, in which FQs are rearranged into different categories according to how their boundaries vary.

FIGURE 3.16: Three types of variance

As shown in Fig. 3.16, FQs on the top have an upper limit. For example, *more than n* has a positive infinity, an open-ended upper limit. *Nearly n* tends to take *n*, or a number similar to it, as its upper limit: any number beyond *n* would not normally be considered as *nearly n* (see Appendix 2, Figs. 2.9 & 2.21). FQs in this category differ in that some may have a lower limit (e.g. *n* in *more than n*, see Appendix 2, Figs. 2.21 & 2.23), but some may not (e.g. *almost all*). FQs in the middle are fuzzier in meaning, because they neither have an obvious upper limit nor a lower one. They may vary radically in both upper and lower bounds. Most FQs in the middle category are Type I or II FQs. For example, *about n* has no obvious limit. Finally, FQs on the bottom may have a lower limit. For example, *fewer than n* may have negative infinity as a lower limit (see Appendix 2, Fig. 2.22), and *n-odd* tends to have *n* as its lower limit, or a number similar to it (see Appendix 2, Figs. 2.12, 2.13 & 2.14). Again, FQs in this category differ in that some may

have an upper limit (e.g. n in *fewer than n*), some may not (e.g. *n -odd*).

Having explored FQs individually, we will next discuss some general tendencies concerning them.

3.3 The numeral and the norm

First, the numeral. Channell (1983) states that in English approximations are likely to occur in *round* numbers, such as multiples of five and ten, hundreds or thousands. That is to say, round numbers are favoured as the numerals in number approximations. It is also the case in Chinese, especially with bigger numbers. For instance, we may say *about 7 people*, but very rarely say *about 200,007 people*. Moreover, Rosch (1975b) claims that multiples of ten are most frequently used for making n or m approximations.

In Chinese data, for *2,000 ren zuoyou* 'about 2,000 people', the numeral is *2,000*, while it is also the mode number. Naturally, its distribution is centred around 2,000, and its mean is pretty similar as well, 1.934. This applies to 13 out of 13 items of *about n* approximations tested in both the English and the Chinese tests shown in Tables 3.12 and Table 3.13 below, reproduced from Appendix 2, Table 2.2, Groups 4, 5 and 6 and Appendix 1, Table 1.2.

TABLE 3.12: The number approximation test in Zhang

No	Item title	Mean	Mode
1	200 ren zuoyou 'about 200 people'	200.48	200
2	2,000 ren zuoyou 'about 2,000 people'	1,934.11	2,000
3	20,000 ren zuoyou 'about 20,000 people'	20,257.32	20,000
4	200 duo ren '200-odd people'	214.17	210
5	2,000 duo ren '2,000-odd people'	2,096.02	2,100
6	20,000 duo ren '20,000-odd people'	22,482.39	20,500
7	jiangjin 200 ren 'nearly 200 people'	188.88	195
8	jiangjin 2,000 ren 'nearly 2,000 people'	1,869.37	1,990
9	jiangjin 20,000 ren 'nearly 20,000 people'	18,766.68	19,500

TABLE 3.13: The number approximation test in Channell

No	Item title	Mean	Mode
1	about 2.00 pounds	1.97	2.00
2	about 4lbs	4.08	4.00
3	about 6pm	6.00	6.00
4	about 10 pages	9.96	10.00
5	about 15 people	15.05	14/15
6	around the 20% mark	20.60	20.00
7	about 40 replies	40.13	40/41
8	about 500 pounds	508.00	500.00
9	about 14,000 pounds	14,033.65	14,000
10	around 10 million	9,942,307.00	10 million

As displayed in Table 3.12 and Table 3.13, it is appropriate to say that a trend for *about n* approximations and the like is that *n* in *about n* is most likely a norm⁷. This is not the case, however, for the other two types of approximations shown in Table 3.12. For *n-odd*, the mean and mode number is normally bigger than *n*. To take *2,000 duo ren* '2,000-odd people' as an example, its mean is 2,096, and the mode is 2,100, bigger than 2,000. Three out of three items in Chinese data were given a bigger mean and mode number than the corresponding *n*. Also, for three items of *nearly n* in the Chinese data, all three of them were given a mean or mode number which was smaller than the *n*.

Furthermore, we consider the issue that *n* is or is not included in an interval. In the case of *about n*, the *n* is included in the interval of FQs. Consider *at least 200*: it is clear that 200 should be in the interval. Another example is *more than 200*: the average ratio of excluding and including 200 is 35.9 vs 64.1 (see Appendix 2, Table 2.4), i.e. a majority of the subjects thought that the *n* in *more than n* is inclusive.

However, this does not necessarily apply to other FQs, such as *nearly n* and *fewer than n*. For *nearly n*, the percentage of subjects who did not mark

⁷In statistics, a norm is usually measured by mean, mode, or median. In this section we compare two of the three—mean and mode. The reason is that the information for median is not available in Channell's English test.

200 into the interval of *jiangjin 200 ren* 'nearly 200 people' was 72% (see Appendix 2, Fig. 2.9). For *fewer than n*, the test results were marginal: the ratio of excluding and including *n* was 46.2% vs 53.8% (see Appendix 2, Table 2.5).

In addition, *n* in semi-FQs plays a more crucial role than in other types of FQs. Our data showed that subjects did not give any number below *n* for *more than n* and above it for *fewer than n* (see Table 3.9 and Table 3.11 above). However, for some other FQs, subjects often marked intervals beyond *n*.

To conclude, the fact that the numerals in number approximations are not always considered as a norm reveals that it should not be assumed that the norm of distribution of an FQ is just the numeral. That is, there is not necessarily an equivalence between the norm and the numeral; the two could play different roles in the designation of an FQ's value. As a result the numeral could be excluded from an interval, as seen in the data. This finding will be represented in my formal treatment of FQs where a norm or prototype is more important than a numeral (see Chapter 6 for further discussion).

3.4 Function of the size of numerals

The designation of an interval is also affected by the size of the numerals. Channell's test results indicate that the length of an interval varies as a function of the size of the numerals. For instance, *500 or so pounds* (mean length,⁸ 77.30) was judged as designating a smaller mean length than *2,000 or so pounds* (mean length, 550). Let us look at Channell's test results for *about n*, *n or so* and *n or m* in Table 3.14 (see Appendix 1, Table 1.2):

⁸ Mean length means the average interval length.

TABLE 3.14: Function of the size of a prototypical number

No	Item title	Mean length
1	about 4lbs	.61
2	about 40 replies	5.96
3	about 500 pounds	51.92
4	about 14,000 pounds	1,067.3
5	around 10 million	1.5 million
6	a bottle or so	.568
7	200 or so people	46.15
8	3,000 or so students	538.46
9	30,000 or so visitors	4,846
10	3 or 4 metres	1.1
11	70 or 80 people	17.15
12	3 or 4 hundred pounds	90.46
13	2 or 3 thousand people	1,260.6

As shown in Table 3.14 the size of the numerals indeed affects the *mean length* of FQs. It shows that as the size of the numerals increases the scores in column three increase as well. To take the *about n* approximation as an example, there is about a ten times difference in terms of the size of the numerals between *about 4 lbs* and *about 40 replies*, and the difference between their scores in column three is also roughly 10 times. The results of *n or so* approximations are also consistent with *about n* approximations. The comparison between *3,000 or so students* and *30,000 or so visitors* shows that approximately 10 times is again the difference between the size of their numerals and mean lengths. Finally, the test results of *n or m* approximations also support the view that there is a direct ratio between the size and the mean length. For instance, the difference between numerals of *3 or 4 metres* and *3 or 4 hundred pounds* is about 100 times, and the difference of their mean lengths is about 100 times as well.

This is also applicable to the Chinese data (see Appendix 2, Table 2.2, Groups 3, 4 & 5). In the following Table 3.15 three groups of FQs are listed, including *nearly n*, *n-odd* and *about n*. In each group FQs have the same approximators but different numerals: 200, 2,000, 20,000.

TABLE 3.15: Chinese data

Item title	Range
jiangjin 200 ren 'nearly 200 people'	100
jiangjin 2,000 ren 'nearly 2,000 people'	700
jiangjin 20,000 ren 'nearly 20,000 people'	20,000
200 duo ren '200-odd people'	125
2,000 duo ren '2,000-odd people'	1,350
20,000 duo ren '20,000-odd people'	15,000
200 ren zuoyou 'about 200 people'	110
2,000 ren zuoyou 'about 2,000 people'	1,000
20,000 ren zuoyou 'about 20,000 people'	20,500

The results in Table 3.15 show that the range increases as the size of the numeral increases. For instance, *nearly 20,000 people*, *20,000-odd people* and *about 20,000 people* were judged as designating a much larger range than *nearly 200 people*, *200-odd people* and *about 200 people*. Their ranges were given as 20,000 vs 100, 15,000 vs 125 and 20,500 vs 110. In other words, as the size of the numeral reduces the length of the corresponding range becomes smaller, e.g. *nearly 20,000 people* (range 20,000), *nearly 2,000 people* (range 700) and *nearly 200 people* (range 100).

Finally, it is shown that whether or not a numeral is a round number makes a substantial difference to the length of interval. Channell claims that in *about* n approximations, the interval designated varies significantly depending on whether an FQ has a round number n . Moxey and Sanford (1993b: 110) also report that round numbers admit to larger approximative boundaries than do other numbers. It appears that an approximation with a round number (e.g. *about 20*) licenses a longer interval than do other numbers (e.g. *about 21*). The reason for this may be that if the hearer tends to think the speaker of *about 21* is more certain about the relevant interval than the speaker of *about 20*, then it would be appropriate to think of a narrower interval for *about 21* (see Section 4.1 for more discussion on this).

3.5 Function of approximators

Our data showed that the length of interval varies according to the different added approximators. Let us consider Table 3.16, reproduced from Appendix 2, Table 2.2, Groups 3, 4 and 5.

TABLE 3.16: Function of approximators

Item title	Range
jiangjin 200 ren 'nearly 200 people'	100
200 duo ren '200-odd people'	125
200 ren zuoyou 'about 200 people'	110

In Table 3.16, three FQs have the same number (200), but different ranges (i.e. the length between maximum and minimum numbers given by subjects): 100, 125 and 110, respectively. It appears that the ranges are not the same due to their different approximators *nearly*, *odd* and *about*. In other words, it is shown here that approximators indeed change the ranges.

Also, the different approximators can change some other scores with regard to the designation of FQs. Consider Channell's data in Table 3.17 (see Appendix 1, Table 1.2).

TABLE 3.17: More function of approximators

Item title	Mean
about 500 pounds	508.00
500 pounds or so	560.96

In Table 3.17, the means are different, although they have the same numeral. Item 2 has a much bigger mean score than that of item 1. That is, the mean of *500 pounds or so* spreads much further upwards than that of *about 500 pounds*. This is again due to their different approximators (*about* and *or so*). It is then suspected that for a number, say 20, the range or mean would be different when combined with different approximators, such as *about 20* and *20 or so*.

3.6 Consistency between the degree of fuzziness, the size of a numeral and the length of a range

It is observed from our data that the degree of fuzziness has a direct ratio to the size of a numeral and the length of a range. In my Chinese test, the degree of fuzziness was represented by *standard deviation*. Considering the degree of fuzziness, the larger the standard deviation of an FQ, the fuzzier the meaning of the FQ. For Type I FQs, we can see from my Chinese data (see Appendix 2, Table 2.2, Group 1) that FQs denoting larger proportions have larger variances. FQs which denote smaller proportions have smaller variances, the point made in Section 3.2.1 above.

For FQs in Type II (see Appendix 2, Table 2.2), the test results also showed

that there was less variance corresponding to FQs having smaller numerals like *200 ren zuoyou* 'about 200 people' than FQs having larger numerals like *20,000 ren zuoyou* 'about 20,000 people'. This is illustrated in Table 3.18, reproduced from Appendix 2, Table 2.2, Groups 3, 4 and 5.

TABLE 3.18: The consistency between the three kinds of results

No	Item title	Std. Dev.	Range
1	jiangjin 200 ren 'nearly 200 people'	9.8	100
2	jiangjin 2,000 ren 'nearly 2,000 people'	115.66	700
3	jiangjin 20,000 ren 'nearly 20,000 people'	2,330.31	20,000
4	200 duo ren '200-odd people'	14.45	125
5	2,000 duo ren '2,000-odd people'	124.18	1,350
6	20,000 duo ren '20,000-odd people'	2626.64	15,000
7	200 ren zuoyou 'about 200 people'	13.42	110
8	2,000 ren zuoyou 'about 2,000 people'	130.89	1,000
9	20,000 ren zuoyou 'about 20,000 people'	2541.85	20,500

As the figures in Table 3.18 show, when the size of the numerals and the length of the ranges become larger, the degree of standard deviation also increases. Let us compare items 7, 8 and 9. The size of the numeral of item 7 is ten times smaller than item 8; so is item 8 to item 9. In terms of range, the range of *200 ren zuoyou* 'about 200 people' is 110 which is again roughly ten times smaller than *2,000 ren zuoyou* 'about 2,000 people' (range = 1,000); for *20,000 ren zuoyou* 'about 20,000 people', the range is 20,500, approximately 20 times bigger than item 8. Then, the standard deviation for the three is 13.42, 130.89 and 2,541.85 respectively. The difference of

the score of standard deviation between the three is again about 10 times between items 7 and 8, and about 20 times between items 8 and 9. So, the trend that the degree of fuzziness has a direct ratio with the size of the numeral and the length of range is empirically illustrated by the type of *about n*, the same relationship for the other two, *nearly n* (items 1, 2 and 3) and *n-odd* (items 4, 5 and 6) is also observed, as shown in Table 3.18.

Finally, let us look at what Channell concludes from her English data. The results of her informant tests show that hearers interpreted the fuzzy expressions as identifying fuzzy sets whose membership is defined by (a) the form and content of an expression; (b) linguistic context and situation of utterance; and (c) world knowledge. Channell concludes the following, based on the results of her elicitation test.

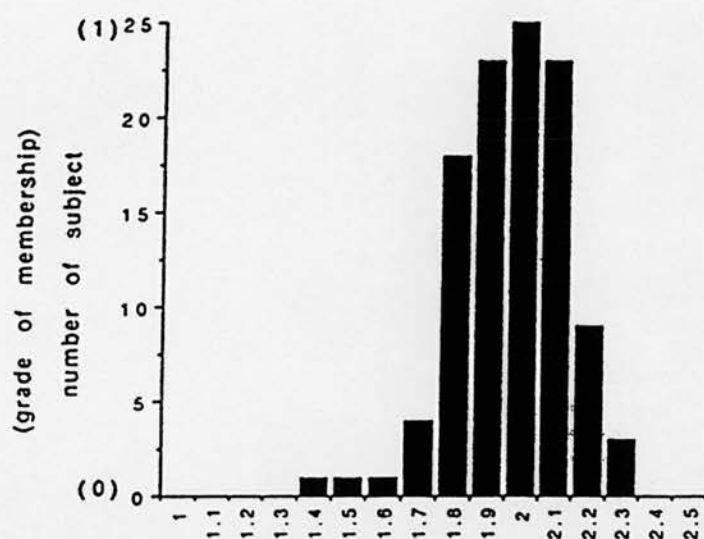
1. There is a set of expressions whose effect is to bring a fuzzy reading (= an approximation) to an utterance containing a number.
2. The resulting approximations are understood as designating continuous intervals of numbers.
3. Different approximators (e.g. *about* in *about 20*) change the interval designated.
4. Although there is a high degree of agreement on numbers that near the exemplar number in an interval (e.g. 20 in *about 20*), there is no consensus about the extent of the interval in any given case.
5. Both the size and form of the exemplar number affect the length of the interval.
6. Whether or not the exemplar number is a round number affects the length of the interval (e.g. *about 31* vs *about 30*).
7. The nature of the item being approximated (e.g. discrete vs non-discrete) affects the length of the interval.
8. The conversational setting in which an approximation occurs affects how it is understood.

9. Sentences containing approximators characteristically have entailments and implicatures. For instance, *He has exactly \$20* entails *He has approximately \$20*.

In general, Channell's findings are similar to what have been observed in Chinese. It is plausible to claim that linguistic context, situation, and world knowledge all play a role in establishing what the set consists of for FQs. However, the claim that sentences containing approximators characteristically have entailments and implicatures is not applicable to all FQs. For instance, *He has exactly \$20* may not entail *He has nearly \$20* (see Section 7.3 for further discussion).

3.7 Some notes on membership function

In this section, we examine the trend for all the membership functions observed from our data. Channell's original English data did not introduce the concept of membership function. However, it is not difficult to convert her data to the form of a membership function. Let us examine Fig. 3.17, reproduced from Appendix 1, Fig. 1.1.

FIGURE 3.17: About 2 pounds

In the figure, the Y-axis is the number of subjects. We may alter it into a range $[1,0]$, where the top (25) gets full membership 1, and the bottom gets 0. Then, if a number was given by all 25 subjects, it has a total membership of 1; if given by fewer subjects, it has a lower grade of membership. This is exactly how I calculate in my test, where membership functions were derived by dividing a particular count by 135 (total number of the subjects in my test), a ratio representing a particular estimate against the best estimate (see page 66 for more explanation).

Let us now compare Fig. 3.17 above and Fig. 3.18 below reproduced from Appendix 2, 2.16.

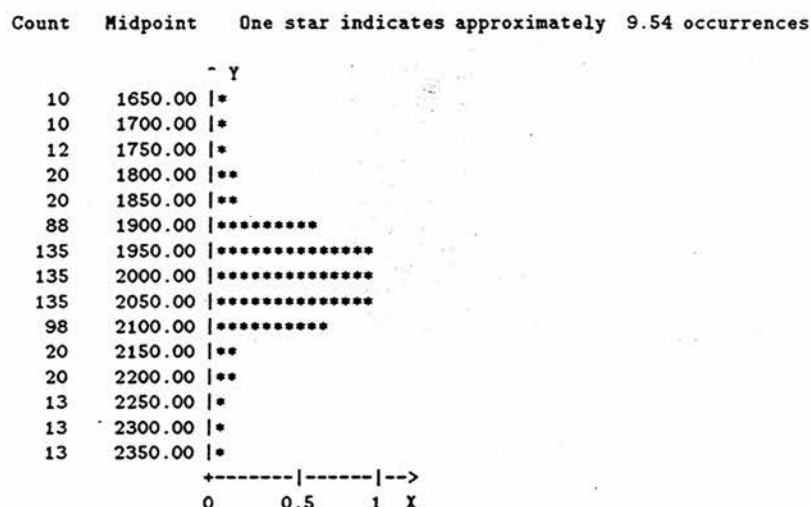


FIGURE 3.18: 2,000 ren zuoyou 'about 2,000 people'

A general trend shown in the two figures is that membership is not simply an all-or-nothing matter, but rather a matter of degree. For example, in *2,000 ren zuoyou* 'about 2,000 people', 2,000 is more typical than 1,900 and 2,100; and in the same way, 1,900 and 2,100 are more typical than 1,500 and 2,500, though all numbers here have the property of *2,000 ren zuoyou* 'about 2,000 people' to some extent. This trend is observed in all types of FQs in both English and Chinese data (see Appendices for the details).

Let us then examine closely the membership function of *liang, san (ge) '2 or 3'*, given as in Table 3.19 and Fig. 3.19, reproduced from my Chinese data (see Appendix 2, Fig. 2.18).

TABLE 3.19: *Liang, san (ge) '2 or 3'*

Numbers(<i>x</i>)	1	2	3	4	5	6	7
$\mu_{liang\ san\ (ge)}(x)$	0.007	0.94	0.99	0.5	0.12	0.02	0.007

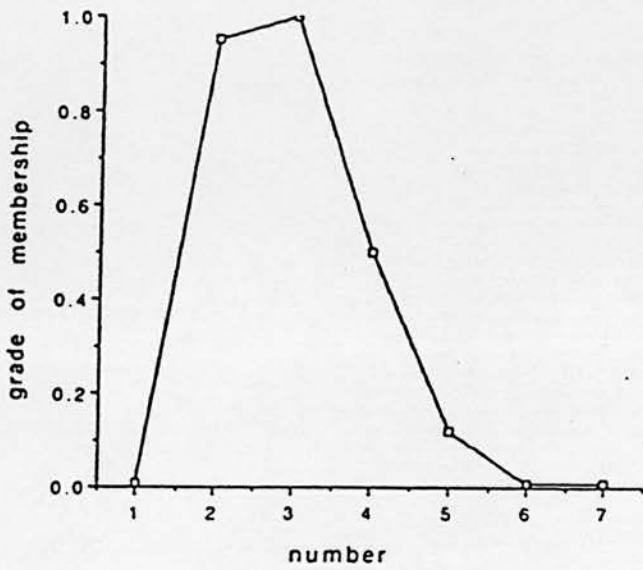
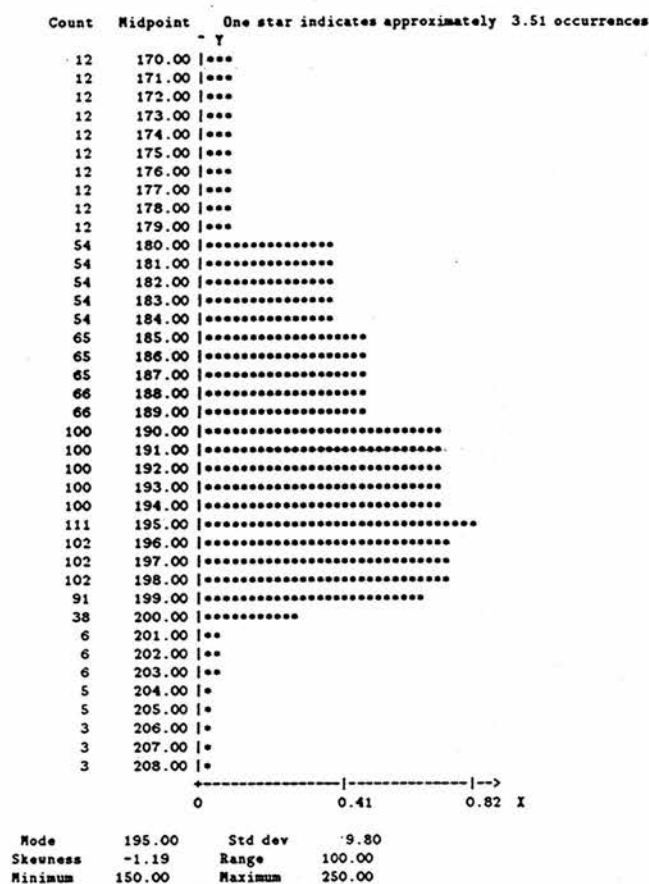


FIGURE 3.19: *Liang, san (ge) '2 or 3'*

It is shown in Table 3.19 and Fig. 3.19 that if a number is fewer than 1 exclusively or greater than 7 exclusively, then it does not belong to *liang, san (ge) '2 or 3'* at all. If a number is in the interval between 1 and 7, then it has the property of *liang, san (ge) '2 or 3'* to a certain degree. For instance, the number 2 belongs to *liang, san (ge) '2 or 3'* to a degree of 0.94; 6 is of *liang, san (ge) '2 or 3'* to a degree of 0.02; and 3 (mode number) belongs to *liang, san (ge) '2 or 3'* almost totally.

A trend guides the actual numbers in Table 3.19, with the requirement that the membership of the set increases to 0.99 and decreases from 0.99, in the

way shown in Fig. 3.19. It rises and falls continuously as it should. For more discussion, see Section 5.2. There is another trend that the closer a number to a norm, the higher its membership. As Table 3.19 shows, 4 is nearer to 3 (the norm) than 5, so 4 has a higher membership (0.5) than 5 (0.12). However, there is a problem in claiming this trend. Empirical evidence in Fig. 3.20 gave a counter-example, reproduced from Appendix 2, Fig. 2.9.

FIGURE 3.20: *Jianqjin 200 ren* ‘nearly 200 people’

As shown in Fig 3.20, the membership of the number 201 is 0.04, 184 is 0.4. However, 201 is closer to the norm 195 than 184 is. This violates the claim.

One way to solve this problem appears to be that we can add a constraint to the claim, that the norm should be an end of scale which consists of a set of numbers in question. Let μ be a grade of membership, the function " $\mu(x) \geq \mu(y)$ iff $|z - x| \leq |z - y|$ " indicates that the closer x is to z , z is a norm, the closer $\mu(x)$ is to 1; on the other hand, the further y away from z , the closer $\mu(y)$ to 0. This can be demonstrated in Fig. 3.21.

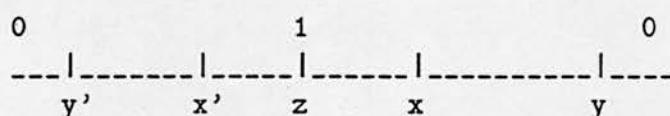
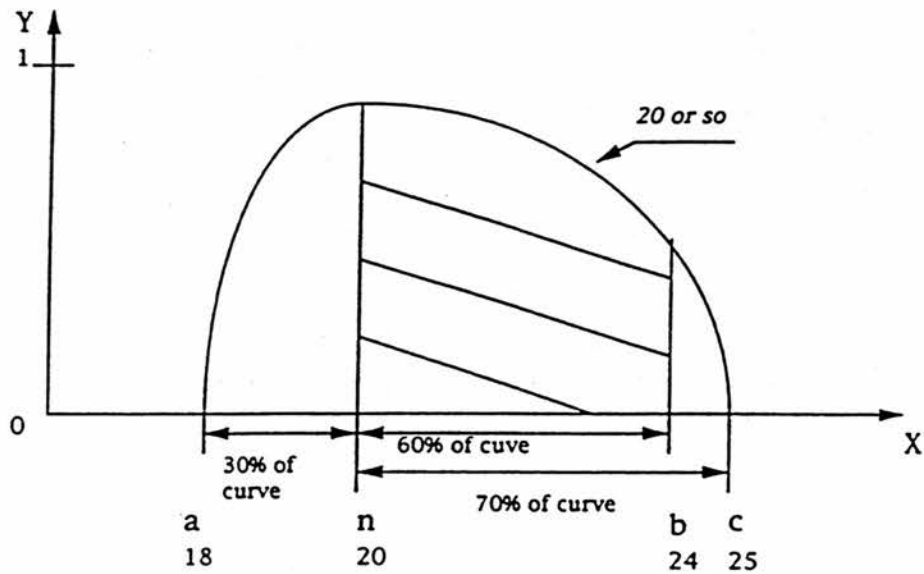


FIGURE 3.21: $\mu(x)$ and $\mu(y)$

As Fig. 3.21 shows, z is a norm, and x is placed closer to z than y , notated by " $|z - x| < |z - y|$ ". Then, $\mu(x)$ is greater than $\mu(y)$. However, the function " $\mu(x) \geq \mu(y)$ iff $|z - x| \leq |z - y|$ " works if and only if z is an end of scale where x and y are allocated, i.e. $\langle y', x', z \rangle$ or $\langle z, x, y \rangle$, where z is either a minimum or maximum. Let us examine Fig. 3.20 again. If the norm 195 is taken as an end of scale for both sides of the interval, then the problem could be solved. On one side of 195, the membership increases as the numbers approach 195. On the other side, the membership decreases as the numbers move away from 195. In the case where z is not an end of scale we have to consider it as one if we want to make the claim that the closer a number to a norm, the higher its grade of membership. For example, with *about n* type, where the norm may not be the end of scale. We have to group numbers involved into two: greater than the norm and less than the norm, as shown in Fig 3.21. That is, we have to divide the individuals on the scale into two groups: x', y', z and z, x, y , where z is the end of scale for both groups. Then, the function " $\mu(x) \geq \mu(y)$ iff $|z - x| \leq |z - y|$ " would work.

Let us then explore if there are some other ways of solving the problem. For example, let the membership function refer to the area under curve. That is, if we talk about the percentage of curve between points, would the constraint of an end of scale not be needed? If so, the rule would be that the higher the percentage of curve for a point to the norm, the higher its membership. This can be represented as in Fig. 3.22.

FIGURE 3.22: *Area under curve*

As shown, although a is closer to the norm (20) in distance than b on the other side, its membership would not be greater than b , because its percentage of curve is lower than that of b . This solves the problem raised earlier, as shown in Fig. 3.20. That is, if we take the percentage of curve into consideration, although a number is closer to a norm in distance, it does not necessarily get a higher grade of membership.

However, there would still be some problem with this solution. For example, at point c , the percentage is 70%, higher than that of b . Then, according to the assumption that the higher a point's percentage of curve, the higher its membership, μ_c would be greater than μ_b . In fact, as shown in Fig. 3.22 above, μ_b should be greater than μ_c rather than in reverse. So, the solution is counter-intuitive and inconsistent, at least in some situations.

It appears that a valid and consistent solution to solve the problem mentioned on page 115 above is to use the constraint that a norm should be considered as an end of scale. Hence, in my formal semantic work below, I will use this constraint to claim that the closer a number is to a norm, the higher its grade of membership is (see Chapter 6 for the details).

3.8 Conclusion

The purpose of this chapter is to do some semantic analysis of FQs in both English and Chinese, based on our empirical data. Our data here show some semantic trends of FQs. It is observed that there are similarities and differences between different types of FQs and between the two languages. Common properties across English and Chinese imply that FQs are systematic. It is observed (e.g. in Sections 3.4, 3.5 and 3.6) that there is indeed regularity in terms of semantics of FQs, which will be discussed theoretically in Section 4.2 concerning compositionality of FQs.

It is also observed that a common feature of FQs is that, from an exemplar of some kind, a set consisting of a selection of possibilities is inferred. This verifies empirically the essence of the FST which will be discussed in Section 5.2. It appears that the denotational meanings of FQs have indeed imprecise boundaries, which may be represented by a membership function. My speculation is that membership functions can provide a sound means for a formal semantics of FQs (see Chapters 5, 6 and 7 for further discussion).

4

Pragmatics, compositionality and FQs

Having discussed the empirical data for FQs, we will go further in this chapter to explore FQs in terms of compositionality. Compositionality claims that the meaning of an expression is a function of the meanings of its parts (Cann, 1993). Regarding the compositionality of FQs, I will concentrate on the requirement of a semantic algebra (i.e. expressions that are manipulated according to special rules of operation) which produces a homomorphism from the semantics of one FQ to another. For instance, *about 20 pounds* and *about 200 people* have similar semantic components. Then, if the two have the same semantic behaviour, they are compositional.

For any adequate semantic theory compositionality is required. The reason is simple—in order to make our language meaningful there must be some way of associating expressions with appropriate meanings. That is, it is plausible to expect that there is a homomorphism between the meaning of an expression and the meanings of other expressions of the same type, as well as the meaning of a sentence and the meanings of its component lexemes. Otherwise, the meaning of every single expression or sentence in a language would have to be listed, and we would not be able to determine the meanings of sentences of any language due to the infinite sentences and the finite

resources of the brain (Cann, 1993). Hence, meanings of expressions are expected to be compositional¹, and sentences should be defined recursively from their components, just as they are defined recursively by syntactic rules. For further discussion, see Partee (1984).

All issues discussed in this chapter will involve one question: whether or not FQs (atomic or combinative) are compositional; if so, in what way. As an illustration, if we claim that compositionality is based on the shape of curves that FQs generate then the claim is highly likely to be invalid. The fact is that *about 50* and *about 3* may not have the same symmetrical curve, as pointed out by Linda Moxey (personal communication). This will be elaborated in the following sections. Then, what is compositionality based on? How can we show FQs of a same type derive their meanings in the same way? We will try to answer these questions by exploring the relation between semantics and pragmatics. In other words, I will tackle the old question of truth conditions versus "meanings" from the perspective of FQs.

4.1 Pragmatics and FQs

It is well-acknowledged that we do not interpret the meaning of FQs in a vacuum; pragmatic factors² play a role. Then, the question is whether or not they affect compositionality. What follows is a discussion of the pragmatic effects on FQs, bearing in mind the question raised here.

¹There are some idiomatic phrases, such as *kick the bucket*, which are not compositional, but they are extremely limited in number.

²The term *pragmatic factor* used here is in a broad sense. It includes not only linguistic discourse, but also social context (world knowledge, background, experience, culture, individual attitude, etc.).

4.1.1 Scale effects

The interpretation of FQs is not just related to expressions themselves, it is partially a result of the scale onto which they are mapped. As an illustration, variance tends to increase as we move towards the centre of a scale from its ends. For instance, on the 0-100% scale, expressions (e.g. *many*) towards the middle of the scale tend to be fuzzier than those in the end (e.g. *few*). This has to do with the scale (i.e. *many* has a bigger scale size), not the expression itself, i.e. there are floor and ceiling effects³. Expressions in the ends have less space to expand, such as *about 3*, so they are less fuzzy than those that have more space to manoeuvre, such as *about 20* (see Section 3.2.1 for empirical evidence on this point). This can be illustrated as in Fig. 4.1 below.

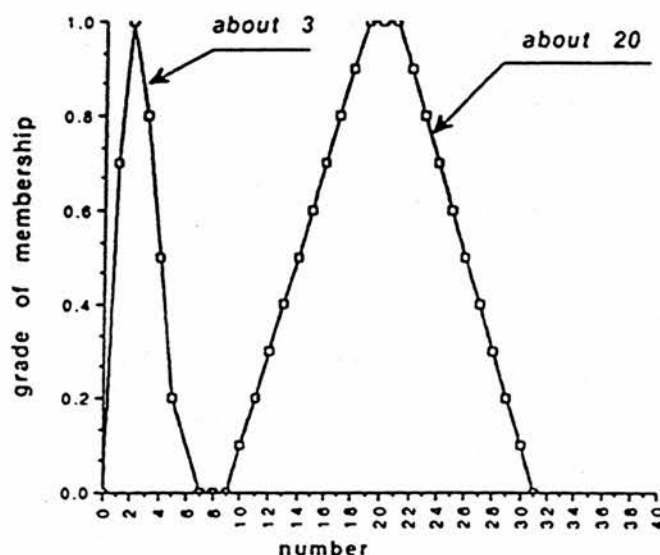


FIGURE 4.1: 'about 3' and 'about 20'

In the figure, we can see that *about 3* does not have a symmetrical curve, as *about 20* has. This has little to do with their semantic components; they have

³A floor effect is defined here as an effect that limits the expansion of an FQ's extension around the lower end of scale. For example, the lower bound of *about 1* would not go very far because 1 is adjacent to 0, the end of the scale. Similarly, in a scale 1-100, the upper bound of *about 99* would also not have much room to expand upwards, because 99 is next to the upper end of scale, 100. This is termed as a ceiling effect.

exactly the same semantic structure (i.e. *about n* structure). What makes a difference here is where on the scale they are located, respectively. That is, *about 3* is located down the lower end of the scale and has less room to expand downwards. On the other hand, *about 20* is not located in an end of scale, so its curve has more freedom to move around than *about 3* has (see Section 3.2.1 for more discussion on this point). This kind of effect is termed as a floor effect. Because of it, *about 3* is suspected of not having a symmetrical curve here.

There is empirical evidence for scale effects observed in my Chinese data. Let us examine the Figs. 4.2, 4.3, and 4.4, reproduced from Appendix 2, Figs. 18, 19 & 20.

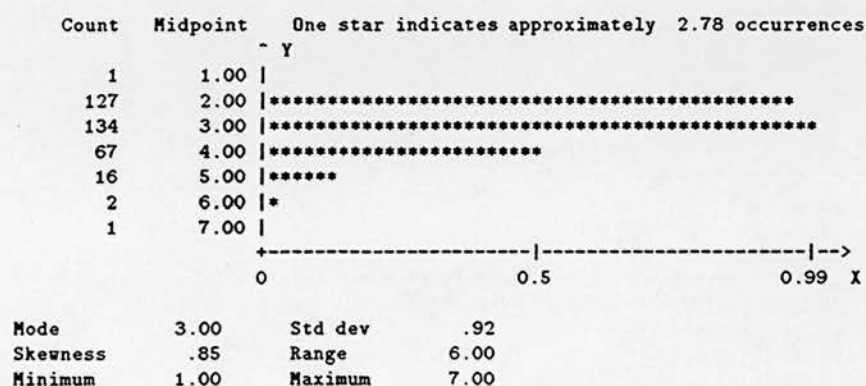
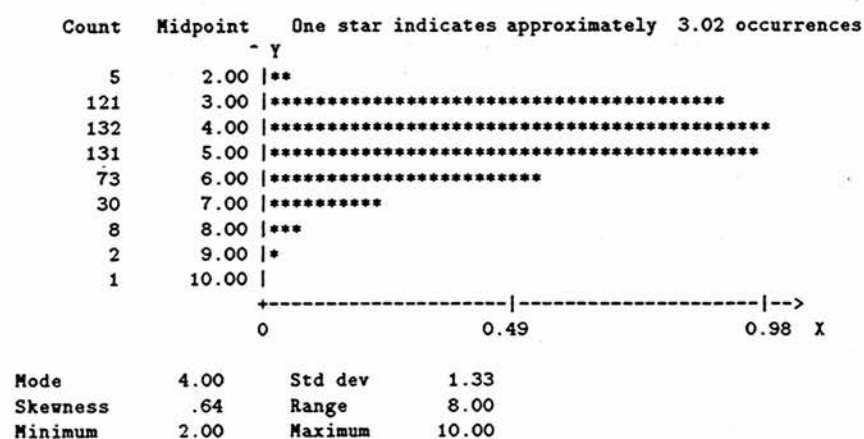
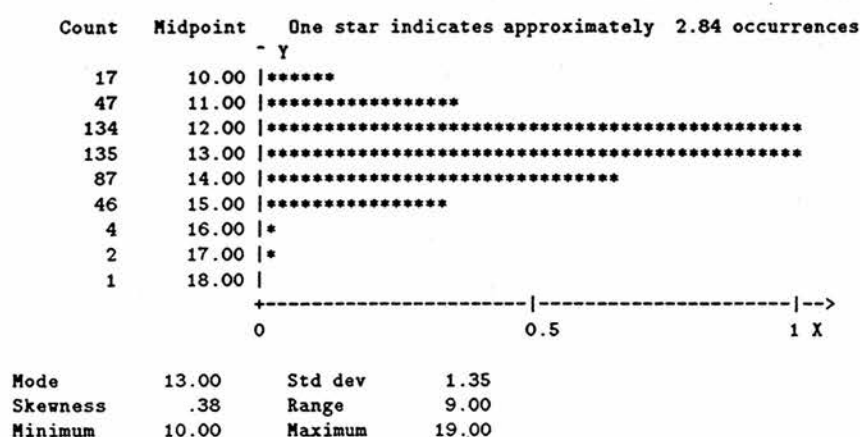


FIGURE 4.2: *Liang, san (ge) '2 or 3'*

FIGURE 4.3: *San, wu (ge) '3 or 5'*FIGURE 4.4: *Shi'er, san (ge) '12 or 13'*

In these figures, the histogram of *12 or 13* has a more symmetrical distribution around 12 and 13 than the other two. That is why its score of skewness is lower. The skewness in these figures is almost certainly a floor effect—a scale effect rather than something to do with the expressions. For *2 or 3*,

there is not much room to expand below 2, whereas there is more room in 12 or 13 than the other two.

Another example, *many*, may have a monotonic curve with an open-ended upper limit. However, if we fix a scale of 1-100%, *many* could be different as I had in my Chinese data (see Appendix 2, Fig. 2.3) where *xuduo ren* 'many people' was given a single-peaked curve. It behaved like *about n* in having a single-peaked distribution (see next section for more discussion on this). This is because a certain scale in my test had been fixed for testing *xuduo ren* 'many people', the subjects were invited to designate an interval for *many* out of 100. Thus, no one would be able to give positive infinity as the upper bound of *xuduo ren* 'many people' under the circumstances. If the constraint had not been provided, some subjects would have considered positive infinity as the upper bound of *xuduo ren* 'many people'. Moreover, it is suspected that when given a 0-100 interval, subjects might compare *many* with other FQs on the scale, like *most*, *almost all* and *all*. Because the quantity of *many* is normally considered as less than those FQs it got a single-peaked curve, rather than a monotonic one with 100 as the best estimate.

There may also be effects of the overall difference between top and bottom of the scale, granularity⁴, etc, though these are perhaps more likely to influence the flatness/pointedness of a curve rather than skewness. For example, *about 20* and *about 21* may have similar symmetrical curves, but they may differ in terms of flatness because of the effect of granularity. It is suspected that

⁴In terms of granularity, Linda Moxey (personal communication) explains that all human descriptions of quantities have vague ranges whether they contain words or numbers. This is because our measurements are limited by the means which we have available for measuring things like time, distance, etc. Therefore, *about 10am* has a series of states of affairs which it might correctly describe, and this is also true of *about 10.02am*. However, if we asked people to give a range of times which might be described by *about 10am*, they are likely to think in terms of the minutes around 10am, possibly even in batches of five minutes; if we ask the same question of *about 10.02am*, people are more likely to think of seconds around 10.02, or at most, minutes. This is because 10am is a point on a coarser scale than 10.02, and subject responses are influenced by the scale used in the question. So-called exact numerical descriptions also have different intervals because of granularity, for example, *He stayed for five minutes* versus *He stayed for five and a half minutes*. In the same vein as 10am vs 10.02am, the curve of the former may be flatter than that of the latter.

about 21 would have a more pointed curve than *about 20* would have. Another example, *a few*, may not be affected by scale in terms of monotonic/single-peaked shapes as *many* would be, because it is less fuzzy than *many* (see Section 3.2.1 for the details). However, *a few* could be affected by scale in terms of symmetrical/skewing curve, as well as flatness/pointedness. It is suspected that if the scale is narrower, then the curve may be more pointed. Or, it may have a skewed curve when the scale is 0-4, as opposed to having a symmetrical curve when the scale is 0-10. All these phenomena demonstrate the importance of scale effects in interpreting FQs.

In conclusion, if we ask subjects to give numbers in response to quantity expressions we cannot assume that the answers they give are determined simply by the expressions themselves. There are more factors involved, such as scale effects. Accordingly, there will be a contextual parameter built in my formal semantics of FQs (see Chapter 6), which is meant to represent this kind of scale effect and other kinds of contextual factors.

4.1.2 Other relevant factors

Apart from scale effects, there are more factors involved in interpreting FQs.

The item being modified

Hörmann (1983) claims, based on a test of *some*, *several* and *a few*, that the numbers assigned to an expression can be decided by a function of the size of the objects and of the spatial situations surrounding the objects. For instance, with *a few*, subjects gave higher numerical values for the situations further down this list:

- | | | |
|-----------------------|-----------------------|-------|
| A few people standing | before the hut. | |
| ... | before the house. | |
| ... | before the city hall. | (4.1) |
| ... | before the building. | |

That is to say that *a few people* was given the longest interval in the situation of *before the building* and the shortest interval in the situation of *before the hut*⁵.

In addition, Sadock (1977) states that interpretation differs from when we are told that a man is approximately six feet tall to when we are told that a cockroach is approximately six feet tall because of our world knowledge about the usual tallness of men and cockroaches. Also, Jim Hurford (personal communication) thinks that *about five* may mean a range of 2 to 9 in the sentence of *I'll be there in about five minutes*; but 3 to 7 in the sentence of *She has about five children*. It appears that the range of *about five* may vary, depending on what it modifies, *minute* or *children*. Moxey and Sanford (1993b) point out that it is scarcely acceptable to speak of about two people, but it is quite acceptable to talk about two volts when making a measurement with a voltmeter, since fractions of volts are quite acceptable.

My own data also showed that the nature of items being modified affect the way an FQ is being interpreted. It is shown in my Chinese data (see Appendix 2, Group 7) that the noun *ren* 'people' has some effect on the designation of *more than n* and *fewer than!! n*. It appears that the subjects understood *more than n* and *fewer than n* differently depending on whether or not it was associated with the noun. For example, responses to *200 yixia* 'fewer than 200' and *200 ren yixia* 'fewer than 200 people' were radically different. A much longer interval (modal interval = $-\infty-200$) was given to the former; for the latter not one subject gave a negative number (modal

⁵Moxey and Sanford (1993b: 28) point out that such an effect could be due to the number of people who might be expected to be standing before a hut or a city hall. Moxey *et al's* data (unpublished) also showed that it is not size alone but also expectation that influences estimates. For example, the number of people *expected* in front of a large building influences estimates for *many* and *some*: the more expected, the higher the values given. Refer to page 127 for further discussion.

interval = 100–200). As for *200 yishang* ‘more than 200’ and *200 ren yishang* ‘more than 200 people’, again the modal interval for the former is 200– $+\infty$, and 200–300 for the latter. The key here is the noun *ren* ‘people’, i.e. with or without it makes a great deal of difference in assigning the meaning for *more than n* and *fewer than n* (see Section 3.2.3 for the details).

Expectation

There is a close relationship between expectation and interpretation of FQs—empirical evidence has been given in Moxey and Sanford (1993b) (see Section 2.4 for the details). Let me give an example. In the sentence *There are many couples who lived together before they got married*, *many* may mean 80% or 90% in a western culture, but perhaps at most 10% in Chinese culture. Due to cultural differences, people from different cultures have different expectations. For instance, in Chinese culture lovers normally do not live together until they get married, but this is not the case in the western world. It is suspected that Chinese people have a lower expectation about the number of unmarried couples who live together, and therefore they would judge *many* in the above sentence a lower percentage. Again, suppose that in the sentence *There are many students who have enrolled for Business Studies at the University of Otago*, where Business Studies is perceived as a favourite subject, then *many* is expected to have a high value. On the other hand, in *There are many students who have enrolled for Classical Chinese Grammar at the University of Otago*, *many* may have a lower value, because a normal expectation for the Classical Chinese Grammar course is lower. It could be that *many students* in the case of Business Studies is 500 students, but only 25 for the Classical Chinese Grammar course.

There are numerous empirical data supporting the claim that the higher the base rate is, the higher the numerical value of an quantity expression is. For instance, Wallsten, Fillenbaum and Cox (1986) demonstrate that with respect to a high relative frequency event meteorologists judged a considerably greater probability to an item compared to a low relative frequency event. Pepper and Prytulak (1974) also found that *frequently* was considered to

mean approximately 70% of the time when it was used to describe the frequency with which Miss Sweden was found attractive, due to a high expected frequency. On the other hand, the term was given only approximately 20% of the time when it was used to describe the frequency of air crashes, due to a low expected frequency. Similarly, Smithson (1987) says that the frequency of 15 times a year might be given a membership of 1.0 to *often* when the events in question are earthquakes, but not when they are sunny days. For an extensive discussion of these frequency expressions, see Pepper (1981).

Apart from those factors discussed above, there are many more contextual factors involved in interpretation of FQs. As an illustration, FQs can be interpreted differently from one time to another. Consider the sentence *Many children were in the school playground*. The proportion denoted by *many children* depends very much on what time this sentence was uttered. If it was break time, the proportion of *many children* would be bigger than that of class time. Hence, the information which the sentence conveys varies corresponding to different time spans.

Moreover, let us look at cultural factors. It is a Chinese New Year custom for children to receive a red package with some cash in it. For Chinese, *a few* in *There are a few yuan⁶ in the red package* is likely to be an even number, which is a symbol of satisfaction, except four. Four has a pronunciation similar to *death* in both Mandarin and Cantonese while the pronunciation of eight resembles that of *fa*, meaning *getting rich*. So, when choosing such things as telephone numbers eight is the most favourite number, while four is the least favoured. Given all these factors, it is expected that the number denoted by *a few* in the above sentence would most likely be eight. Nevertheless, this would not have any significance to those brought up in a different cultural environment.

The finding that FQs are interpreted differently because of pragmatic factors is assumed to be valid for other fuzzy expressions also. One may have to consider a few contextual factors simultaneously to interpret fuzzy expressions adequately. For example, to work out the membership of *tallness* there are several contextual factors that could be taken into account: sex,

⁶Yuan is a unit of Chinese dollars.

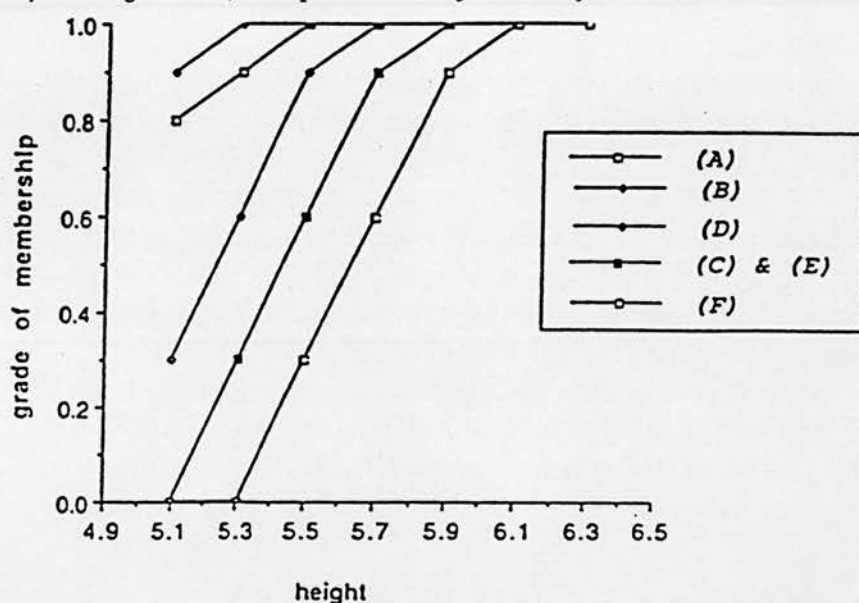
location, occupation, etc. Generally speaking, men are taller than women; British are taller than Japanese; and basketball players are taller than other people. On the basis of these assumptions I present the following Table 4.1, where membership of the set denoted by *tallness* is represented by the scale of $[0, 1]$. The numbers given in the table are hypothetical for illustrating the contextual effects on FQs.

TABLE 4.1: Effect of contextual factors

	5'1"	5'3"	5'5"	5'7"	5'9"	6'1"	6'3"
female (in Britain) (A)	0.8	0.9	1	1	1	1	1
female (in Japan) (B)	0.9	1	1	1	1	1	1
male (in Britain) (C)	0	0.3	0.6	0.9	1	1	1
male (in Japan) (D)	0.3	0.6	0.9	1	1	1	1
female (B. Players) (E)	0	0.3	0.6	0.9	1	1	1
male (B. Players) (F)	0	0	0.3	0.6	0.9	1	1

Note: B. Player stands for basketball players.

According to Table 4.1, for females the lower boundary for full *tallness* in Britain is 5'5" onwards, whereas in Japan it is 5'3" onwards. The latter is less than the former by 2". For males in Britain, it is 5'9" onwards which is 4" more than for females in the same location. Moreover, the lower boundary for full *tallness* of female basketball players and males in Britain is 5'9", which is 6" more than for females in Japan. To examine a fixed height, say 5'3", A's degree is 0.9, B is 1, C is 0.3, D is 0.6, E is 0.3 and F is 0. From Table 4.1, *tallness* can be represented graphically as in Fig. 4.5.

FIGURE 4.5: Tallness

The diversity of these five curves corresponds to the impact of different contextual factors. In addition to the factors discussed, there may well be more factors which need to be taken into account. The more one considers the contextual factors, the more appropriate the interpretation of fuzzy expressions becomes.

Having discussed those pragmatic factors above, it becomes certain that compositionality is not about shape of curves. Our previous discussion shows that shapes of FQs do change from context to context, e.g. *about 3* versus *about 20* and *more than 200* versus *more than 200 people* (see Section 4.1.1 for the details). As discussed in Section 3.2.3, semantically speaking *more than n* or *fewer than n* types could have a monotonic (or hillside-shaped) curve with infinity as an upper/lower bound. However, pragmatically both FQs would have a single-peaked curve, which has been empirically verified in both Chinese and English data here (see Appendix 1, Figs 1.6 & 1.7; Appendix 2, Figs 2.21-2.24). An explanation for this phenomenon would come from the difference between semantics and pragmatics. From the point of view of truth conditional semantics, *He has more than 200 dollars* is still true even though his money is so much that it may be considered as infinity. However, from the point of view of pragmatics, Grice's (1975) maxim of quantity requires us to be not only truthful but maximally informative. Saying *more than 200*

dollars when the actual number approaches infinity is simply not cooperative in a pragmatic scene. That is, it is not informative and appropriate to use it.

So, it appears that the difference between semantics and pragmatics makes us consider FQs in different ways. Pragmatic factors are, however, suspected of not impacting significantly on the semantic pattern FQs induce. For example, it does not matter whether or not *more than 200* has a single-peaked or monotonic curve, i.e. the interpretation of *more than 200* could be 250 in a given context, or infinity in another; they are all sensible as far as its core meaning is concerned (see next section for further discussion on this point).

4.2 Compositionality of FQs

At the beginning of this chapter, we discussed the definition of compositionality and the need for compositionality in semantic theory. An important question is whether or not those pragmatic factors affect compositionality. What follows is a discussion about how compositionality works with FQs taking pragmatic factors into account, i.e. in what way we can claim that FQs are compositional.

4.2.1 Invariant core meaning/truth condition and variant peripheral meaning

The concept of a core meaning is defined here as a meaning under which a set of entities are selected as the extensional meaning of an expression. A core meaning is similar to an intension representing common property conveyed by an expression (see Section 7.2 for more discussion). We term it as *core meaning* in the sense that it is invariant over context, more a concern for semantics. Conversely, interpretation of an expression varies over context, termed *peripheral meaning*, which is more pragmatic-oriented.

The core meaning of an FQ is more determinate than its peripheral meaning is. For example, *about n* type, its core meaning is *a number approximate to n*. From this its peripheral meaning can be derived, which may vary from context to context, from individual to individual, and from time to time. Nevertheless, no matter what number is designated in a given context it must be in accordance with the core meanings, otherwise it would not be meaningful. For example, for *about 20*, its interval could be 18-22, 17-25, 15-22, but not 100-120. The last one is suspected not sensible, because it does not follow the core meaning, i.e. is not an interval containing 20.

In the same vein, truth condition of propositions containing an FQ tends to be invariant, though the interpretations of the propositions would be variable. For example, in the sentence *There are many people in my room*, its truth condition is *there is a significant number of people in my room compared to a norm*, which would be constant over contexts. From context to context, the number of people in my room may vary, but it has to be licensed by the truth condition. That is, it could be 10, even 20, but not 1,000; because 1,000 would not conform with the truth condition, i.e. usually a room would not accommodate up to 1,000 people.

As discussed in Section 4.1.1, scale effects influence the shape of curves, but it is less obvious that the core meaning of an FQ, or a truth condition of the propositions with an FQ, is affected by the scale in a significant way. For example, it does not matter whether or not the shape for *about 20* is symmetrical, its core meaning would not change over context.

It appears that the semantic meaning of an FQ has two components: (a) a constant component representing the core, (b) an variable component representing the periphery, which can be defined as:

$$S = c + p \quad (4.2)$$

Formula (4.2) shows that the semantics of an FQ (S) consists of two parts: core (c) and periphery (p). The definition in (4.2) is a case for Mosier's (1941) assumption of word meaning. Mosier assumes that the meaning of a

word may be considered as containing: (a) a constant component reflecting the overall meaning value; and (b), a variable component representing the variation in the meaning of the word due to context, speaker, and the like. He defines the meaning (M) of a word as:

$$M = x + i + c \quad (4.3)$$

where x stands for the constant component over people and context, i the variation in meaning due to the individual, and c the variation in meaning due to the context. Mosier claims that any one of the components could be zero for certain words, and for ambiguous words x could be multiple values. In the case of FQs it appears that x is a single-value, but i and c could be multiple values.

In conclusion, the claim that invariant core meaning/truth condition and variant periphery is plausible with respect to FQs like *about 20* and propositions with an FQ. The distinction of the invariant and the variant and the relationship between them are significant in terms of setting a norm for the assignment of semantic values in a given situation. It is also claimed that the interpretation of FQs, or a proposition with an FQ, must follow what the core meaning/truth condition licenses. This lays a sound foundation for the claim of compositionality, which will be elaborated below.

4.2.2 Motivation

From the discussion above, it is shown that specific numerical values of FQs vary over contexts and individuals. However, this kind of variance is not arbitrary, but is motivated.

Motivation is used here in the same sense as Lakoff (1987). He defines that something in language is *motivated* when it is neither arbitrary nor predictable. Motivated phenomena, according to him, include category extensions, polysemy, etc. Harrinton (1994) comments that in terms of polysemy,

motivation provides a partial reason why certain senses appear, and introduces more systematicity to our understanding of polysemy than previously assumed. However, he states that systematicity is not total: to say that a sense extension is motivated does not mean that the appearance of a particular sense can be predicted. There is no way to predict, *a priori*, that English speakers would develop and conventionalize—for instance, the use of *stand* to mean asserting one's rights, as in *stand up for your rights*. However, *standing up for your rights* is assumed to be sensible to the English speaker by virtue of the embodied nature of the *stand* concept and the linking devices (e.g. metaphor, metonymy) used to realize specific sense extensions. Motivation does not completely explain why a particular sense appears or is used, but rather serves as an important source for why the sense exists. Lakoff points out that in natural language, motivation seems to be more the norm than the exception.

Now, let us examine FQs in particular. Although we may be certain that 200 is in the interval of *about 200*, it cannot be construed that the designation of actual numbers to the expression is also determined. That is, the application of the actual number is not completely predictable. The implication of motivation on FQs is, as Lakoff points out, that motivation is not the kind of phenomenon that algorithms were designed to characterize, because with respect to an algorithm things are either predictable (that is, computable from an input) or they are arbitrary. For example, the actual computation of the value of *many* will depend on the situation in question, and what is the critical criterion (or criteria) under consideration. To interpret a specific meaning for an FQ in a particular utterance is a business that interacts a number of aspects, such as semantics, pragmatics and psychology.

Yet, it cannot be said that the application of the actual number is completely arbitrary either. It appears that the application follows some kind of basic membership function pattern, which has to comply with the requirement set by its core meaning. For instance, *about n* type tends to generate a basic membership function where *n* is a prototypical number. It would be counter-intuitive if, say for *about 2000*, the number 3,000 was given a membership of 1, but the number 2,100 was given 0.

In conclusion, FQs are *motivated* in terms of the application of actual numbers for a particular FQ's meaning. That is to say the assignment of specific values to each FQ in a particular utterance is neither predictable nor arbitrary. It is not predictable in the sense that the specification of an FQ's meaning is context-driven. It is not arbitrary in the sense that the designation of the specific values has to follow some kind of pattern. Furthermore, the motivational characteristic implies that although fuzziness is inherent in languages, it can be handled systematically. As a result of motivation, my formal treatment defines that the specification of some values of FQs has to be done empirically, i.e. they are not computable (see Chapter 6 for details).

4.2.3 Compositional FQs and propositions containing one

Compositionality of FQs, or propositions containing one, is claimed on the basis of constant core meaning/truth condition and motivated variance, as demonstrated above. That is, compositionality is about the fact that an FQ generates a number of peripheral meanings (or interpretations) which are all derived from the same core meaning (or truth condition in terms of a proposition with an FQ). Put another way, the core meaning of an FQ licenses a basic membership function which may vary within the limit that the core meaning permits. That is, we do not go beyond a certain degree of divisibility. For example, the core meaning for *1 or 2* generates a basic membership function, a function designates a set of appropriate numbers, $\{n\}$, which approximate to 1 and 2. It is likely to be the case that n is 1 or 2 (or may be 3), but no less. The reason is that 0 is not 1 or 2.

It is important to note that a basic membership function is governed by the core meaning of each type of FQ. Under this basic membership function, each FQ can be then assigned a specific value corresponding to a given context. For example, for *about n* type, its basic membership function has to follow the core meaning, i.e. *about n* denotes an interval of numbers that precede or follow n . Another example, the basic membership function for *many* would have to be constrained by the core meaning *a significant number compared to*

a norm. It is this kind of basic membership function that captures regularity of the meaning of FQs and lays a foundation for a formal framework to build on. That is the reason that the basic membership function is taken as a basis in my formal work (see Chapter 6 for the details).

In conclusion, compositionality is sustained in that the same type of FQ has the same way to derive a specific value corresponding to a particular context. Although a basic membership function may change, the change is motivated and has a limit. For example, the shape for *about n* may not be precisely symmetrical but it would be odd if it had a monotonic curve.

4.3 Combinative FQs

In this section combinative FQs are in focus. We will look at their semantic behaviours to find out whether or not combinative FQs are also compositional.

4.3.1 Concentration and dilatation

In this section Hersh and Caramazza's (1976) data in English is adopted to illustrate the effect of approximators (see Section 1.3.2 for its definition). Hersh and Caramazza (1976) conducted a series of experiments to explore an application of FST to a study of the meaning of expressions, such as *very small*, *sort of large*. The test results supported the hypothesis that natural language concepts and operators can be described by FST.

In the test, 19 undergraduates at Johns Hopkins University served as paid subjects. Twelve slides, each containing a black square on a white background, were used as the stimuli. The squares measured 4, 6, 8, 10, 12, 16, 20, 24, 28, 32, 40, and 48 in. (10.2 cm to 121.9 cm). Thirteen phrases tested were made up by the two adjectives *large* and *small* paired with various combinations of *not* and *very*, etc. Subjects were put in one group.

Each received an answer sheet containing the 13 phrases in one of 10 random orders. Below each phrase were 12 spaces. When subjects were instructed to look at the first phrase, they were told that they would see 12 squares in a random order, and simply to look at each square and decide whether the phrase applied to it. If it was appropriate, enter *yes* in the appropriate space; otherwise enter *no*. This procedure was repeated for each phrase, with a different order of squares in each block. Although Hersh and Caramazza's data are not for FQs per se, we may use them to examine the two types of functions approximators have: concentration and dilatation.

Concentration: This kind of approximator, exemplified by *very* and *extremely*, makes the distribution of an expression converge to end of scale. It appears that *very* acts as an intensifier, a point empirically verified by Hersh and Caramazza's work shown in Fig. 4.6. The curve of *large* is hillside-shaped. *Very*, as an intensifier, shifts the curve to the right, making the distribution of *large* converge to the end of scale.

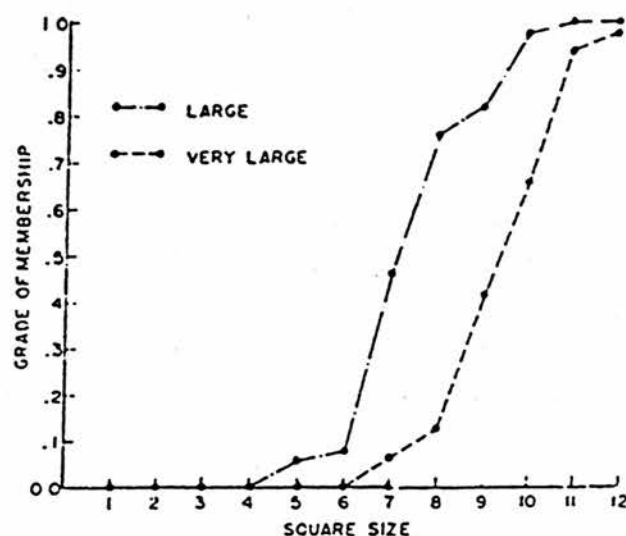


FIGURE 4.6: The effect of *very* on *large*

where the Y-axis indicates the grade of membership, the X-axis indicates the ordinal square size.

Supposing a square is 48 in. (i.e. at the point 12), then it is definitely *large*.

That is, it has the property *large* to the highest degree 1. However, as shown in Fig. 4.6, it has the property *very large* to a degree of 0.98. As a result, for any number picked up from the X-axis, its membership of *very large* would always be less than or equal to its membership of *large*.

Dilatation: Contrary to concentration the function of dilatational approximators exemplified by *somewhat*, *slightly* and *sort of* is to dilate a distribution of FQs. This is empirically tested by Hersh and Caramazza's data on *large* and *sort of large*, as shown in Fig. 4.7:

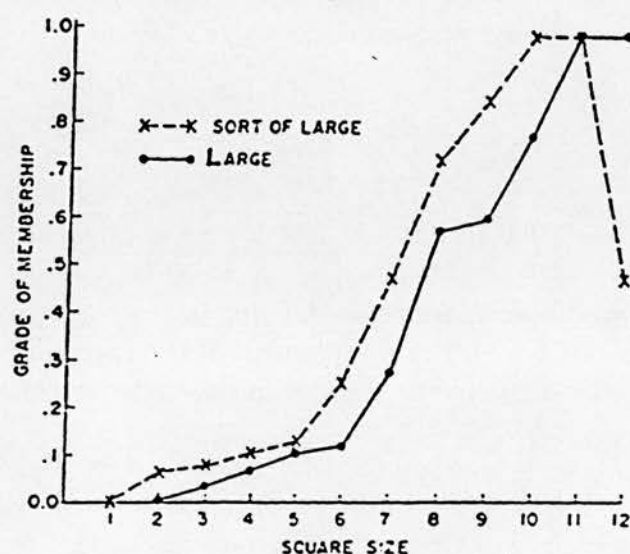


FIGURE 4.7: The effect of sort of on large

As shown in Fig. 4.7, at the square size 10, *large* is the degree 0.75, but *sort of large* increases to 1. Look then at the square size 12: it is *large* to the degree 1, but 0.45 for *sort of large*. It appears that *sort of* tends to raise those values fewer than 1 and to lower those values that close to 1. The curve of *large* in Fig. 4.7 is hillside-shaped, but *sort of* shifts it to the left-hand side and changes its shape to a single-peaked one. This function of *sort of* is different from the function of *very*, i.e. *very* tends to keep the same shape, as shown in Fig. 4.6.

Moreover, the assumption that the boundary between *A* and *not A* is fuzzy is empirically justified by Hersh and Caramazza's (1976) data shown in Fig. 4.8:

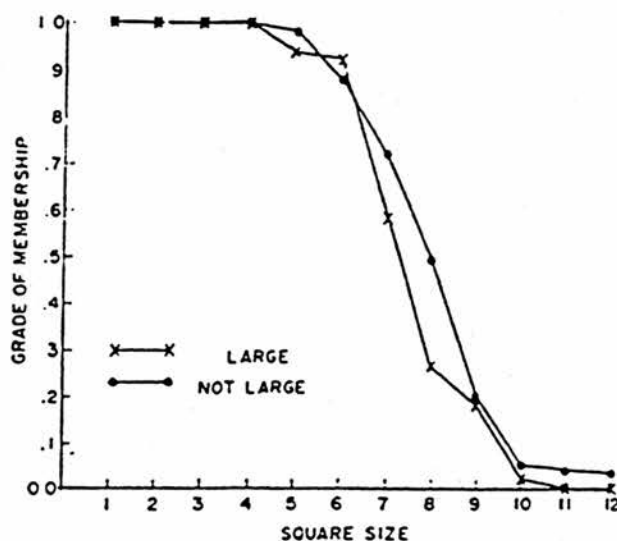


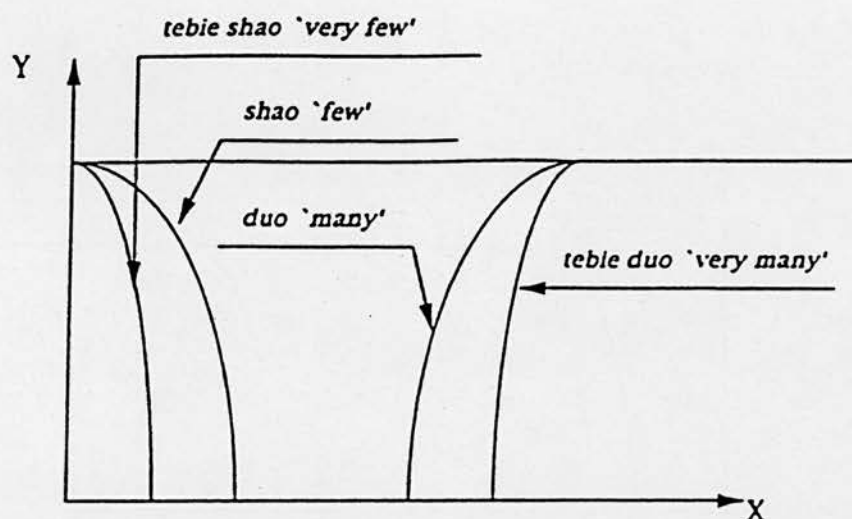
FIGURE 4.8: *Not large and large*

Fig. 4.8 shows membership functions for *not large* and *large*, where the negative phrase is plotted with the complement of the corresponding affirmative phrase. The boundary between the two intermingles together.

The functions of *very*, *sort of* and *not* illustrated by the data have been discussed. We will now look at how these functions work with FQs, both in Chinese and English. It appears that *sort of* can rarely combine with FQs; we will discuss *quite* instead.

4.3.2 *Tebie* 'very', *hao* 'quite' and *bu/mci* 'not'

Tebie 'very': *Tebie* 'very' in Chinese may intensify the values of FQs it modifies. For example, the value of *tebie duo* 'very many' would be greater than that of *duo* 'many'. Similarly, the value of *tebie shao* 'very few' would be less than that of *shao* 'few'. This can be illustrated in Fig. 4.9.

FIGURE 4.9: *Tebie* 'very'

As shown, if an FQ skews downwards like *shao* 'few', *tebie* 'very' tends to reduce its value. On the other hand, if an FQ skews upwards like *duo* 'many', then conversely *tebie* 'very' raises its value. This is also expected to be the case in English. It is speculated that *very* in both Chinese and English tends to push the numerical value of an FQ further along its original expanding direction. In other words, *tebie* 'very' intensifies towards ends of scale, the same as *very* in *very large* shown in Fig 4.6 above.

Hao 'quite': *Quite* differs between English and Chinese. In English, it moderates towards middle of scale. This can be seen in Fig. 4.10.

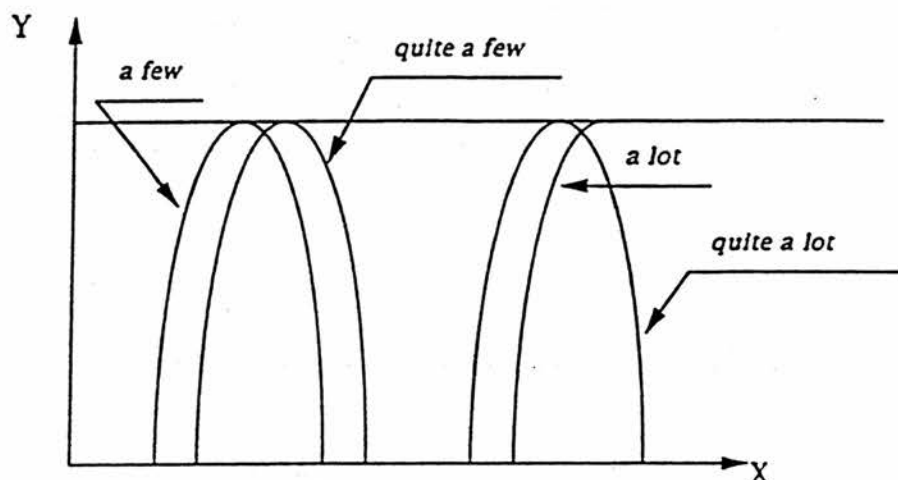
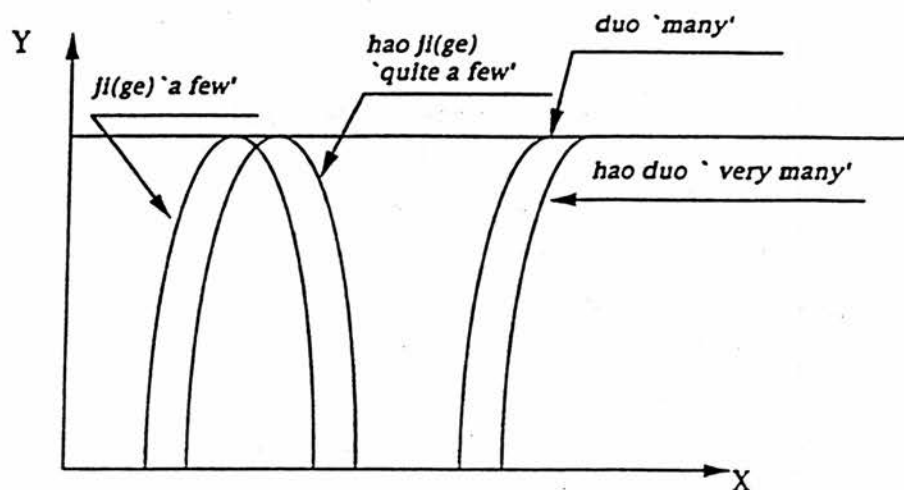


FIGURE 4.10: *Quite*

As shown, *quite a few* has the same shape as *a few*, but not *quite a lot* and *a lot*. In the latter, *quite* changes the shape of *a lot*'s curve, as is *sort of* that changes the shape of *large* shown in Fig. 4.7 above. In general, the function of *quite* appears to push values to the centre of the scale.

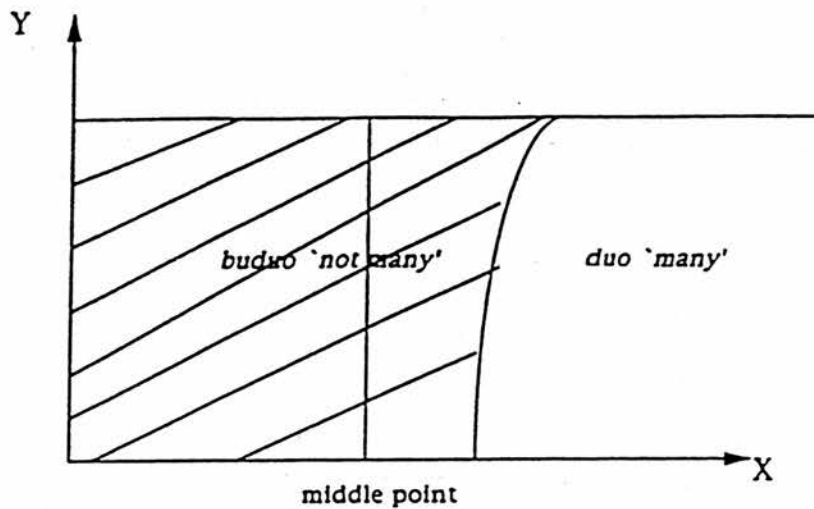
However, *quite* in English does not have its equivalent in Chinese, i.e. *hao* in Chinese does not mean exactly the same thing as *quite* in English⁷. It can function similarly as *quite* in English, with *ji* 'a few' by moderating it towards middle of scale. However, when *hao* combines with *duo* 'many/a lot', it pushes the value towards end of scale, rather than middle of scale, unlike *quite* in English. In the latter case, *hao* means *very*. This can be illustrated in Fig. 4.11.

⁷Here is a translation problem. Between two languages, sometimes there is simply no precise mapping between two words.

FIGURE 4.11: *Hao* 'quite'

As shown in Fig. 4.11, there is some difference between *hao* in Chinese and *quite* in English. The common thing between them is that they both just moderate, unlike *not* which can do more than that.

Bu/mei 'not': In Chinese, both *bu* and *mei* are negative words, the latter tending to be used for the past tense. *Bu/mei* 'not' displays a crucial difference to *tebie* 'very' and *hao* 'quite', in that it not only pushes the values to the opposite direction, but goes to the other side of the middle-point. It does not just moderate, which can be illustrated in Fig. 4.12.

FIGURE 4.12: *Bu/mei 'not'*

Not reduces the value of *many* in *not many*. In addition, can we say *not many* = *a few*, or *not a few* = *many*? In fact, due to fuzzy semantic boundaries, one can hardly be certain about this. Apart from the fact that the two pairs cannot be measured in a mathematical sense, even in a linguistic sense, it is not certain that the two expressions in each pair mean the same.

Here are some examples of the Type II FQs' combination. Channell's (1983: 85) data showed that combinations like *around four or five regions* and *about sevenish or a bit later* are acceptable. However, expressions like *approximately some* are not acceptable. It seems that *about* and *approximately* require a numerical kind to combine with, and the resulting meaning is even fuzzier. The effect of adding an approximator to a numeral is to give it a fuzzy reading. What happens if we add more is that the reading would become even fuzzier. Let us examine the following sentences:

- a. She is *ten*.
 - b. She is *about ten*.
 - c. She is *about ten or a bit younger than that*.
- (4.4)

The sentences from (a) to (c) seem to allow a progressively greater interval. In other words, the meaning of (c) is fuzzier than (b); and (b) is fuzzier than (a).

Combination occurs mainly among FQs in Type I. Type II FQs do not combine as often as other types do. As an illustration, in English *not* does not combine with *about 200* in the same way as with Types I and III. One would not start a sentence with *not about 200*, i.e. we normally would not say *Not about 200 students went to the party*. This is also the case in Chinese, we may say *Meiyou henduo ren lai* 'Not many people came', but would not say *Meiyou erbairen zuoyou lai* 'Not about 200 people came'. The reason presumably is that FQs in Type II have their own approximators, such as *about* in *about 200*. In this case, it sounds somewhat redundant to add another approximator. It sounds odd to say, for example, **very about 200*⁸.

Having discussed combinative FQs, a crucial question is whether or not combinative FQs are compositional. The answer is positive. For example, the way *very* combines with other FQs tends to be intensifying towards ends of scale. On the contrary, *quite* moderates towards middle of scale in English. In Chinese, *hao* 'quite' can be an intensifier (e.g. in *hao duo* 'quite a lot') or a modifier (e.g. in *hao ji (ge)* 'quite a few'). The function of *not* is expected to push in the opposite direction, and possibly goes to the other side of the mid-point. So, the semantic pattern of the same type of combinative FQs in a language, such as *hen x* 'very x' in Chinese, can be predicated, i.e. *hen* 'very' has the same intensifying function over context. Thus, it is speculated that combinative FQs are compositional.

It is important to note that Moxey and Sanford (1993b) found that in a between-subject test⁹ *few* and *very few* were not differentiated in numbers. Hence, they argue plausibly that we should examine the difference between *few* and *very few* in terms of strength of claim instead. This does not undermine the claim of compositionality. The point is that when *few* and *very few*

⁸It is also syntactically ill-formed, because *about 200* is not a typical gradable adjective.

⁹In a between-subject test, a subject is exposed to just one stimulus. On the contrary, in a within-subject design, a subject is exposed to more than one stimulus.

are different numerically, *very* behaves as an intensifier on the scale of numerical values it modifies. Even when the two FQs are not different numerically, *very* still behaves as an intensifier, but this time on the strength of claim instead. The consistency here is vital in terms of compositionality, because *very* would have the same intensifying function in all situations regardless of what kind of scale (numerical or non-numerical) it intensifies.

Finally, a formal representation of compositionality can be represented as in (4.5) below,

$$||f(x_1, \dots, x_n)||(c) = ||f(||x_1||(\dots ||x_n||(\dots ||c||))|| \quad (4.5)$$

which means that the function f has the same effect, subject to some modification depending on context denoted by (c) . For instance, *very* is a function on the interval denoted by *many* such that the interval is shortened between the lowest truth value and 1. Also, *about n* tends to have a basic membership function constrained by *a number approximate to n*. These can be represented as: $||\text{about } n||(\dots ||c||) = ||\text{about } n||(\dots ||c||)$ and $||\text{very FQ}||(\dots ||c||) = ||\text{very}||(\dots ||FQ||(\dots ||c||))$. The important point in (4.5) is that in the case of FQs context factors have a certain effect, but it is definable and functional. The formula in (4.5) can be a representation for all compositional FQs, atomic or combinative. In the same vein, (4.5) can also represent compositionality of propositions containing an FQ, i.e. $||f(\phi_1, \dots, \phi_n)||(\dots ||c||) = ||f(||\phi_1||(\dots ||c||), \dots, ||\phi_n||(\dots ||c||))||$, where ϕ stands for a proposition containing an FQ.

4.4 Conclusion

What has been attempted here is to develop a compositional theory from the perspective of FQs. It claims that compositionality is valid because of FQs' constant core meanings, or truth condition for propositions with an FQ, and motivated peripheral meanings. While acknowledging context plays a role in obtaining peripheral meanings, it is argued that from the point of view of semantics, formal semantics in particular, context is not significant

in the way that it does not affect compositionality or truth conditionality, i.e. they have no significant impact on general semantic patterns of FQs. The fact that the general patterns remain the same over context and individuals implies that the heterogeneity is functional. Pragmatic factors do affect the interpretation of FQs, but do not go beyond a certain degree of divisibility. In this way we could build a compositional theory whereby certain basic membership functions of FQs are subject to modification depending on context, and the modification has to be constrained by the core meaning of an FQ or the truth condition of a proposition containing one. This would explain why such functions are variable without denying compositionality or truth conditionality.

Compositionality of FQs, together with motivation, enables a provision of formal treatment of FQs in my work to be carried out. Also, in my formal semantic framework below, I will employ the concept of membership function as a means to tackle FQs.

5

Formal semantics of FQs (I)

Having discussed relevant semantic characteristics of FQs, an attempt is made in this chapter, as well as in the following chapters, to provide a formal treatment for FQs in which those semantic characteristics explored above will be formally addressed.

We have speculated in the previous chapters that the semantics of FQs can be represented by membership functions based on a compositional theory. What will be shown below is how basic membership functions of FQs can be formally defined, how they vary over types of FQs, how the pragmatic factors can be taken into account, above all how our formal semantics of FQs works. We will first discuss applications of FST and GQT to FQs in this chapter. Then, a formal semantic framework of FQs will be developed in the next two chapters.

5.1 Generalized Quantifier Theory

In this section we will discuss GQT with FQs in focus. Two issues are explored here: in what way FQs are similar to generalised quantifiers, and

whether or not GQT can treat FQs adequately.

For decades the two logical quantifiers, \forall and \exists , have been in a dominant position, regardless how atypical they are in representing the quantifiers in natural languages. In the late 1950s, Mostowski (1957) originated the notion of a *generalized quantifier*, which had little influence until Montague's (1974) work. In truth-conditional semantics, Montague's work—"The proper treatment of quantification in ordinary English" (PTQ)—has in a classical way contributed much to our understanding of the formal treatment of the quantifiers, such as *all*, *some* and *any*. Montague shows that natural language can be elegantly formalized, including natural language quantifiers.

Based on Montague's theory and influenced mathematically by Mostowski, Barwise and Cooper (1981) have devised GQT, which is frequently quoted in papers concerning generalized quantifiers, exemplified by *most* and *many*. Ever since then there has been rapid development on generalized quantifiers, such as Westerståhl's (1989) work. From a slightly different perspective Keenan and Stavi (1986) among others, explore determiners, mainly in terms of their semantic properties. In logic, van Benthem (1984) proposes a logic with a provision for natural language quantification.

5.1.1 Semantic universals

A quantifier is interpreted as a set of sets, thus called a generalized quantifier. It should be noted that a quantifier in GQT is a combination of a determiner and a noun (e.g. *more than 200 + people*), and properties of quantifiers may not apply to a determiner alone. This differs from standard practice in traditional logic, where quantifiers are associated only with determiners.

A *generalized quantifier* is defined as follows (van Benthem, 1982: 61).

By a *generalized quantifier*, we mean a functor D assigning, to each set E , a binary relation $D_E AB$ between subsets A, B of E . (5.1)

Research on generalized quantifiers has explored a number of semantic properties. Here, we will discuss four of the fundamental semantic universals with respect to FQs: conservativity, extension (constancy), quantity, and variation, based on Cann (1993) and Partee, Meulen and Wall (1990). The original proposal given in Barwise and Cooper (1981) on these universals has been revised extensively. The version discussed below is relatively less controversial.

U1: Conservativity

A quantifier is conservative, if it is a function which assigns a set of subsets of E with the property $B, B \subseteq A$. This can be defined in (5.2), and exemplified in (5.3).

$$\text{If } A, B \subseteq E, \text{ then } D_E AB \leftrightarrow D_E A(A \cap B). \quad (5.2)$$

- a. Several students left iff several students are students who left.
 - b. About 200 students left iff about 200 students are students who left.
- (5.3)

Conservativity: Every natural language quantifier is conservative. (5.4)

The definition in (5.2) requires that the extension of the VP denoted by B is contained in the set of sets denoted by $D(A)$. That is, in (5.3a), if *leave'* is in the set of *several' (student')*, then *several' (student')* contains the intersection of the extension of *student'* and *leave'*. The same analysis applies to (5.3b).

It is assumed in (5.4) that for any quantifier, if it is a natural language quantifier, then it is conservative. Take *Many students left* as an example; it is analytically equivalent to *Many students are students who left*. It might be felt that this universal is too obvious to be significant, but as pointed out by Cann (1993) it captures the fact that the denotation of natural language

quantifiers is not logically necessary. Also, conservativity rules out many logically possible quantifiers, and ensures that the interpretation of a quantifier containing a common noun is not affected by those sets of entities not in the extension of the common noun (see Cann (1993: 192-193) for examples).

Conservativity claims that for the interpretation of an FQ, only part of E is relevant. For instance, for the interpretation of a proposition *About 200 students left*, all we need to know is whether there is a subset of E in which there are about 200 students who left. In this case, sets like *apples* and *earthquakes* in E are not our interest.

U2: Extension (Constancy)

A quantifier is extensional or constant if the extensions in a domain do not affect the interpretation of the quantifier. For instance, if the proposition *Every student left* is true, then it is still true if we add more earthquakes or apples to the domain of that model, or take away some from it. This is represented in (5.5).

$$\text{If } A, B \subseteq E \subseteq E' \text{ then } D_E AB \leftrightarrow D_{E'} AB. \quad (5.5)$$

What this universal says is that if we want to interpret a quantified proposition formed with a $NP + VP$, then anything else that does not belong to the extensions of the common noun or the VP can be ignored. For instance, if we want to check the truth value of *A few students left*, the size of other sets in the domain is irrelevant, say the set of apples. The truth value is only dependent on the number of students who left, but not on the number of apples.

If Conservativity and Extension are the case, then the two amount to another condition: Strong Conservativity. This can be defined in (5.6).

$$\text{If } A, B \subseteq E \text{ then } D_E AB \leftrightarrow D_A A(A \cap B). \quad (5.6)$$

Strong Conservativity says that the size of the domain is irrelevant in interpreting a quantifier, and only A (*common noun*) and B (*VP*) are relevant. This is built up on the assumption that both Conservativity and Extension are the case.

U3: Quantity

Quantity claims that the interpretation of a quantifier is only determined by the number of elements in the relevant sets, and the nature of these elements is not relevant. This can be represented as in (5.7).

$$\text{If } F \text{ is a bijection from } M_1 \text{ to } M_2, \text{ then } D_{E_1}AB \leftrightarrow D_{E_2}F(A)F(B). \quad (5.7)$$

Quantity requires sameness of interpretation up to isomorphic models. This condition appears workable with FQs. For instance, *About 20 students left* is true if and only if we know how many students left. The matter of who they actually are is not relevant, since the truth value of the proposition expressed by the proposition is not affected by the fact that John is chosen, rather than Jim, as one of the students in the set who left.

U4: Variation

Variation claims that when more entities are added to a domain in a given model, there can be some set in the domain which is not generated by a quantifier. This is defined in (5.8).

$$\begin{aligned} &\text{For each domain } E \text{ there is a domain } E' \text{ such that } E \subseteq E', \\ &A, B, \text{ and } C \subseteq E', \text{ such that } D_{E'}AB \text{ and } \sim D_{E'}AC. \end{aligned} \quad (5.8)$$

It is shown that set C does not associate at all with set A by D . What this condition tells us is that quantifiers in natural language must comply with the requirement that a quantifier does not generate any sets arbitrarily.

That is, the Principle of Compositionality has to be followed in generating the meaning of a quantifier (see Section 4.2 for further discussion).

The four universals, Conservativity, Extension, Quantity, and Variation, define a generalised quantifier. It appears that one of the four constraints is particularly robust and valid: Conservativity (Cann, 1993). Our discussion above, where FQs are used as examples, shows that FQs fit all the four universals.

5.1.2 Monotonicity

Monotonicity is one of the central issues in GQT, and it presupposes all the four universals discussed in the last section. Monotonicity characterises the properties of different subsets generated by quantifiers. For instance, if *Every student left* is true in some model, then *every*'(student')(leave') is true whether or not more people who left are added to the model. The proposition is still true if we take away one of the students, or even a subset of the students. However, the truth value will be affected if we add more students to the model, should this be the case that some students added are not in the set of *leave*'. What will be demonstrated below is how monotonicity works with FQs.

In general, a quantifier in the formula $D(N)(VP)$ is monotone increasing if the adding of more entities to the N or VP has no effect on the truth value of the formula. On the other hand, a quantifier is monotone decreasing if subtraction of entities from either of the two sets does not affect the truth value of the formula. The property of monotonicity is formally defined in Barwise and Cooper (1981: 184-185) as,

A quantifier Q is *monotone increasing* ($mon \uparrow$) if $X \in Q$ and $X \subseteq Y \subseteq E$ implies $Y \in Q$ (i.e. for any set $X \in Q$, Q also contains all the supersets of X). (5.9)

A quantifier Q is *monotone decreasing* ($mon \downarrow$) if $X \in Q$ and $Y \subseteq X \subseteq E$ implies $Y \in Q$ (i.e. for any set $X \in Q$, Q also contains all the subsets of X). (5.10)

The definitions show how truth values are affected by an increase or a decrease of the number of entities in the extension of an expression. To test an NP for monotonicity, let VP_1 and VP_2 be two verb-phrases that the extension of VP_1 is a subset of the extension of VP_2 . We aim to ascertain if the assumption in (5.11) is logically valid:

- a. If $NPVP_1$, then $NPVP_2$. (NP is *mon* \uparrow)
 b. If $NPVP_2$, then $NPVP_1$. (NP is *mon* \downarrow) (5.11)

Example 1: (5.11a) is valid, where VP_1 is *are men*, and VP_2 is *are human*.

- If $\left[\begin{array}{l} \text{all students} \\ \{ \text{most students} \\ \text{about 200 students} \end{array} \right]$ are men,
 then $\left[\begin{array}{l} \text{all students} \\ \{ \text{most students} \\ \text{about 200 students} \end{array} \right]$ are human. (5.12)

Notice that the reverse implication does not hold, since there could be a person who is human but not a man. The validity of these implications follows from the fact that these NP's are $mon\uparrow$. To exhibit $mon\downarrow$ NP's, we examine (5.13).

$$\begin{array}{lcl}
 \text{If} & \left[\begin{array}{l} \text{no students} \\ \text{few students} \\ \text{neither student} \end{array} \right] & \text{is/are human,} \\
 & & \\
 \text{then} & \left[\begin{array}{l} \text{no students} \\ \text{few students} \\ \text{neither student} \end{array} \right] & \text{is/are a man/men.}
 \end{array} \tag{5.13}$$

In this example, VP_2 swaps place with VP_1 , compared to propositions containing $\text{mon}\uparrow$ NPs in (5.12).

In (5.12), it is shown that an NP is $\text{mon}\uparrow$, if $NPVP_1$, then $NPVP_2$, where VP_1 is a subset of VP_2 . For instance, if *About 200 students are men*, then *About 200 students are human*. On the other hand, in (5.13) an NP is $\text{mon}\downarrow$, if $NPVP_2$, then $NPVP_1$. For instance, if *Few students are human*, then *Few students are men*.

Furthermore, monotonicity can be examined in two types: subject monotone and predicate monotone, as in Cann (1993: 193-194). A quantifier is subject monotone if the truth value of a quantified formula is unaffected by an increase or decrease in its common noun extension. A predicate monotone quantifier is defined in that the truth value of a quantified formula is unaffected by an increase or decrease in its VP extension. Strictly speaking, the definitions given in (5.9) and (5.10) above are for predicate monotone quantifiers only.

Formula (5.14) below gives definitions of the four properties, (5.15) gives inference patterns validated by the definitions in (5.14). Then the validity and invalidity of (5.14) are illustrated in (5.16), (5.17) and (5.18) by examples from all three types of FQs.

- a. A quantifier $D(N)$ is *subject monotone increasing*
iff $D(N_1)VP \rightarrow D(N_2)VP$, where $N_1 \subseteq N_2$.
- b. A quantifier $D(N)$ is *subject monotone decreasing*
iff $D(N_1)VP \rightarrow D(N_2)VP$, where $N_2 \subseteq N_1$.
- c. A quantifier $D(N)$ is *predicate monotone increasing*
iff $D(N)VP_1 \rightarrow D(N)VP_2$, where $VP_1 \subseteq VP_2$.
- d. A quantifier $D(N)$ is *predicate monotone decreasing*
iff $D(N)VP_1 \rightarrow D(N)VP_2$, where $VP_2 \subseteq VP_1$.

(5.14)

- a. $\phi(\chi \wedge \psi)(\varphi) \rightarrow \phi(\chi)(\varphi)$
- b. $\phi(\chi)(\varphi) \rightarrow \phi(\chi \wedge \psi)(\varphi)$
- c. $\phi(\chi)(\varphi \wedge \psi) \rightarrow \phi(\chi)(\varphi)$
- d. $\phi(\chi)(\varphi) \rightarrow \phi(\chi)(\varphi \wedge \psi)$

(5.15)

- a. If several Chinese students left, then several students left.
- b. * If several students left, then several Chinese students left.
- c. If several students left early, then several students left.
- d. * If several students left, then several students left early.

(5.16)

- a. If about 200 Chinese students left, then about 200 students left.
- b. *If about 200 students left, then about 200 Chinese students left.
- c. If about 200 students left early, then about 200 students left.
- d. *If about 200 students left, then about 200 students left early.

(5.17)

- a. *If fewer than 200 Chinese students left, then fewer than 200 students left.
- b. If fewer than 200 students left, then fewer than 200 Chinese students left.
- c. *If fewer than 200 students left early, then fewer than 200 students left.
- d. If fewer than 200 students left, then fewer than 200 students left early.

(5.18)

Let me explain the inference patterns in (5.15), taking (a) as an example. If we match it with the proposition *If several Chinese students left, then several students left* in (5.16a), then it would be like this: $\phi(\text{several})(\chi(\text{students}) \wedge \psi(\text{Chinese}))(\varphi)(\text{left}) \rightarrow \phi(\text{several})(\chi)(\text{students})(\varphi)(\text{left})$. Next, we consider (5.15d) and the proposition *If fewer than 200 students left, then fewer than 200 students left early* in (5.18d). That is, $\phi(\text{fewer than } 200)(\chi)(\text{students})(\varphi)(\text{left}) \rightarrow \phi(\text{fewer than } 200)(\chi)(\text{students})(\varphi(\text{left}) \wedge \psi(\text{early}))$. The same principle applies to the rest of inference patterns given in (5.15).

In (5.16), (5.17) and (5.18), those propositions that do not have certain properties are indicated by an asterisk. Each illustration contains four proposition correlating to the four properties in (5.14). For instance, in (5.16) and (5.17), *several* and *about* propositions are both subject and predicate increasing and neither is monotone decreasing. However, in (5.18) propositions with *fewer than* are monotone decreasing for both subject and predicate, and has no monotone increasing property at all.

FQs may be sorted corresponding to different inference patterns to which they give rise. Some Type I FQs are displayed in Table 5.1 below, where they are examined in terms of the four properties, defined in (5.14) above.

TABLE 5.1: Four properties of FQs in Type I

FQs	subject monotone increasing	subject monotone decreasing	predicate monotone increasing	predicate monotone decreasing
very many	?	No	Yes	No
a lot	?	No	Yes	No
many	?	No	Yes	No
quite a lot	?	No	Yes	No
quite a few	?	No	Yes	No
a few	Yes	No	Yes	No
several	Yes	No	Yes	No
not many	No	?	No	Yes
few	No	?	No	Yes
very few	No	?	No	Yes

It is shown in Table 5.1 that for the two FQs in the middle, *a few* and *several*, their properties are relatively easy to identify. However, for other FQs it is less obvious whether or not they possess certain properties. This goes back to the claim that FQs which may be used to denote an interval rather than proportion are less fuzzy (see Section 3.2.1 for more discussion). *A few* and *several* can be used in this way (i.e. they may denote a meaning that is less proportional), therefore they are less uncertain. For instance, we tend to interpret *several* as a number, say six, rather than 5%. On the other hand, with other FQs in Type I, a percentage is often used. Take *many* as an example; it is unclear whether or not it is subject monotone increasing in the proposition like *If many Chinese students left, then many students left*. The reason is that *many Chinese students left* does not necessarily entail *Many students left*¹. There may be a situation in which *many* in the first clause is counted as a small proportion of students in the second clause, i.e. the number of Chinese students who left is proportionally greater than that of other students of non-Chinese nationality. But it does not guarantee that

¹If the extensions of common nouns (*Chinese students* vs *students*) are equivalent, then the proposition is monotone increasing. The problem is that we normally do not know whether or not they are (see Section 7.3 for further discussion on this).

the number of students who left was more than that of students who did not leave. Those FQs with a question mark in Table 5.1 are semantically more complex than FQs without.

It shows that all FQs listed above *a few* and *several*, i.e. *very many*, *a lot*, *many*, *quite a lot* and *quite a few*, are predicate monotone increasing only, less obvious in terms of subject monotone increasing, and have no monotone decreasing property of any sort. They are all positive FQs. *A few* and *several* are positive as well, but it is clear that they are both subject and predicate monotone increasing. Below *a few* and *several* are negative FQs; they are *only a few*, *not many*, *few*, and *very few*, all predicate monotone decreasing. Again, it is not clear if they are subject monotone decreasing.

A preliminary conclusion may be drawn here. In terms of FQs in Type I, all the positive FQs are monotone increasing, and all the negative FQs are monotone decreasing. Let us check this with FQs in Types II and III in Table 5.2 below.

TABLE 5.2: Four properties of FQs in Types II and III

FQs	subject monotone increasing	subject monotone decreasing	predicate monotone increasing	predicate monotone decreasing
about n	Yes	No	Yes	No
n-odd	Yes	No	Yes	No
n or so	Yes	No	Yes	No
nearly n	Yes	No	Yes	No
more than n	Yes	No	Yes	No
at least n	Yes	No	Yes	No
fewer than n	No	Yes	No	Yes
at most n	No	Yes	No	Yes

Similar to Table 5.1, Table 5.2 shows the four properties of FQs in Types II and III. It verifies the conclusion that all positive FQs are monotone increasing, and all the negative FQs are monotone decreasing. Compared with Table 5.1, FQs in Table 5.2 are simpler in terms of identification of the four

properties. As opposed to *more than n* and *at least n*, *fewer than n* and *at most n* are negative and monotone decreasing in both subject and predicate. It appears that all FQs in Type II, plus two positive FQs in Type III, are monotone increasing in both subject and predicate. Interestingly, although distributional shapes of the four FQs in Type II, *about n*, *n or so*, *n-odd* and *nearly n*, are different as illustrated in Chapter 3 by our empirical data in Chinese and English, they behave similarly in terms of monotonicity. This implies that different shapes of FQs do not affect their unity in terms of the inferential pattern, as they do not affect compositionality of FQs (see Section 4.2 for the details).

Quantifiers analysed so far are all FQs, and there might be some interesting difference between FQs and non-FQs. Thus, three typical non-FQs are tested with the four properties in Table 5.3.

TABLE 5.3: Four properties of non-FQs

FQs	subject monotone increasing	subject monotone decreasing	predicate monotone increasing	predicate monotone decreasing
every	No	Yes	Yes	No
a	Yes	No	Yes	No
no	No	Yes	No	Yes

As shown, one exception to the claim made with FQs above is that although *every* is positive, it is subject monotone decreasing. However, the claim that all positive quantifiers are monotone increasing is still valid as far as FQs are concerned.

Comparing Table 5.3 with Table 5.1 and Table 5.2, there are three interesting points. Firstly, non-FQs all have two properties of the four, but some FQs have only one (e.g. all type I FQs, except *a few* and *several*, shown in Table 5.1). This might be due to the uncertainty of FQs' meanings. Secondly, *every* has two opposite properties: subject decreasing and predicate increasing. It thus seems inaccurate if we simply say that *every* is monotone increasing and a specification of predicate monotone increasing is more

appropriate. In terms of FQs having two opposite properties, this hardly happens, i.e. an FQ normally would be either monotone increasing or monotone decreasing, but not both. Again, this might be due to their imprecise semantics, i.e. they are not universal quantifiers like *every*. Thirdly, most Type I FQs do not have the property of subject monotonicity except *a few* and *several*, but it is not the case for other types of FQs and non-FQs. As far as FQs in Type I are concerned predicate monotonicity is a more relevant property and subject monotonicity is less interesting.

Monotonicity reveals the kind of inference licensed by quantifiers, illustrated in (5.19). If (a) is true, due to the property of *a few* ($\text{mon}\uparrow$) (d) follows from (b); whereas the property of *few* ($\text{mon}\downarrow$) does not imply that (e) follows from (c).

- a. All politicians are polite.
 - b. A few men are politicians.
 - c. Few men are politicians.
 - d. A few men are polite.
 - e. Few men are polite.
- (5.19)

Some non-FQs, like *exactly 20*, are not monotone quantifiers. In English, all phrases in the form of *exactly n* are assumed to be non-monotone. However, Barwise and Cooper (1981) claim that quantifiers like *exactly 20* can be expressed as a conjunction of expressions which are monotone. For instance, *exactly 20* could be considered as a conjunction of *not more than 20* and *not less than 20*. This suggests that any quantifiers might be considered monotone, because a quantifier, like *exactly 20*, can be expressed as a conjunction of other quantifiers.

There is some uncertainty about whether an FQ is monotone increasing or monotone decreasing. For instance, Moxey and Sanford (1993b: 73) think that *a few* is monotone increasing. But, Barwise and Cooper (1981) are less sure about it and think that FQs like *a few* have two readings: *some but not many* and *at least a few*. If the former is the case, then *a few* is not monotone; if the latter is the case, then it is $\text{mon}\uparrow$. This also applies to *several*. It appears to me that *a few* is monotone increasing, e.g. *If a few*

students are men, then a few students are human, no matter which of the two readings is the case. This kind of disagreement shows that the monotone types of inferences are not analytical, since our intuitions play an important part in it (see Section 7.3 for further discussion on this).

Monotonicity gives rise to a number of semantic universals. As an illustration compound FQs formed, for example by *and*, permit only those FQs with the same monotone direction. For instance,

- a. Many students and several lecturers left.
 - b. *Many students and few lecturers left.
 - c. Few students and fewer than 3 lecturers left.
 - d. *Few students and at least 3 lecturers left.
- (5.20)

In (5.20), (a) has two conjoined FQs, *many students* and *several lecturers*, of which both are monotone increasing. However, (b) sounds odd, because *few* is monotone decreasing. The same applies to (c), where both FQs are monotone decreasing, and (d), where the requirement of the same monotone direction is not fulfilled.

The constraint can be formulated as in (5.21) (Barwise & Cooper, 1981),

Co-ordination constraint: A compound FQ formed by conjunction *and* or disjunction *or* must contain FQs with the same monotone direction.

(5.21)

According to (5.21), any combination containing a decreasing quantifier and an increasing one is excluded on a semantic ground. This constraint is expected to be valid for all compound FQs with *and/or*. The validity can be tested by checking through FQs in Table 5.1. For instance, the combination of *very many* and *a few* (e.g. *Very many students and a few lecturers left*) and *not many* or *only a few* (e.g. *Not many students or only a few lecturers left*) are fine, because the two FQs combined are monotone in the same

direction. If this requirement is not fulfilled, then two FQs are not likely to be conjoined. For instance, the combinations of *very many and few* and *not many or a lot* sound odd. So, in Type I FQs the constraint in (5.21) indeed works. Type II FQs in Table 5.2 are simpler, because they are all in the same monotone direction. With Type III FQs in Table 5.2, the constraint works as well as it does with FQs in Type I. Finally, both conjunction and disjunction preserve monotonicity. For instance, the compound FQ, *many students and a few lecturers*, must also be monotone increasing. The same argument applies to disjunction of FQs.

How well the constraint works with a compound FQ formed by FQs from different types has still to be questioned, however. It seems that the combination of *about n and few* (e.g. *About 5 students and few lecturers left*) is workable as is that of *fewer than n and a few* (e.g. *Fewer than 20 students and a few lecturers left*). But, this validity is against the constraint set in (5.21), since the two FQs in both combinations are monotonely opposite. Barwise and Cooper (1981: 217) claim that this kind of variation is difficult to explain. Thus, it appears that the constraint in (5.21) applies only to the FQs of the same type.

5.1.3 Comments on GQT

GQT is certainly better compared with the first-order predicate calculus which can only deal with few atypical quantifiers like *all* and *some*. As Barwise and Cooper (1981) point out, there are two respects in which the standard first-order logic is inadequate in dealing with the quantified propositions. First, natural language quantifiers are too complex to be symbolized by either \forall or \exists . For instance, the meaning of *many students* cannot be represented by \exists in a satisfactory way. Secondly, there is a big gap between the syntactic structure of quantified propositions in predicate calculus and that in natural language. As an illustration, for *Many students left*, it fits neither \rightarrow for \forall , nor \cap for \exists . On the other hand, GQT permits a logical syntax to relate more closely to a natural language syntax. With GQT the serious limitation of first-order logic can be overcome by a higher-order concept denoted by a generalised quantifier. Also, it can treat a wider range of

natural language quantifiers.

Van Benthem (1982: 61) comments: "Generalized quantifiers' discover a whole fine-structure of Montague Grammar, so to speak. ... most conspicuously, general conjectures have been formulated about the occurrence of, or connections between, certain types of determiner in all human languages. This development of 'semantic universals' not only enriches the Montagovian fund of semantic themes, it also provides a promising rapprochement between earlier 'fragmentary' approaches in formal semantics and more common 'global' linguistic habits of semantic description." I subscribe to the point made by van Benthem that Barwise and Cooper's interest is located at the level of "meaning postulates", in between global "categorical fit" and elaborate lexical details.

Moxey and Sanford (1993b:107) also comment on Barwise and Cooper's work:

"...they concentrate upon specifying how natural language expressions fit into the Generalized Quantifier Theory, which is about the kinds of inferences licensed by expressions, rather than being about the (numerical) values that may be associated with expressions. If a particular natural language expression (quantifier) denotes a state of affairs that corresponds to some category within the theory, then it is possible to say what inferences it licenses: That is, what necessarily true things follow if the quantified assertion is true."

Barwise and Cooper's approach is indeed a promising one in analyzing the properties like semantic universals and monotonicity. However, it has its limitation. Although the semantic universals tackle the semantics of generalized quantifiers with some success, it assumes a bivalent theory of truth. That is, a proposition is either true or false. It is then difficult for GQT to capture the degree of truth represented by propositions containing an FQ.

Any propositions which contain an FQ have a degree of truth, such as the proposition *Many people are girls*. GQT has no provision for the degree of

truth. In order to deal with fuzziness, GQT confines that any interpretations of generalized quantifiers should be done in a fixed context. The problem with FQs is that even within a fixed context their interpretations may still be fuzzy.

In conclusion, GQT is not totally incompatible with FST approach, particularly the idea of a set of subsets, but it needs to be determined how fuzziness, e.g. degree of truth, can fit in GQT (see Section 7.1 for further discussion on this). This is where FST comes in, which is the issue discussed next.

5.2 Fuzzy Set Theory

For the past two decades, GQT has been studied extensively in linguistics. but FST (Zadeh, 1965, 1971-73, 1983) is limited in its influence, although it is well-established in many other fields. Ever since the 1960s, the emergence of FST has provoked new insights into our study of fuzzy phenomena. FST has extended theories developed on the basis of Zadeh's original ideas; it also has many versions. I intend to introduce Zadeh's basic ideas and normal version, and importantly, show how they work with FQs linguistically.

5.2.1 The definition

FST is a fuzzy set-based theory with the basic idea that, instead of either being in a set or not, an individual is in the set to a certain degree, say some number between zero (non-membership) and one (full membership), notated as $[0, 1]^2$. This is called a membership (characteristic) function.

FST can deal with a proposition with fuzzy characteristics. With respect to a fuzzy predicate, FST is concerned with the degree of satisfaction of *gradual* properties expressed by the fuzzy predicate, such as *about 20* and *red*, graded

²Smithson (1987: 300) proposes that the value may also take on other kinds, such as linguistic values.

on the scale $[0, 1]$. For instance, the proposition *Mary is about 20 years old* might be neither totally true, nor totally false, but true to a degree notated by some number between 0 and 1.

Formally, a *fuzzy set* is defined as follows (Zadeh, 1965, 1975):

In a universe of discourse $E = \{x, \dots\}$, a fuzzy set A , $A \in E$, can be characterized in terms of a set of ordered pairs $\{x, \mu_A(x)\}$, where $\mu_A(x)$ is understood to be the degree of membership of x in A . $\mu_A(x)$ is usually taken to have values in the interval $[0, 1]$.

(5.22)

An element of the fuzzy set A can, in this way, be designated by the ordered pair:

$$\langle x, \mu_A(x) \rangle \quad (5.23)$$

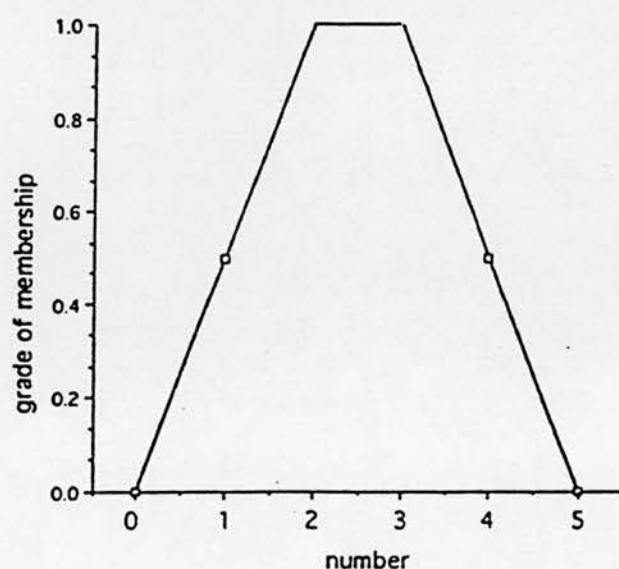
When A in (5.23) is a non-fuzzy set, its membership function can take on only 1 and 0 corresponding to whether x does or does not belong to A , respectively. So, FST is a generalization of the conventional two-valued set theory (see Section 8.2.1 for an elaboration on this). Smithson (1987: 9) comments that the definition of a fuzzy set has some intuitive appeal since ordinary common sense presents us with sets which fit this description. For instance, for *about 200*, 215 might be given a membership, say 0.8.

Applying the definition in (5.22) to the semantics of FQs, a FQ, like *2 or 3*, may be considered as a fuzzy set to which individuals belong to a certain degree. *2 or 3* can be represented in terms of its membership function $\mu_{2 \text{ or } 3}(x)$. Instead of taking just two values, 1 or 0, $\mu_{2 \text{ or } 3}(x)$ can take values in the interval $[0, 1]$, each representing a degree of membership. Note that the nearer the value of $\mu_{2 \text{ or } 3}(x)$ to 1, the higher the grade of membership of x ; this has been the case in my Chinese data (see Section 3.7 for more discussion).

A hypothetical membership function of *2 or 3* is given here as in Table 5.4, and graphically represented in Fig. 5.1.

TABLE 5.4: *2 or 3*

Numbers(x)	0	1	2	3	4	5
$\mu_{2 \text{ or } 3}(x)$	0	0.5	1	1	0.5	0

FIGURE 5.1: *2 or 3*

It is shown in Table 5.4 and Fig. 5.1 that if a number is fewer than 0 inclusively or greater than 5 inclusively, then it does not belong to *2 or 3* at all. If a number is in the interval between 1 and 4, then it has the property of *2 or 3* to a certain degree. For instance, the number 2 belongs to *2 or 3* to a degree of 1, as does 3; 4 is of *2 or 3* to a degree of 0.5. This membership function indicates the meaning of *2 or 3* in this particular model. It appears that the application of the actual numbers for an FQ is not predictable, rather it is pragmatically determined. However, the application is not arbitrary either. This is because FQs are compositional and motivated, as argued earlier (see Chapter 4 for the details).

5.2.2 Fuzzy set operation

To define a fuzzy set is to specify its membership function. To specify the complement of a fuzzy set or the union/intersection of two fuzzy sets we have to specify how the membership function of the complement, union or intersection is related to the membership function of the fuzzy set(s) from which it is derived. The following definitions are proposed by Zadeh (1965).

- a. Union: $\mu_{A \cup B} = \max(\mu_A, \mu_B)$
 - b. Intersection: $\mu_{A \cap B} = \min(\mu_A, \mu_B)$
 - c. Complement: $\mu_{A'} = 1 - \mu_A$
 - d. Subset: $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ for all $x \in X$.
- (5.24)

These definitions represent the relationship of fuzzy sets to their union, intersection, complement and subset. To see how they work, let us examine Fig. 5.2.

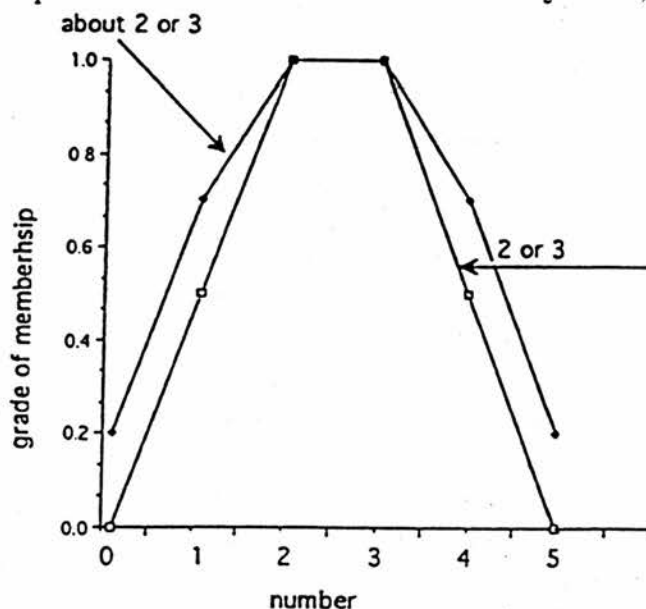


FIGURE 5.2: 2 or 3 and about 2 or 3

Let $A = 2 \text{ or } 3$ and $B = \text{about } 2 \text{ or } 3$. If $\mu_{2 \text{ or } 3}(x) = 0.6$, $\mu_{\text{about } 2 \text{ or } 3}(x) = 0.8$, then the union for them is 0.8, and the intersection is 0.6. The complement

for $\mu_{2 \text{ or } 3}(x)$ is 0.4, for $\mu_{\text{about } 2 \text{ or } 3}(x)$ is 0.2. Given a number in Fig. 5.2, its grade of membership of *2 or 3* is expected to be less than or equal to its grade of membership of *about 2 or 3*. For instance, the number 4 has the property of *2 or 3* to the degree 0.5, whereas it has the property of *about 2 or 3* to the degree 0.7. Also, given the number 3, the membership of both is 1. The trend here is that the membership of *about 2 or 3* for a number in the interval will always be greater than or equal to the membership of *2 or 3* for the number.

It is evident that the formula of (5.24d) is valid. However, it appears that while all other operations of fuzzy sets in (5.24) can have a grade of membership, (5.24d) cannot, which takes only 0 and 1. It would be consistent if it can also take on intermediate values. There are several ways by which this can be done:

(a) Lukasiewicz (described in McCawley 1981: 366):

$$\begin{aligned} |A \subseteq B| &= 1 && \text{iff } |A| \leq |B| \\ &= |B| && \text{iff } |A| > |B| \end{aligned} \quad (5.25)$$

(b) Ronnie Cann (personal communication):

$$\begin{aligned} |A \subseteq B| &= 1 && \text{iff } |A| \leq |B| \\ &= |A| - |B| && \text{iff } |A| > |B| \end{aligned} \quad (5.26)$$

(c) McCawley (1981: 480):

$$\begin{aligned} |A \subseteq B| &= 1 && \text{iff } |A| \leq |B| \\ &= 1 - |A| + |B| && \text{iff } |A| > |B| \end{aligned} \quad (5.27)$$

Or, (5.27) could be represented as in (5.28):

$$|A \subseteq B| = \min(1, 1 - |A| + |B|) \quad (5.28)$$

since $|A| \leq |B|$ is equivalent to " $1 - |A| + |B| \geq 1$ ".

To explain how the formulas work, let us reuse Fig. 5.2 above, but this time we examine the degree that $| \text{about } 2 \text{ or } 3 | (A)$ entails $| \text{about } 2 \text{ or } 3 | (B)$, rather than the other way round as in (5.24d). In Fig. 5.2, to take the number 4, $|A| = 0.7$, and $|B| = 0.5$. Then, $|A \subseteq B| = 0.5$ according to (5.25), 0.2 according to (5.26), and 0.8 according to (5.27).

To make sense of these results, we need to discuss the rationales behind the three formulas. In (5.25), the formula represents the intersection of the membership values of A and B . Formulas (5.26) and (5.27) represent the difference between the values of A and B . Note that (5.26) and (5.27) describe the difference in a reverse way. In terms of (5.26), the more trivial the difference between $|A|$ and $|B|$, shown by the value $|A - B|$, the higher the degree of $|A| \subseteq |B|$. On the other hand, for (5.27), the bigger the value of $(1 - |A| + |B|)$, the higher the degree of $|A| \subseteq |B|$. The values from (5.26) and (5.27) are complementary to each other (e.g. $0.2 + 0.8 = 1$). The three formulas in (5.25), (5.26) and (5.27) all provide a way of representing a degree of set inclusion, making the formulas in (5.24) more consistent. It appears to me that (5.25) is more straightforward than the other two, although all of them can offer a degree of membership. For example, In Fig. 5.2, let $|A| = 0.7$, and $|B| = 0.5$. As shown, from 0 to 0.5, both A and B are true, the minimum of the two values or the intersection of the two sets.

5.2.3 Fuzzy proposition operation

The treatment for fuzzy set operations is also applicable to propositions containing a fuzzy set. In terms of the propositional logic, FST is a generalization of standard propositional logic in that it makes a move from two truth values *false* and *true* to degrees of truth notated in $[0, 1]$. If we take two fuzzy propositions P and Q , and let " \neg ", " \wedge ", " \vee " and " \rightarrow " be connectives, then valuations for the connectives are defined as follows, quoted from Zadeh (1965, 1971):

$$\begin{array}{ll}
 a. & |\neg P| = 1 - |P| \\
 b. & |P \wedge Q| = \min(|P|, |Q|) \\
 c. & |P \vee Q| = \max(|P|, |Q|) \\
 d. & |P \rightarrow Q| = 1 \text{ iff } |P| \leq |Q|
 \end{array}
 \tag{5.29}$$

Note that when $|P|$ and $|Q|$ are both 1 or 0, or one is 1, the other is 0, we would get the classical truth value. That is, if one puts binary values into (5.29), one gets binary values back.

To see how (5.29) works, let us examine Fig. 5.3.

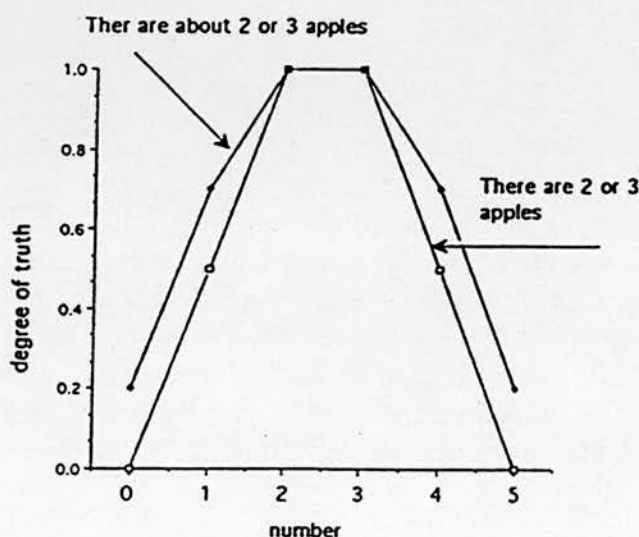


FIGURE 5.3: Two propositions with 2 or 3 and about 2 or 3

Let $P = \text{There are 2 or 3 apples}$, and $Q = \text{There are about 2 or 3 apples}$. To take the number 4 in Fig. 5.3 as an example, $|P| = 0.5$, and $|Q| = 0.7$. Then, the proposition *There are not 2 or 3 apples* is true to the degree 0.5, and *There are not about 2 or 3 apples* is true to the degree 0.3. As a result, *There are 2 or 3 apples and there are about 2 or 3 apples* is true to the degree 0.5—the minimum of 0.5 and 0.7; whereas *There are 2 or 3 apples or there are about 2 or 3 apples* is true to the degree 0.7—the maximum of 0.5 and 0.7.

In terms of entailment³, the formula is $|P \rightarrow Q| = 1$ iff $|P| \leq |Q|$. Thus, here $|P \rightarrow Q| = 1$, since $|P| < |Q|$, i.e. $0.5 < 0.7$. That is, if there are *2 or 3* apples, then there are *about 2 or 3* apples. From the trend shown in Fig. 5.2 above, the grade of membership of *2 or 3* for a given number is always less than or equal to its grade of membership of *about 2 or 3*. We can thus derive that the truth value of a proposition containing the former would be always equal to or less than that of a proposition containing the latter as shown in Fig. 5.3.

It is again worth noting that other formulas in (5.29) all represent a degree of truth, except (5.29d). This invokes the same question discussed about (5.24d) above, i.e. we need a formula to offer a degree of truth for \rightarrow . Godel (described in Rescher 1969: 44) proposed,

$$\begin{aligned} |P \rightarrow Q| &= 1 && \text{iff } |P| \leq |Q| \\ &= |Q| && \text{iff } |P| > |Q| \end{aligned} \quad (5.30)$$

The formula is similar to (5.25) above. This time let $|P| = \text{There are about 2 or 3 apples}$, and $|Q| = \text{There are 2 or 3 apples}$. Take the number 4, as shown in Fig. 5.3, $|P| = 0.7$ and $|Q| = 0.5$. Then $|P \rightarrow Q| = 0.5$, which is the same result from \wedge , i.e. to take the minimum value of the two values, when $|P| > |Q|$. According to (5.30), the greater the value of Q , the greater the value of an entailment. This can be illustrated in Fig. 5.3, where the bigger $|Q|$ is, the more true the entailment.

5.2.4 FST and prototype theory

FST rejects a number of well-known standard theories, such as *The Principle of Bivalence* and *The Law of Excluded Middle*, which have attempted to fit the real world to an all-or-none assumption, giving no place and provision for fuzziness.

³Here, the notion of *entailment* is used in a straightforward logical sense.

Prototype theory developed by Rosch (1973, 1975) has some bearing on FST in that they both have a provision for fuzziness in natural languages. Prototype theory is meant to represent the fuzziness by virtue of defining the relation between a prototype for a fuzzy set and the rest of the elements in the set. A prototype⁴ is, as defined in Smithson (1987: 301), an exemplary member of a set which may be virtual or imagined. The standard notion of a prototype, as stated in Smithson (1987: 58), claims two points: (1) the prototype itself is a member of the category which it represents; (2) the more dissimilar a given item is from a prototype, the lower that item's membership value in the corresponding category.

On this theory, Lyons (1986: 71) says:

"Generally speaking, we operate with what have come to be called *proto-types*, or stereotypes; and usually what we want to refer to conforms to the proto-type. For example, the proto-type for 'dog' might be rather like the Longman definition, ...: 'a common four-legged flesh-eating animal, especially any of the many varieties used by man as a companion or for hunting, working, guarding, etc.'. I have now quoted the definition in full; and it will be observed that the additional part of the definition, running from 'especially' to 'etc.', indicates that there are several varieties of dogs and that some of these fall within the *focal extension* of 'dog' (that is, they are more typical sub-classes of the class than other, non-focal, varieties are). As for the varieties, we could all name a few, and dog-fanciers a lot more: spaniels, terriers, poodles etc. When we say that someone knows the meaning of 'dog', we imply that he has just this kind of knowledge."

Coleman and Kay (1981) assert that an individual is assigned a degree of membership according to how similar it is to a prototype. For instance,

⁴The prototype is termed differently. For instance, it is called an exemplar in Channell (1983). She terms 500 an exemplar in *about 500 pounds*. The prototype is called *paradigm* in Sainsbury (1991).

when we try to judge the applicability of an FQ *about 200*, and if the prototype is the number 200, then we are able to judge individuals through their similarities to the prototypical number 200. The similarities here come in degrees, and the truth value for a proposition containing a fuzzy expression appears to be a matter of degree as well.

However, Chierchia and McConnell-Ginet (1990: 390) say that not all imprecise concepts are graded in terms of similarity to a prototype. For example:

- a. Eloise is tall.
 - b. Peggy is tall.
- (5.31)

They believe that there is no plausible prototype associated with *tall* even with recourse to context. The truth value for the propositions in (5.31) does not depend on how closely Eloise and Peggy resemble the prototype. Suppose that a prototypical tall American woman is 5'9", Eloise is 6'2" tall and Peggy is 5'5" tall, then Peggy's height differs by only 4" from that of the prototype, but Eloise's height differs by 5". Then, it appears that Peggy is more similar in height to the prototype than Eloise is, and what follows is that Peggy is more typical in tallness than Eloise. This result is obviously counter-intuitive. In fact, (a) is absolutely true, but (b) is, at best, *nearly* true. Based on this instance, Chierchia and McConnell-Ginet claim that prototypes are not always relevant when we are dealing with semantic imprecision.

There are two ways by which we can say that prototype theory can actually deal with the case in (5.31). Firstly, allow the prototype to have more than one entity. For instance, a range of heights [5'9", 6'2"...] could all be prototypes. Since Eloise is 6'2" tall, which is in the set of prototypes already, so there is no question that Eloise is tall. On the other hand, Peggy is only 5'5" tall, which differs from the prototype by 4". The upshot then is that Peggy is less similar in height to the prototype than Eloise, because Eloise's height is a prototype itself. So, Peggy is less typical in tallness than Eloise, and we can solve the problem by getting the intuitive result that (a) is truer than (b) in (5.31). The claim that the prototype could be a set of entities is verified empirically in my Chinese data, e.g. *200 ren zuoyou* 'about 200 people' was

given a set of numbers from 195 to 205 as its prototypes (see Appendix 2, Fig. 2.15). Zadeh (1982: 293) also claims that the prototype could be a set of more than one single object. He goes further to say that the prototype could be an infinite collection of objects, which is mathematically plausible.

The second way is to put a constraint on the claim that the closer a given item to a prototype, the higher the item's membership value. The constraint is that where compared items are placed, a prototype must be an end of scale. With this constraint, the claim works universally (see Section 3.7 for more discussion). In (5.31) the prototype does not act as an end of the scale, where Eloise and Peggy are compared. Consequently, the claim that the prototype theory does not work in this case is invalid, as far as the constraint is concerned. That is, in the example given, the scale would be [5'5" (Peggy) ... 5'9" (prototype) ... 6'2" (Eloise)], where the prototype is not the end of the scale.

It is suspected that people do not make judgments about tallness in general. If we ask "*How tall is tall?*", people would feel hard to answer this question. We may have to be more specific, asking questions like "*How tall is tall for a ten year old girl?*". Tallness is a relative concept, it varies from context to context. In a given context, *A* who is 4'8" may be said relatively *tall* if the only compared person is *B* who is 4'5". Also, if we compare two children of three years old, a prototype for them would be much shorter than a prototype for adults. A tall person, a tall building, and a tall tree require different prototypes. Therefore, a prototype or typical value appropriate in a given context cannot be the basis of judgments about tallness in general. Of course, in the case of FQs this claim may or may not be the case. It may be the case for *many*, but not for *about n* type where *n* tends to be regarded as a prototypical number or at least very similar to it.

To conclude: the two theories, FST and the prototype theory both provide an account for fuzziness. They are compatible in that both reveal the property of degree by looking into the relation between a prototype and the rest of the related individuals in the model. They both claim that the closer an individual to a prototype, the higher the membership it has of the set. The prototype in a fuzzy set acts as a norm and influences the membership of the related individuals in the set. Although FST and prototype theory are

not mutually exclusive they employ different methods to represent fuzziness. FST accounts for fuzziness by using intermediate values in $[1, 0]$, which are not part of the means used in prototype theory. FST focuses on the continuous nature of membership, prototype theory on representativeness, as commented by Fuhrmann (1988: 195).

In my work, FST is adapted to explore FQs, membership function in particular. The reason I use membership function is that it is more suitable to represent the meaning of FQs (see next chapter for a detailed demonstration).

5.2.5 Fuzzy grammar

There is a wide range of application of FST in linguistics and a brief discussion of fuzzy grammar is given here. Through the following discussion, it will become evident that FST applies equally well to grammar as to semantics.

Fuzzy grammar has been advocated since the early 1970s (see Matthews 1981: Ch. 1, & Newmeyer 1986: Ch. 5 for more discussion). FST makes grammars capable of generating sentences with a degree of grammatical appropriateness. Fuzzy grammar is based on the assumption that grammar is a non-discrete phenomenon, rather than discrete. For example, Comrie (1989) discusses the issues of fuzzy degree of subjecthood, nouniness and adjectiveness. More recently, Meyer (1992) treats the category of apposition as gradient, because of borderline cases between apposition, complementation and coordination.

The most well-known work on the fuzziness in grammar is Ross's (1973) paper. He discusses category squishes and claims that there is a grey area about the noun phrase. Ross considers the notion of *squish* as an important part of his notion of non-discrete grammar. He states that a lexical item could be placed somewhere on a continuum with verb and noun as the two extremes, and that this kind of continuity also applies to other grammatical categories. As an illustration, Ross provides a hierarchy of *noun-phrasiness* as in Fig. 5.4:

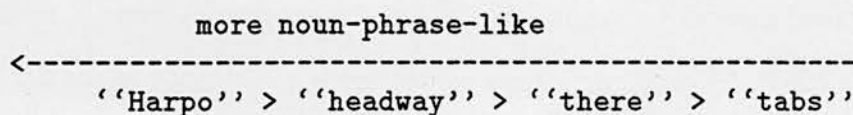


FIGURE 5.4: Ross: nouniness

The “greater than” sign here is to be interpreted as an implication. For any two items, *A* and *B*, if *B* “passes” some test, then *A* will also pass it. In other words, the items under discussion are pseudo-NPs: they are a subset of typical NPs like *Harpo* in Fig. 5.4. This point is compatible with the FST in that a fuzzy expression defines a set of entities with a degree of membership. Ross suggests that the core/patch approach proposed by Morgan (1972) is promising in dealing with syntactical fuzziness. It is suggested that instead of discrete categories we talk about degrees of *nouniness* and *verbi-ness*. Hence, in this theory grammatical categories and rules are considered as having fuzzy boundaries. Hooper (1994) also points out that in dealing with descriptions of the nominalisation types, it is clear that at certain points they form continua rather than discrete categories. Her data from Tokelauan empirically undermines the notion of strict categoriality.

Lakoff (1973b) subscribes to Ross’s claim, suggesting that speakers do not always make clear or uniform judgments about whether a sentence is well-formed or clear in meaning. Almost every syntactic or semantic phenomenon has a shadowy area in which speakers become unclear with respect to judgments about meaning and well-formedness. He points out that Ross has made the following claims in the absence of a theory of fuzzy grammar. No current theory of grammar can even begin to accommodate the facts that Ross has observed.

1. Rules of grammar do not simply apply or fail to apply; but rather apply to a degree.

2. Grammatical elements are not simply members or nonmembers of grammatical categories; rather they are members to a degree.
3. Grammatical constructions are not simply islands or nonislands; rather they may be islands to a degree.
4. Grammatical constructions are not simply environments or nonenvironments for rules; rather they may be environments to a degree.
5. Grammatical phenomena form hierarchies which are largely constant from speaker to speaker, and in many cases, from language to language.
6. Different speakers (and different languages) will have different acceptability thresholds along these hierarchies.

In the framework of cognitive grammar developed by Langacker (1987, 1991) and Lakoff (1987), the notion of prototype categories is assumed, producing degrees of grammatical categoriality like *nouniness*. In addition, Hopper and Thompson (1985) criticize the absoluteness of the categories and claim the fuzziness of the categories by suggesting that grammatical categories are derived from the needs of discourse rather than being ontologically given.

As far as application of FST is concerned, Labov's (1978) work seems to be representative. His work applies the theory to explore non-discrete categorial boundaries. Like his fellow Berkeleyans G. Lakoff and L. Zadeh, David Palmer (*Linguist-list*, Vol-5-190) has been working with a computational model of fuzziness as it applies to corpus analysis, specifically with *fuzzy* part-of-speech categories. He treats a word's POS (part-of-speech) as a series of probabilities based on its occurrence in a corpus of different POS categories. Using degrees of *nouniness* and *verbiness* facilitates corpus analysis by not requiring a discrete POS categorization. He claims that he has obtained encouraging results by applying this approach to sentence boundary disambiguation.

Although syntactic fuzziness is not of concern here, our discussion in the section shows that fuzziness is widely spread, and is indeed a characteristic of natural language. More importantly, it demonstrates that FST works

well to represent fuzziness in grammar. Apart from the application to the semantics of FQs, FST is indeed proved capable of dealing with fuzziness in other linguistic fields.

5.2.6 Development of FST

FST is a favourite with some linguists and logicians for dealing with the semantics of fuzzy expressions in an approximate fashion. Some work has been done by Lakoff (1973a). He proposes a treatment of semantic imprecision by applying FST linguistically. He stresses that one obviously cannot specify the points on a number continuum at which any particular approximation ceases to be true. Putnam (1975) says that if one really wanted to formalize fuzzy expressions, it would be necessary to employ *fuzzy sets* or something similar, rather than sets in the classical sense. McCawley (1981) discusses extensively the application of FST in linguistics, using some numerical values in the interval $[1, 0]$ to describe membership of a fuzzy expression.

Sadock (1977) proposes a way to deal with a proposition containing a fuzzifier like *approximately*, by ignoring the degree of truth generated by *approximately*. Take *approximately 200* as an example. According to Sadock, all the numbers in the assumed applicable domain, say from 150 to 250, would receive the same true value 1, whether a prototype or not. As far as he is concerned, things like the acceptability of an approximation are a matter of pragmatics. It seems to me that Sadock's account does not represent natural language, such as the degree of truth for a fuzzy proposition. For instance, the number 195 is certainly a better example of *approximately 200* than 150. Any adequate semantic theory must take this fact into account. Channell (1983: 205) points out another limitation of Sadock's account. It cannot deal in an acceptable manner with logical operations like entailment. For instance, it appears that *There are exactly 200 people* entails *There are approximately 200 people*. However, because Sadock does not assume that the truth value assignment to the latter proposition could be a matter of degree, there is no point in talking about entailment between the two propositions, i.e. they are logically equivalent. This is counter-intuitive.

FST is not only influential in theories but its application is also widely implemented. Why is FST useful in practice? Generally speaking, because of ubiquitous fuzziness, to design a complex system like a control system flexibility is greatly demanded. FST can represent things in degree, providing a treatment to deal with fuzziness, and consequently producing the much needed flexibility. It is proven in practice that the system using FST can indeed operate in a much more efficient way. Let me give a few examples here. According to Bartjan Wattel's (19th Jan. 1994 on fuzzy-mail network) e-mail report, the first application of FST was done by Dr Mamdani of the University of London, who in 1974 designed an experimental fuzzy control for a steam engine. In 1987, a subway system which was controlled by FST started operating in Sendai, Japan. The FST in this subway system made the journey more comfortable with smooth braking and acceleration. In 1989 Omron Corp. demonstrated fuzzy workstations at the Business Show in Harumi, Japan. This workstation was simply a RISC-based computer equipped with a fuzzy inference board. The fuzzy inference board was used to store and retrieve fuzzy information and to make fuzzy inference. Recently, Sony introduced the Sony Palmtop, which uses an FST decision tree algorithm to perform handwritten (using a computer lightpen) Kanji character recognition. For instance, if one writes 250, the Sony Palmtop can distinguish the number 5 from the letter S.

Since 1965, many researchers have been applying FST to their fields, like Artificial Intelligence-Robotics (Goguen 1975, Uragami *et al* 1976), Image Processing (Chang 1971, Jain & Nagel 1977), Speech Recognition (Coppo & Saitta 1976, Pal & Majumder 1978), Biological and Medical Science (Lee & Lee 1974, Butnariu 1977), Applied Operation Research (Jones 1974, Dubois 1978), Economics and Geography (Gottinger 1973, Taranu 1977), Sociology (Zadeh 1973b, Dimitrov 1977), Psychology (Oden & Anderson 1974, Kochen 1975), Linguistics (Lakoff 1973a, Joyce 1976), Semiotics (Nowakowska 1976, Vaina 1978), Damage Assessment of Structure (Blockley 1975, Blockley 1978), Analysis of Scientific Literature (Allen 1973, Jones 1976). So, it appears that FST is worthwhile exploring both in theory and in practice.

5.2.7 Some questions about FST

The most frequently asked question about FST is the use of precise numbers. As an illustration, if we define *about 20* by the membership function in Table 5.5 below,

TABLE 5.5: *about 20*

Numbers(x)	15	16	17	18	19	20	21	22	23	24	25
$\mu_{20r3}(x)$	0	0.3	0.5	0.7	0.9	1	0.9	0.7	0.5	0.3	0

then the question is why 16 receives 0.3, but not 0.4 or 0.2. In fact, this kind of argument misses the point of FST, since the precise numbers here are hypothetical and for illustration only. The determination of membership function to a specific term has to be treated as an empirical matter, which may vary from individual to individual and from context to context. It is neither a major concern for FST nor for my work here. What I am interested in is the structure of the membership function, and how it varies from one type of FQ to another. Whether 16 receives 0.3 or 0.4 is not a real issue here, provided that *about 20* generates a set of numbers that conforms to the core meaning for *about n*, that is a number approximate to n . To be more specific, it does not really matter whether 16 receives 0.3 or 0.4, as long as the values do not go beyond a certain threshold. For example, 200 would not be an appropriate member for *about 20*. The real problem would be that in a given context one gave 16 a membership of 0.4, but 0 for 19; or 21 for 0, but 24 for 1, because the pattern of basic membership function for *about n* would seem to be violated.

Another concern is about Zadeh's (1972, 1973a) hypotheses on language operators. For instance, Zadeh assumes that the function of *very* is a power function, i.e. $|very P| = |P|^2$. However, in Hersh and Caramazza's (1976) data (see Section 4.3) this was not the case. The subjects gave 0.75 as the grade of membership of the square size 8 of *large*; 0.12 of *very large*. According to Zadeh's power function, however, the value of *very large* should be $|0.75|^2$, i.e. 0.56 rather than 0.12.

It is also argued by Budescu and Wallsten (1990) that those mathematically prescribed operations in FST, shown in (5.24), should not be applied indiscriminately in natural language because they are empirically unsupported. For instance, their data showed that *and* judgments were best fitted by some sort of an average of the two stimulus terms, rather than a minimum operation. Oden (1977a) also claims on empirical grounds that the truthfulness of a conjunction of fuzzy statements is judged to be equal to the product of the truthfulness of its component statements. The falsity of a disjunction of fuzzy statements is judged to be equal to the product of the falsity of its statements. It is believed that these rules provide a better account of the data than rules based on the minimum and maximum truthfulness of the component statements as proposed in FST (see (5.29) above).

However, it appears that even if one were to do empirical studies of standard linguistic judgments, I suspect that there would be variability, as there is with grammaticality judgment. But from a theoretical point of view the empirical variance may be explained by things outside the theory, such as pragmatic factors. The use of empirical data allows the basis of a theory to be laid, but does not provide an absolute testing ground for it. The point is that we should not take the actual value of power function seriously; what should be taken seriously is the relation between *X* and *very X* denoted by the function. The power function is meant to convey the idea that the function of *very* is to intensify the value of the item being modified. The actual exponential could be adjusted according to a given context. For example, it could be 3 in one situation, or 1/2 in another. This goes back to the issue of compositionality of FQs (see Section 4.2 for the details). That is, certain basic membership functions may be subject to modification depending on context, but the core meanings keep constant and the variance is motivated, which make the semantics of FQs operative, functional, and more importantly, sensible.

The conclusion is that the precise numbers adopted in FST should not be taken seriously. The real power of FST is that it generates, through the notion of membership function and degree of truth, semantic and inference patterns of fuzzy expressions that capture human and linguistic behaviours. In particular, its basic idea gives rise to new insights in the study of FQs, and it provides a means for dealing with fuzziness. What makes FST so striking is its non-standard treatment of fuzzy sets—an individual may be in a set to

a certain degree and so a proposition containing a fuzzy term may be true to an extent.

Finally, there is a question about whether or not FST can be limited to a relatively small finite number of values. I agree generally with Lakoff's (1973a: 492-493) idea that the fuzzy logic function is continuous in principle. However, because of our limited perceptual capacity in a given model the function shall be considered to locate, in an approximate manner, a finite set. This becomes a significant point when we come to distinguish a paradoxical case from a fuzzy case.

It has been proposed that the question of fuzziness can be considered in the form of the paradoxes of *sorites* (e.g. *heap*) and *falakros* (e.g. *a bald man*), such as in Hersh and Caramazza (1976). For example, the criterion for the concept of *a bald man* is not sharp enough to enable us to decide whether a man is bald or not. That is, if a man has 10,000 hairs he is not bald, then if he loses one hair ($10,000 - 1 = 9,999$) he is still not considered to be bald. If we let this repeat 9,999 times then he would have no hair at all, but still not be bald.

In fact, the similarity between a classical paradox like *a bald man* and fuzziness is that both raise difficulties for the Law of the Excluded Middle, because in both cases there is no clear-cut boundary to be drawn. However, it seems to me that the two phenomena are not the same. Consider the example of *a bald man* again. The situation where a man is still not bald even though he has no hair at all would not be the case in terms of fuzziness, because the concept shall only be fuzzy to an extent. There is a threshold beyond which the concept would not be fuzzy at all. That is, there is a tolerance rule in defining a fuzzy term which sets the threshold for the fuzzy term. For instance, if a man has no hair at all he is definitely bald, which is not fuzzy at all. The claim that the variance of fuzzy terms does not go beyond certain divisibility is a condition for compositionality of FQs (see Chapter 4 for the details). Thus, it is argued that the FST is capable of making a finite set concerning a particular context which draws a line between paradox and fuzziness.

5.3 Conclusion

In this chapter we have discussed the application of GQT and FST to FQs. It appears that FQs are similar to general quantifiers in terms of semantic patterns and structural relations. In particular, the idea that a quantifier is a set of subsets is applicable for FQs, which will be incorporated into my formal treatment of FQs in the following chapters. However, GQT cannot be employed here as my approach since it assumes the theory of bivalent truth, hence lacks an explicit provision for fuzziness that FQs convey.

FST is chosen as my approach for its provision of fuzziness—membership function and degree of truth. It is argued that precise numbers used in FST is not a real problem. The adoption of numbers, as far as a formal semantics is concerned, has significance only with respect to semantic patterns, i.e. the number is not significant via the number itself, but in terms of the way it is ordered.

Since it appears that any fixed values in functions or operations of FST cause suspicion on empirical grounds, what could be done perhaps is to leave specific values to be determined in a given context, i.e. do not use precise numbers to define a general operation, which will be the case in my formal framework (see Chapter 6 for the details).

6

Formal semantics of FQs (II)

As mentioned earlier, the problem with the conventional approach is that it leaves FQs, such as *about n*, as an unanalysed primitive and does little to capture the meaning of it. As Klein (1980) points out, the treatment utilizes a semantic metalanguage in which FQs occur. This kind of treatment is not particularly informative or useful. For decades fuzziness has been a neglected area in linguistics, partly because of its complexity and inconvenience for classical, all-or-none, approaches. In this chapter, a framework for formal semantics of FQs will be discussed. Models here aim to capture intuitions about the properties and relations of FQs. The ultimate goal is to provide a rigorous and intuitive formal semantics for FQs.

Let us reconsider the three types of FQs that are being dealt with in this thesis. The three are listed in Table 6.1, reproduced from page 21.

TABLE 6.1: Types

- Type I: few, a few, many, a lot
- Type II: about n , n or m , n -ish, nearly n , n or so, n -odd
- Type III (semi-FQs): fewer than n , more than n , at least n .

The three types will be dealt with in turn. It should be noted that FQs in Type III differ from expressions like *more N than M* which is a two-place determiner. Type III FQs in my terms are all one-place determiners.

Furthermore, in the case of FQs, a determiner consists of two subclasses: atomic determiner and compound determiner. The structure of FQs is represented in Fig. 6.1.

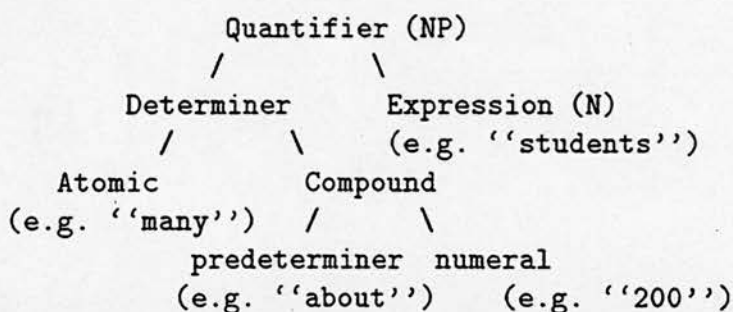


FIGURE 6.1: The Structure of an FQ

In Fig. 6.1, we can see that an FQ with a compound determiner has more a complex structure than an FQ with an atomic determiner.

The symbols of L_{FQ} (formal language of FQs) used here are:

- | | | |
|--------------------------|---|-------|
| 1. constants: | a, b, c, \dots | |
| 2. variables: | x, y, z, \dots | |
| 3. set expressions: | A, B, C, \dots | |
| 4. predicates: | X, Y, Z, \dots | |
| 5. formulas: | ϕ, φ, \dots | (6.1) |
| 6. operators: | $\subseteq, \in, =, \cap, \cup, \dots$ | |
| 7. some of the following | non-logical determiners: <i>many, about n, more than n, ...</i> | |

Next, we need to have syntactic formation rules to define a translation for FQs and specify the procedure for translating fuzzy quantifications. Then,

we need to have interpretation rules to provide a formal semantic treatment for FQs and propositions containing them.

6.1 Syntactic formation rules

We define four kinds of expressions of L_{FQ} : *set expressions*, *determiners*, *FQs*, and *formulas*.

- G1. A, B, or C is a set expression.
- G2. If ϕ is a formula, and x is a variable then $\lambda x[\phi]$ is a set expression.
- G3. If D is primitive, then D is an atomic determiner (AD).
If $D = \text{predeterminer} + \text{numeral}$, then D is a compound determiner (CD).
- G4. If D is a determiner (AD or CD) and A is a set expression, then $D(A)$ is an FQ.
- G5. If R is an n-ary relation symbol and i_1, \dots, i_n are constants or variables, then $R(i_1, \dots, i_n)$ is a formula. Similarly, if B is a set expression and i is a variable or constant then $B(i)$ is a formula.
- G6. If FQ is a fuzzy quantifier and C is a set expression, then $FQ(C)$ is a formula.
- G7. If ϕ and φ are formulas, then $\phi \cap \varphi, \phi \cup \varphi, \neg\phi/\neg\varphi$, and $\phi \rightarrow \varphi$ are all formulas.

(6.2)

We provide seven syntactic formation rules in (6.2). Set expressions are formed by G1 and G2. Determiners are formed by G3: ADs correspond to determiners in Type I FQs (e.g. *many*); and CDs are the determiners in Types II and III (e.g. *about(20)*, *more_than(20)*). G4 forms an FQ, i.e. given a set expression A and a determiner D, we write an FQ, $D(A)$. Finally, formulas are built up by G5-G7, where G7 are combinations of quantified propositions with FQs.

6.2 Semantic rules

Having specified syntactic formation rules for L_{FQ} , here are semantic rules. A model can be built that represents precisely what events, properties and relations make up the situation being modelled, i.e. a description of the denotations of all the basic expressions in an object language. The next step is to set up the rules for interpreting expressions in the object language. In doing this, one must consider any arbitrary model that specifies how the denotations of composite expressions are constructed from those of their component parts, using recursive definitions. This indicates that compositionality of FQs is presupposed here.

A model for L_{FQ} provides a means for determining the meaning of expressions in L_{FQ} . It assigns interpretations to set expressions, which are some subsets of E , the universe of discourse. The recursive clauses below state formally the truth conditions of formulas in L_{FQ} by defining what must be the case for a formula with a particular structure to be true in any model, i.e. a formula ϕ is true in a model M iff $|\phi|^M = 1$, false iff $|\phi|^M = 0$. Otherwise, ϕ is true to a degree iff $|\phi|^M \in [0,1]$ exclusive. A distinctive feature of formal semantics of FQs is that the truth values are in degree, i.e. a formula formed with an FQ can be true to a degree, a value in $[1,0]$, rather than just 1 or 0.

Given a model, $M = \langle E, | \rangle$, then,

- S1. For any constant a , $a \in E$; For any variable i , $g(i) \in E$.
- S2. For any set expressions or predicates, $A, X, |A|, |X| \subseteq E$.
- S3. If R is an n -ary relation symbol, then $|R| \subseteq E_1 \times \dots \times E_n$.
- S4. $|D|$ assigns to A a set of sets FQ , such that
 $X \subseteq FQ \leftrightarrow (A \cap X) \subseteq FQ$.
- S5. If R is an n -ary relation symbol, then $|R(i_1, \dots, i_n)|$ is 1,
 iff $\langle |i_1|, \dots, |i_n| \rangle \in |R|$ completely.
 Otherwise, $|R(i_1, \dots, i_n)| \in [1, 0]$ excluding 1.
 Also, if A is a set expression, then $|A(i)| = 1$, iff $|i| \in |A| = 1$
 Otherwise, $|A(i)| \in [1, 0]$ excluding 1.
- S6. If D is an atomic determiner and A a set, then $|D(A)| = |D|(|A|)$
 and $|(\text{PreD}(\text{Num}))(A)| = (|\text{PreD}|(|\text{Num}|))(|A|)$, where PreD
 stands for a predeterminer, and Num stands for a numeral.
- S7. If FQ is a fuzzy quantifier, X is a predicate, then $|FQ(X)| = 1$,
 iff $X \in |FQ|$ completely. Otherwise, $|FQ(X)| \in [1, 0]$ excluding 1.
- S8. If ϕ is a formula, then $|\neg\phi| = 1$, iff $|\phi| = 0$.
 Otherwise, $|\neg\phi| \in [1, 0]$ excluding 1.
- S9. If ϕ and φ are formulas, then $|\phi \wedge \varphi| = 1$, iff $|\phi| = 1$ and $|\varphi| = 1$.
 Otherwise, $|\phi \wedge \varphi| \in [1, 0]$ excluding 1.
- S10. If ϕ and φ are formulas, then $|\phi \vee \varphi| = 0$, iff $|\phi| = 0$ and $|\varphi| = 0$.
 Otherwise, $|\phi \vee \varphi| \in [1, 0]$ excluding 0.
- S11. If ϕ and φ are formulas, then $|\phi \rightarrow \varphi| = 0$, iff $|\phi| = 1$ and $|\varphi| = 0$.
 Otherwise, $|\phi \rightarrow \varphi| \in [1, 0]$ excluding 1.

(6.3)

S4 states that all FQs are conservative. That is, for any FQs, $|D|(A)$ is a set of sets with the property that $X \in |D|(A)$ iff $(X \cap A) \in |D|(A)$ (see page 149 for more discussion of this point). From S9 to S11 we deal with the interpretation of complex formulas. Apart from the classical truth conditions for those connectives we introduce a degree of truth. With intermediate truth values in the treatment we do not need to draw any arbitrary distinction between 1 and 0, and can capture the fuzziness in the form of true to a degree.

To see an application of the formal rules set above, let us look at examples in Fig. 6.2 and Fig. 6.3. These illustrate how the translation procedure for L_{FQ} works.

- a. About 200 students left.
b. translation and interpretation

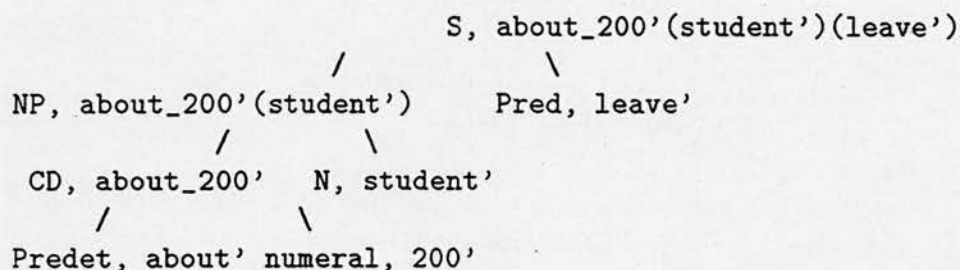


FIGURE 6.2: Illustration I

Fig. (6.2) is an analysis tree for the sentence *About 200 students left*. The tree can be looked at as a top-down, or bottom-up tree. As an illustration, for the top-down analysis the sentence consists of two main elements: an NP (*about_200'(student')*) and a predicate (*leave'*). Next, one level lower, NP has two elements: a complex determiner (*about_200'*) and a noun (*student'*). Finally, the complex determiner consists of two elements: a predeterminer (*about'*) and a numeral (*200'*). The procedure of the bottom-up analysis is the same, except in a different order.

- a. Many students and a few lecturers left.
 b. translation and interpretation

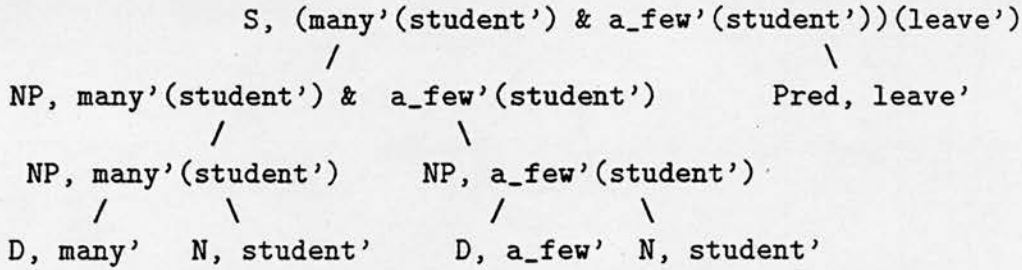


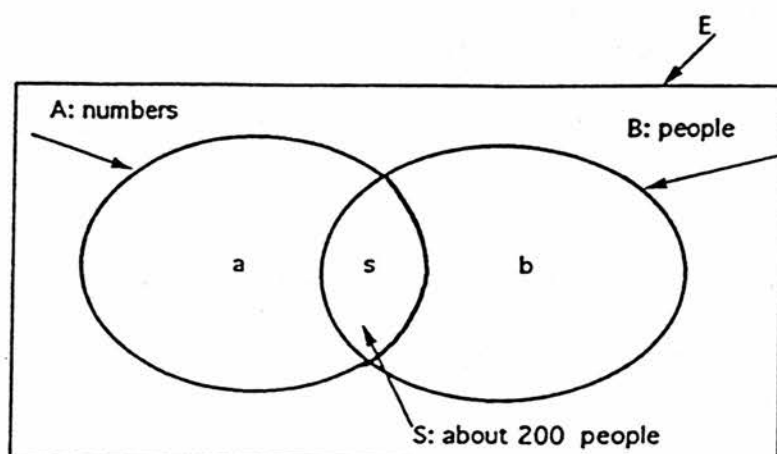
FIGURE 6.3: Illustration II

Again, the analysis tree illustrated in (6.3) for the sentence *Many students and a few lecturers left* is similar to the one in (6.2) above, except that this one has a more complex NP. The NP is a conjunction of two NPs: *many'(student')* and *a_few'(student')*. On the second level there are an NP and a predicate, on the third level two NPs. Finally, a determiner and a noun are derived from each of the two NPs.

What follow are formal models of FQs using the rules given. Membership functions of FQs are our focus here.

6.3 Fuzzy quantifier

As defined in Section 5.2, membership function is specified by a mapping from a domain to a set in question. By an FQ, we mean a functor D assigning, to a domain E , a *fuzzy* binary relation $D_E A B$ between subsets A , B of E in a given context. This can be illustrated in Fig. 6.4:

FIGURE 6.4: About 200 people

In Fig. 6.4, the boundary of intersection set S of A and B is fuzzy. This represents the result of the fuzzy binary relation between A and B , which is ascribed by FQs' fuzzy denotation, like that of *about 200 people*. This can be defined formally in (6.4) below.

Definition: Let E be a non-empty domain: $S, A, B \subset E$. $S = A \cap B$: $x, y, z \in S$. Let C be a set of contexts, $c \in C$. Let $[0, m, 1]$ be a set of membership values, where 1 corresponds to a total membership, 0 corresponds to a non-membership, and m corresponds to an intermediate value between 1 and 0. Then, an FQ is a function from S in E and C onto $[0, m, 1]$, which can be defined as below: For all $x, y, z \in S$ and all $c \in C$,

$$\begin{aligned}
 FQ((E, S)c)(x) &= 1 \quad \text{iff } x \in N, \\
 &= m \quad \text{iff } x \in M, \\
 &\quad \mu_x \geq \mu_y \text{ iff } |z - x| \leq |z - y|, z \in N \quad (6.4) \\
 &= 0 \quad \text{otherwise,} \\
 \% (na) \star \% (nb).
 \end{aligned}$$

In (6.4), N stands for a norm, a central tendency, which could be a number or

a set of numbers. It is a prototype for a membership function. For instance, for *about n* type, n may be a prototypical number. If $x \in N$, then $\mu_x = 1$, where μ_x stands for the membership of x . All members in the set of N receive the same membership—a total membership of 1.

The intermediate value m is determined by whether x is in the area of M , an area between a threshold t and a norm. The threshold t sets a limit: any element outside t is no longer in the interval denoted by an FQ. For instance, we may set the t as 5th percentile in a given context—anything which falls out the 5th percentile will receive a membership of 0. This says that an entity is eligible for consideration if and only if it is within t which can be graphically represented in Fig. 6.5 below: the numbers are hypothetical.

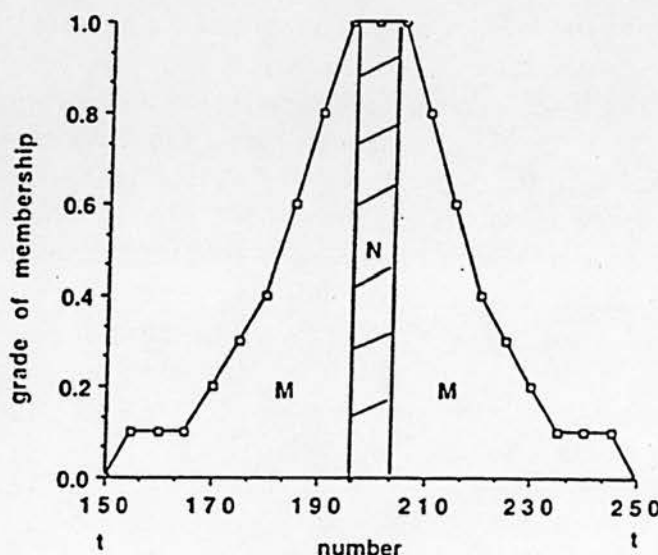


FIGURE 6.5: *About 200 pcople*

From Fig. 6.5 we can see that if x is in the central area dashed (N), then $\mu_x = 1$. If x is in the non-central area (M), $0 < \mu_x < 1$. If x is outside the threshold t , then $\mu_x = 0$. The function " $\mu(x) \geq \mu(y)$ iff $|z - x| \leq |z - y|$, $z \in N$ " says that the closer x to z , the closer μ_x to 1; on the other hand, the further y away from z , the closer μ_y to 0. It must be emphasized again that a norm here must be an end of scale otherwise the function would not work universally, which has been explained in Section 3.7 above.

The function “ $\%(na) \star \%(nb)$ ”, where \star stands for operations like “=”, “>” or “<”, says that the percentage of curve above or below a norm indicates different types of FQs. For example, for *about n* type, the percentage of curve above a norm ($\%(na)$) tends to be equal, or similar, to the percentage of curve below the norm ($\%(nb)$); for *n-odd* type, $\%(na) > \%(nb)$; for *nearly n* type, $\%(na) < \%(nb)$.

In summary, (6.4) defines that the nearer a number to N , the higher the grade of membership it gets. In (6.4), N , M , and the threshold t , are all variables. Any specification of their values, i.e. the actual values of a membership function for a particular FQ, has to be determined empirically.

As an illustration, from Fig. 6.5 we can see that if a number is fewer than 150 exclusively or greater than 250 exclusively, then it does not belong to *about 200 people* at all. If a number is in the interval between 150 and 250, then it has the property of the FQ to a certain degree. For instance, the number 210 belongs to the FQ to the degree 0.8; 180 is 0.4; and 200 (the norm) belongs to *about 200 people* totally. The shape of the curve is symmetrical, so that $\%(na) = \%(nb)$. Moreover, if $x = 190$, $y = 180$, then $\mu_x = 0.8$ and $\mu_y = 0.4$, because 190 is closer to the norm (200) than 180. This relation is defined in (6.4) as $|\mu_x| \geq |\mu_y|$ iff $|z - x| \leq |z - y|$, i.e. $0.8 > 0.4$, as $|200 - 190| < |200 - 180|$.

Then, the FQ in Fig 6.5 can be interpreted as,

$$\begin{aligned}
 \text{about}_{200}((E, S)c)(x) &= 1 && \text{iff } x \in 195 - 205, \\
 &= m && \text{iff } x \in 150-194 \text{ or } 206-250, \\
 &&& \mu_x \geq \mu_y \text{ iff } |200 - x| \leq |200 - y|, \\
 &= 0 && \text{otherwise,}
 \end{aligned} \tag{6.5}$$

$$\%(na) = \%(nb).$$

Moreover, the definition in (6.4) is assumed to be applicable to other types of FQs (e.g. *n-odd* and *nearly n*) and semi-FQs (e.g. *more than n* and *fewer than n*). In terms of *nearly n* and *fewer than n*, n may not be a norm, and even not in the interval denoted by the FQ. On the other hand, for *more*

than n and over n , n may not be excluded. For n -odd, n is more certainly in the interval. For FQs in Type I, such as *many* and *a few*, the norm for a membership function is less obvious than FQs in other types. For those FQs which may have a monotonic curve, such as *almost all* and *very few*, the norm would be end of scale. For example, *all* would be a norm for *almost all*, and zero would be a norm for *very few*. Finally, for those FQs which may have a bimodal curve, such as *10 or 20*, the same principle of membership function still applies. What differs is that they may have two norms. i.e. a membership function with two peaks (see Fig. 3.8 above).

More provision for formal representation of FQs may be attempted. e.g. a modal interval of FQs. Take *about 200 people*, *about 2,000 people* and *about 20,000 people* as examples. Supposing, as my Chinese data showed (see Appendix 2, Figs. 2.15, 2.16, and 2.17), their modal intervals are 190-210, 1,900-2,100, and 19,000-21,000, respectively. Then, a formula for *about n* can be given as follows:

$$\begin{aligned} \text{Mode}_{\text{about } n} : \\ V_{\min} &= n - (n \times 5\%) \\ V_{\max} &= n + (n \times 5\%) \end{aligned} \quad (6.6)$$

where n denotes the numeral, V_{\min} stands for the minimum value of the modal interval, and V_{\max} is the maximum value of the modal interval.

From (6.6), we can get that modal intervals of *about 200 people*, *about 2,000 people* and *about 20,000 people* are: 190 ($200 - 200 \times 5\%$)-210 ($200 + 200 \times 5\%$); 1,900 ($2,000 - 2,000 \times 5\%$)-2,100 ($2,000 + 2,000 \times 5\%$) and 19,000 ($20,000 - 20,000 \times 5\%$)-21,000 ($20,000 + 20,000 \times 5\%$), respectively. Certainly, we could calculate a modal interval for any *about n* approximations by using the formula in (6.6). Take *about 10* as an example. $V_{\min} = 10 - 10 \times 5\% = 9.5$; $V_{\max} = 10 + 10 \times 5\% = 10.5$, i.e. the modal interval of *about 10* is 9.5-10.5.

It appears that the percentage in (6.6) may vary from context to context¹, i.e.

¹Specifications of a membership function can be influenced by contextual factors, such as scale effects (see Section 4.1.1 for further discussion of this point). A formal provision

we might need a "rounding function" which has certain fineness or coarseness built in. For example, *about 10 people* may mean 8–12, rather than 9.5–10.5, as predicted by (6.6). So, (6.6) can be modified as:

$$Mode_{about\ n} : (n \pm n \times f(p)\%) \quad (6.7)$$

where n is a norm, f is rounding function depending on context and p is a base determined empirically. For example, let n be the numeral in *about n*; $p = 5\%$, we may have,

$$\begin{aligned} \text{about } 10 &= 9 - 11 &= < n - n \times (2 \times 5)\%, n + n \times (2 \times 5)\% > \\ & &\text{i.e. } < n - n \times 10\%, n + n \times 10\% > \\ \text{about } 20 &= 17 - 23 &= < n - n \times (3 \times 5)\%, n + n \times (3 \times 5)\% > \\ & &\text{i.e. } < n - n \times 15\%, n + n \times 15\% > \\ \text{about } 100 &= 80 - 120 &= < n - n \times (4 \times 5)\%, n + n \times (4 \times 5)\% > \\ & &\text{i.e. } < n - n \times 20\%, n + n \times 20\% > . \end{aligned} \quad (6.8)$$

In (6.8), everything else keeps consistent, except f . This f represents variance due to different contexts or models. For example, for *about 10* it is 2; for *about 20* it is 3 and 4 for *about 100*. So that one works out a proportion of a norm and scales the minimal and maximal appropriately.

As demonstrated above, the formula in (6.8) works for FQs like *about n*; it is also applicable to more complex FQs. Take *many people* as an example. It is well-known that *many* is not necessarily above 50% (see page 75 for illustrations). Supposing that n is 70%, p is 14%, we might have:

for this kind of scale effect on membership function is proposed in Sadock (1977) (see his paper for the details).

$$\begin{aligned}
\text{many1} &= 60\% - 80\% = \langle n - n \times (1 \times 14)\%, n + n \times (1 \times 14)\% \rangle \\
\text{many2} &= 10\% - 20\% = \langle n - n \times (6.15 \times 14)\%, n + n \times (-5.15 \times 14)\% \rangle
\end{aligned}
\tag{6.9}$$

where standard half-rounding, to one decimal place, has been applied. In (6.9), using function f *many* is assigned to have two different interpretations: 60%-80% or 10%-20%. As shown, the function f could be a natural number (e.g. 1), a fraction (e.g. 6.15) or even a negative number (e.g. -5.15). By using f , the modal interval may vary from one model to another. Similarly, we may apply (6.9) to other types of FQs.

6.4 Fuzzy quantification

Fuzzy quantification is formed by a proposition containing an FQ. Instead of asking about the grade of membership of an FQ we ask about the truth of a proposition that asserts the grade of membership. The general rule is, if x is a member of X only to a certain degree, then the proposition x is X should be true only to that degree, rather than being absolutely true or false. A formal definition of *fuzzy quantification* is given in (6.10):

Suppose there is a non-empty domain E . An FQ is involved in the interpretation of an **FQ** (*fuzzy quantification*) in E . The **FQ** is a function from subsets A, X, P of E and a set of contexts C onto a set of truth values $[0, m, 1]$, where 1 corresponds to absolutely true; 0 absolutely false; m corresponds to some intermediate truth values between 1 and 0. For all $A, X, P \subset E, c \in C$,

$$\begin{aligned}
\text{FQ}((A, X)c) &= 1 \quad \text{iff } A \cap X = Pc, \\
&= m \quad \text{iff } A \cap X = Pp, \\
&= 0 \quad \text{otherwise.}
\end{aligned}
\tag{6.10}$$

where Pc is a central area of an intersection of A and X . Pp is a peripheral

area of the intersection of A and X , which is a complement area of P_c . The formula (6.10) can be graphically represented in Fig. 6.6:

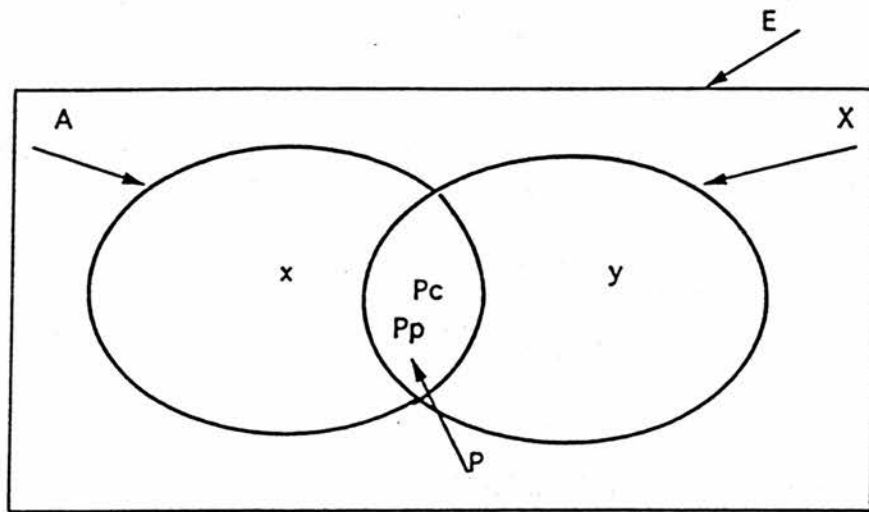


FIGURE 6.6: Fuzzy quantification

As shown in Fig. 6.6, the intersection of the two subsets A and X is the set P . P in turn consists of two subsets: P_c and P_p . Now, consider the proposition *About 200 people are girls*, based on the membership function of *about 200 people* in Fig. 6.5 above. It can be interpreted by:

$$\begin{aligned}
 \text{about } 200((A, X)_c) &= 1 \quad \text{iff } |A \cap X| = 195-205, \\
 &= m \quad \text{iff } |A \cap X| = 150-204 \text{ or } 206-250, \\
 &= 0 \quad \text{otherwise.}
 \end{aligned} \tag{6.11}$$

in which the value of A will be fixed as

$$\{x \mid \text{People}(x)\},$$

while that of X will be

$\{y \mid \text{Girls}(y)\}$.

What (6.11) means is that this proposition will be absolutely true, if the set of people who are girls has exactly 195–205 members; true to an extent if it has members which fall into 150–194 or 206–250; and false otherwise. The truth value should be 1 at 195–205 and gradually decreases from 195 downwards and 205 upwards. The statement approaches truth as the numbers move up to the norm, and approaches falsity as the numbers move away from the norm.

There is a regularity here about the semantics of fuzzy quantification. On the one hand, numbers increase from 150 to 250 continuously, and on the other hand, truth values decrease from 1 both upwards and downwards. The trend of the movements for both numbers and truth values makes the whole fuzzy quantification sensible.

Furthermore, we discuss the question of whether A and X in (6.6) are ordered. This question is related to the issue of *sortal quantification* raised by Altham and Tennant (1975). They claim that frequently the range of quantification is restricted, in which quantifiers are *sortal*. A proposition *About Xs are Ys* is one in which the quantifier is not sortal, because it is similar to *About Ys are Xs*. In contrast, *Many Xs are Ys* involves a sortal quantifier, because it is not equivalent to *Many Ys are Xs*. In *Many Xs are Ys*, the quantifier's range is restricted to the set of Xs . Thus, the set of Xs determines a norm. Consequently, since the set of Xs may not be the same cardinal number as the set of Ys , then the norm determined by one may be different from that determined by the other. The non-equivalence of the two sortally quantified propositions is a consequence of this fact. Let us consider the following examples:

- a. About 20 men are lawyers.
 - b. About 20 lawyers are men.
 - c. Many lawyers are men.
 - d. Many men are lawyers.
- (6.12)

It appears that (a) and (b) are truth-conditionally equivalent, but not (c)

and (d). The proposition (c) may be true, but (d) may not. There are far more men than male lawyers—the norm for *many men* is correspondingly larger than that for *many lawyers*.

How can this be represented in the treatment of fuzzy quantification then? The formula in (6.6) deals well with FQs in Types II and III, because FQs of these types are non-sortal. However, with respect to the fact that some FQs in Type I, such as *many*, are sortal, we need to find some way to solve the problem. One solution is to assume that the norm of X and Y in (6.10) would be the same. Suppose n is the norm for *many men*, and m for *many lawyers*, then (c) and (d) in (6.12) would both be true if $m = n$. However, this is not likely to be the case, because the size of *many lawyers* would be smaller than the size of *many men*. Then, another solution is to put a constraint over the formula in (6.10), as shown in (6.13):

$$\begin{aligned} \text{FQ}(\langle A, X \rangle c) &= 1 \quad \text{iff } A \cap X = Pc, \\ &= m \quad \text{iff } A \cap X = Pp, \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{6.13}$$

where the pair of $\langle A, X \rangle$ is ordered, representing sortal quantifications; that is, A determines the norm, not X . Supposing A is *lawyers* and X is *men*. Then, that *Many lawyers are men* is true does not necessarily entail that *Many men are lawyers*. The norm determined by A would in this case be much smaller than that of X .

In conclusion, fuzzy quantifications containing FQs in Types II and III are not sortal, whereas those containing some FQs in Type I, such as *many*, are sortal and may be treated with the extra constraint as shown in (6.13). Put another way, syntactic disposition such as word order does not affect the interpretation of non-sortal fuzzy quantifications, but it has impact on those sortal fuzzy quantifications.

6.5 Compound proposition

A formal treatment of compound propositions containing FQs needs to capture the fact that the truth values for a compound proposition should have a degree of truth, since the constituents from which they are derived have the same feature.

Let P be assigned a numerical value $|P|$, called the degree of truth of P , such that $0 \leq |P| \leq 1$. Let “ \neg ”, “ \wedge ”, “ \vee ” and “ \rightarrow ” be connectives. Valuations for the connectives are defined as follows,

$$\begin{aligned}
 a. \quad & |\neg P| &= f|P| \\
 b. \quad & |P \wedge Q| &= f(|P|, |Q|) \\
 c. \quad & |P \vee Q| &= f(|P|, |Q|) \\
 d. \quad & |P \rightarrow Q| &= 1 \text{ iff } |P| \leq |Q| \\
 & &= |Q| \text{ iff } |P| > |Q|
 \end{aligned} \tag{6.14}$$

Let us compare (6.14) with operations proposed in Zadeh (1965, 1971), reproduced from (5.29) above,

$$\begin{aligned}
 a. \quad & |\neg P| &= 1 - |P| \\
 b. \quad & |P \wedge Q| &= \min(|P|, |Q|) \\
 c. \quad & |P \vee Q| &= \max(|P|, |Q|) \\
 d. \quad & |P \rightarrow Q| &= 1 \text{ iff } |P| \leq |Q|
 \end{aligned} \tag{6.15}$$

There are two differences between my treatment and Zadeh's. One is that I do not use specific function in (a), (b) and (c). The reason is that they may not be empirically appropriate (see Section 5.2.7 for the details). Instead, I use a general function f to indicate that these operations generate some kind of function between those components, and any specific functions have to be determined empirically.

The other difference concerns entailment. I promote the idea that entailment can be in degree, but this is not the case in Zadeh's model. In my

treatment, an intermediate value can be given, as shown in (6.14d). This makes the treatment more consistent and adequate. For further discussion, see Sections 5.2.2 and 5.2.3.

6.6 Conclusion

It is shown that FQs can be treated in formal semantics. The formulas here convey the claims made previously, that an FQ can be interpreted as a fuzzy operator, and a norm functions as a prototype and other numbers in the set are ordered depending on how closely they approximate to a given norm.

My formal treatment of FQs is based on the conclusion drawn previously that FQs are compositional and motivated. On the one hand, compositionality of FQs guarantees that the semantic structure of the same type of FQs is preserved. In defining compositionality membership function is deployed. That is, functions of approximators, such as *about*, *or so* and *very*, give a mechanism by which the same type of approximator operates in the same way towards expressions being modified.

On the other hand, the characteristic of motivation for assignment of specific values requires that some values of my formal treatment have to be determined empirically, i.e. they are not computable. In my treatment contextual factors are taken into account, due to the fact that the meaning of an FQ may vary from context to context.

Hence, my formal treatment is made unique by the two characteristics—compositionality and motivation—of FQs. It is different from a conventional approach (e.g. algorithmic approach which only permits a phenomenon either predictable or arbitrary) in that the treatment here is partially determined empirically. That is, in the treatment the general semantic pattern of an FQ is predictable, i.e. any basic membership functions for a certain type of FQ is functional, although it may be moderated by context, as shown for example in (6.4) and (6.7) above. However, the specifications have to be done empirically.

Standard FST (Zadeh, 1965) has been developed here to deal with the formal semantics of FQs. My treatment differs from Zadeh's in several aspects. Apart from the two mentioned on page 200, in my formal framework the membership function takes account of new information, such as the constraint that a norm has to be taken as an end of scale. Otherwise, it would not necessarily work if we want to claim that the closer a number to a norm the greater its membership. Also, we look at percentage of curve to examine types of FQs, i.e. whether or not they are symmetrical (see Section 6.3 for the details).

7

Formal semantics of FQs (III)

This chapter continues our discussion of the formal semantics of FQs. We will show how the notion of membership function of FST can be introduced into GQT. Then, the issues of intensional models and entailment with FQs will be tackled.

7.1 GQT models incorporating with membership function

As shown in Chapter 5, the pattern of FQs is similar to generalised quantifiers for the most part. As far as the semantics is concerned, GQT would be able to deal with fuzziness adequately, if it does not assume the bivalent theory. In the following models it is attempted to alter GQT theory to take account of new information—membership function.

Generally, generalised quantifiers can be modelled as sets containing entities or functions over entities. Let us examine:

- a. Several students left.
- b. Many students left.
- c. About 200 students left.
- d. 200-odd students left. (7.1)
- e. Nearly 200 students left.
- f. More than 200 students left.
- g. Fewer than 200 students left.

Sentences in (7.1) can be semantically analysed in terms of set theory. As an illustration, an NP like *several students* denotes a set of sets of entities. Then, (7.1a) can be translated as *several' (student') (leave')*. The semantics of generalised quantifiers can be directly defined by the model theory, without the use of variables and value assignments. That is, a generalised quantifier is considered as a function which assigns to each A of E a set of subsets of E . As an illustration, the definition of *some* is given in (7.2).

some is the function which assigns to each $A \subseteq E$, the set of sets such that $\{X \subseteq E \mid X \cap A \neq \emptyset\}$. (7.2)

It follows then that a proposition whose subject NP formed with *some* is true if the VP of the proposition denotes a set that is in the set denoted by the subject. That is, the proposition conveyed by *Some student left* is true iff $(\text{leave}') \cap (\text{student}') \neq \emptyset$. In what follows, we will show how membership function works with such GQT models.

7.1.1 Type III FQs (semi-FQs)

The interpretation of Type III FQs can be done in terms of the cardinality of a set. This means that we are able to interpret Type III FQs by looking at the number of elements they contain. A proposition expressed by a sentence like *At least 200 students left* may be interpreted as in (7.3).

$$\begin{aligned} \text{at_least_200}'(\text{student}')(\text{leave}') &= 1 \text{ iff } |\text{leave}' \cap \text{student}'| = 200. \\ \text{Otherwise, at_least_200}'(\text{student}')(\text{leave}') &\in [1,0] \text{ exclusive of } 1. \end{aligned} \quad (7.3)$$

Formula (7.3) says that *At least 200 students left* is true just in the case where the intersection of the extension of *student* with that of *leave* has 200 members, i.e. the set of students who left has a cardinality that is equal to 200¹. Otherwise, the truth value would be true to a degree. For instance, 250 may be true to a degree of 0.8.

If the set actually has 195 students who left, then *At least 200 students left* may also be true, say, to the degree 0.1. Of course, one might say that in a strict mathematical sense, if there are 195 students who left, then the proposition would be totally false. However, it seems that if we are talking about ordinary language, then a degree of truth might be more intuitive. In the same vein, Channell(1994) states that the interval for *not less than n* could go below *n*. For example, *the cost of repair was not less than 60 pounds*; she does not think it would be odd if the cost turned out as something like 58 pounds. In other words, the use of a fuzzy term fuzzifies the number.

What I am interested in is if the value could go below *n*, how would this affect the semantic pattern generated by *at least n*, *not less than n* and the like? It appears that this does not significantly affect the basic membership function these FQs license. It is just a matter of whether an interval could or could not take a number below *n*. As stated in Section 4.2, in my system the basic membership function may be moderated over context, so the possibility of having some numbers below the *n* is not rejected. In fact, in my data, similar phenomena occurred, such as *jiangjin 200 ren* 'nearly 200 people', *jiangjin 2,000 ren* 'nearly 2,000 people', *jiangjin 20,000 ren* 'nearly 20,000 people', where subjects gave an interval beyond the *n* (see Appendix 2, Figs. 2.9, 2.10 and 2.11).

On the other hand, for *200 yishang* 'more than 200' and *200 ren yishang*

¹ Here, we hypothetically take 200 as a norm, though it may vary. This will be the case throughout this section, i.e. any specific value occurred in the models is my best guess, but it may be subject to change from individual to individual.

'more than 200 people', no subject in the Chinese test gave an interval which was fewer than n (see Section 3.2.3 for the details). The trend in my test was that n in *more than n* was the limit, i.e. its basic membership function varies with this limit in the particular context.

Similarly, for *At most 200 students left*, the formula would be the same to that of *At least 200 students left* in (7.3), except that $|\text{leave}' \cap \text{student}'| \leq 200$. Also, other Type III FQs like (f) and (g) in (7.1) can be interpreted as in (7.4).

- a. $\text{more_than_200}'(\text{student}')(\text{leave}') = 1$ iff $|\text{leave}' \cap \text{student}'| = 201$
Otherwise, $\text{more_than_200}'(\text{student}')(\text{leave}') \in [1,0]$ exclusive of 1.
 - b. $\text{fewer_than_200}'(\text{student}')(\text{leave}') = 1$ iff $|\text{leave}' \cap \text{student}'| = 199$.
Otherwise, $\text{fewer_than_200}'(\text{student}')(\text{leave}') \in [1,0]$ exclusive of 1.
- (7.4)

As shown in above models, degree of truth play a role in representing fuzziness. By doing this GQT framework can provide more intuitive semantic interpretations for expressions like FQs.

7.1.2 Type I FQs

If we try to interpret a sentence like *Several students left* in (7.1a), it will not be so straightforward. For *several students*, its norm is less obvious than a norm for *at least 200 students* is. So, apart from appealing to membership function, we need to take pragmatic factors into consideration, i.e. *several* may be interpreted with respect to some pragmatically determined values (Cann, 1993). Let v represent this kind of value, *several* can be interpreted as in (7.5a):

- a. $\text{several}'(\text{student}') = \{X \subseteq E \mid |X \cap \text{student}'| = v\}.$
 b. Let $v = 5$, then $\text{several}'(\text{student}')(\text{leave}') = 1$ iff $|\text{leave}' \cap \text{student}'| = 5$. Otherwise, $\text{several}'(\text{student}')(\text{leave}') \in [1,0]$ exclusive of 1. (7.5)

What (7.5) says is that if in a given model *several* requires a norm of 5 entities denoted by the common noun (i.e. *students*), and those entities have the property denoted by the verb phrase (i.e. *left*), then the sentence in (7.1a) has the interpretation in (7.5).

In the same vein, we may interpret propositions involving *many*, *few*, and *a lot* as all needing some reference determined contextually, and a degree of truth. In addition, v can be an interval or a proportion. The interpretation of *many* is more complex, as illustrated in (7.6).

- a. $\text{many}^{M1}(\text{student}')(\text{leave}') = |\text{student}' \cap \text{leave}'| \geq v \cdot |E|.$
 b. $\text{many}^{M2}(\text{student}')(\text{leave}') = |\text{student}' \cap \text{leave}'| \geq v \cdot |A|.$ (7.6)

For (a), suppose there are two subsets in E , the set of students and the set of lecturers, whose properties are denoted by the predicate *left*. Many^{M1} interprets *Many students left* by comparing the set of students to the set of lecturers. If the set of students who left is larger than the set of lecturers who left, then many^{M1} is true. In (b), the interpretation has little to do with the set of lecturers, but is determined by comparing the set of students who left and the set of students who did not, i.e. the set of lecturers is not relevant. Many^{M2} is true if the set of students who left is larger than its complement set.

Suppose the value v for *many* in a given context is 60%. Then, (7.1b) has the interpretation in (7.7).

- a. $\text{many}'(\text{student}') = \{X \subseteq E \mid |X \cap \text{student}'| \geq v \cdot |\text{student}'|\}$.
 b. Let $v = 60\%$, and $|\text{student}| = 200$, then, $\text{many}'(\text{student}')(\text{leave}') = 1$ if $|\text{leave}' \cap \text{student}'| = 120$.
 Otherwise, $\text{many}'(\text{student}')(\text{leave}') \in [1,0]$ exclusive of 1.
- (7.7)

7.1.3 Type II FQs

A more complex situation arises if we look at sentences like (7.1c), (7.1d), and (7.1e). For instance, if we try to interpret *about 200 students* three steps are needed, rather than two steps for FQs like *several students*. That is, *about* takes on two arguments, translated as $(\text{about}(200))(\text{student})$, while *several* in $\text{several}(\text{student})$ takes only one. This can be graphically represented in Fig. 7.1,

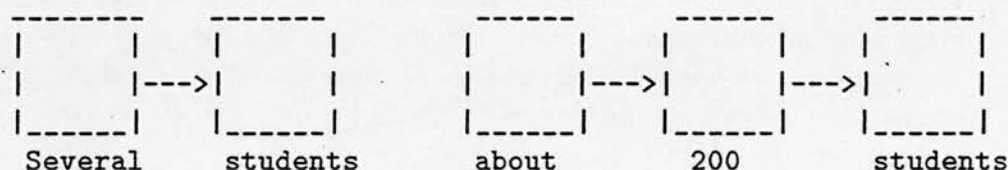


FIGURE 7.1: Several students and about 200 students

The figure (7.1) can be symbolised in (7.8):

$$(\text{about})^{n^{NP}} \quad (7.8)$$

It says that *about* applies to n (a numeral) to form a determiner, then *about* n applies to a common noun to form an NP. That is, the interpretation of an NP with *about* is likely to take two mappings to complete, which has more steps than FQs like *several students*. This is also the case for Type III FQs, such as *more than 200 students*. Furthermore, *about* in *About 20 students danced*

decomposes at least three sets: dancers who were not students, students who did not dance, and students who danced. In a way, *about*, along with other determiners like *many*, behaves like a set filter to select those individuals who are qualified to be in the relevant set.

In terms of the interpretation of (7.1c), it appears that in a number continuum our judgments become increasingly positive as the number of students who left moves towards (from either side) 200. That is, *about* denotes a membership function of $200 \pm v$. The sentence *About 200 students left* can be interpreted as in (7.9).

- a. $\text{about_200}'(\text{student}') = \{X \subseteq E \mid |X \cap \text{student}'| = v \pm 200\}$.
 b. Let $v = 2\%$ then, $\text{about_200}'(\text{student}')(\text{leave}')$
 $= 1$ iff $|\text{leave}' \cap \text{student}'| \in [196, \dots, 204]$. (7.9)
 Otherwise, $\text{about_200}'(\text{student}')(\text{leave}') \in [1, 0]$ exclusive of 1.

What (7.9a) shows is that *about 200 students* is interpreted as a set whose members are approximate to 200. Note that this time, v is a proportion. The formula in (7.9b) defines *About 200 students left* as totally true if the intersection of *student'* and *leave'* belongs to the interval. Otherwise, a degree of truth should be the case.

The sentence in (7.1e) may be interpreted as in (7.10),

- a. $\text{nearly_200}'(\text{student}') = \{X \subseteq E \mid |X \cap \text{student}'| = v\}$.
 b. Let $v = [196-199]$, $\text{nearly_200}'(\text{student}')(\text{leave}') = 1$ iff
 $|\text{leave}' \cap \text{student}'| \in [196 - -199]$.
 Otherwise, $\text{nearly_200}'(\text{student}')(\text{leave}') \in [1, 0]$ exclusive of 1. (7.10)

Conversely, the sentence in (7.1d) may be interpreted as in (7.11).

- a. $200_odd'(student') = \{X \subseteq E \mid |X \cap student'| = v\}.$
 b. Let $v = [200-204]$, $200_odd'(student')(leave') = 1$
 iff $|leave' \cap student'| \in [200-204].$ (7.11)
 Otherwise, $200_odd'(student')(leave') \in [1,0]$ exclusive of 1.

In (7.10) and (7.11), v is an interval. The interpretations in (7.10) and (7.11) differ only in that the distribution of the former expands from n downwards, whereas for the latter it is upwards from n .

A general formula for interpretation of FQs and a proposition formed with an FQ is given in (7.12),

- a. $D(CN) = \{X \subseteq E \mid |X \cap CN| \star v\}.$
 b. $D(CN)(VP) = 1$ iff $|VP \cap CN| = x.$ (7.12)
 Otherwise, $D(CN)(VP) \in [1,0]$ exclusive of 1.

where \star stands for an operation such as $=, \leq, \geq$; the values of v and x are pragmatically determined, and they could be a natural number, a percentage, or an interval.

In conclusion, the models derived from GQT can be integrated with the notion of membership function, and the integration works well with FQs. Importantly, these models capture our intuitions about what these FQs mean in natural language by using a membership function.

7.2 An intensional treatment

In my work, FQs are explored in terms of their denotations. Denotation here is defined as consisting of *intension* and *extension*, which are intended to refer to things in the world (Cann, 1993: 267-268). The latter is the entity or function that an expression refers to in a model. The former is

more abstract, and it is this common thing that causes each of the elements in the reference of an expression to be identified as such. For instance, the extension of *many people* is the set of people which is considered to be *a significant number compared to a norm*. Its intension is the property of being *a significant number compared to a norm*, the *significance* of the set of entities abstracted away from the individuals themselves.

The purpose of making a distinction between extensional and intensional denotations is to allow a distinction to be made between referentially transparent and referentially opaque contexts. In the latter case straightforward determination of an extension may fail. In the case of FQs there may not be a transparent reference between an FQ and entities it denotes. As in the illustration—*Many students went to the party*—a precise extension of *many students* is not accessible, i.e. we do not know precisely how many is many. Thus, any claim of a fixed extension has to be a result of some kind of idealization.

On the one hand, the intension is known to us, i.e. they are objective in a way that agreement is possible. Intension of FQs is relatively consistent over individuals and contexts. For instance, the intension of *many is a significant number compared to a norm*; although the norm has to be set pragmatically, the abstract property indicated by the intension remains constant. However, extensions determined by the same intension are subjective in that extensions of FQs vary from individual to individual. For instance, we may agree on that *many* means *a significant number*, an intension. But, it is possible for us to come up with different interpretations of it. What is the norm? It could be three when I talk about the Chinese students of English each year at the University of Otago. However, it may change to millions when I talk about the Chinese population of China. So, a norm differs depending on contexts. A norm which is suitable in one context may not be in another. We can raise or lower the norm to serve our purpose, as we wish. In fact, extensional meaning is hardly fixed, even in a fixed context. This is because there is great diversity among people, namely individuality of language users plays a crucial role in determination of the extension of FQs (see Section 1.4 for further discussion on the causal factors of fuzziness). The fact that FQs has an invariable intension, but variable extension, can be represented as follows:

$$I \Rightarrow \{E_i, \dots, E_n\}, \text{ where } \{E_i, \dots, E_n\} \in [1, 0]. \quad (7.13)$$

Formula (7.13) says that the intension of an FQ generates a set of extensions with respect to different contexts or individuals. More importantly, those extensions belong to the FQ to an extent, notated in $[1, 0]$. As an illustration, *about 20 students* can be defined as in (7.14):

$$n \approx 20 \Rightarrow \{\dots, 0.5_{17}, 0.7_{18}, 0.9_{19}, 1_{20}, 0.9_{21}, 0.7_{22}, 0.5_{23}, \dots\} \quad (7.14)$$

where the intension *a number that approximates to 20* produces a set of numbers (notated by subscripts) in which each individual belongs to the FQ to a degree.

It is attempted in this section to look at the relation between intension and extension, to show how the contextually determined manipulation of extensions could be handled in a formal theory by examining some intensional models. Intensional models normally contain times, possible worlds and other things (e.g. places), along with entities. Let intensional models here allow extensions to vary along times (t), contexts (c) and individuals (i). That is, extensions are defined by an index consisting of time, context and individual. If one of the parameters changes, the extension may also change. Hence, $[\phi]^{M,g,t,c,i}$, the truth values of a proposition with an FQ at the index $\langle t, c, i \rangle$ with respect to model M and variable assignment g , may differ from $[\phi]^{M,g,ta,ca,ia}$ to $[\phi]^{M,g,tb,cb,ib}$. Here are formulations of intensional models:

- I1. If x is a constant or variable, then $[x]^{M,g,t,c,i} = g(a)$.
 - I2. If $x = R(t_1, \dots, t_n)$, then $[x]^{M,g,t,c,i} = 1$ iff $\langle [t_1]^{M,g,t,c,i}, \dots, [t_n]^{M,g,t,c,i} \rangle \in [R]^{M,g,t,c,i}$. Otherwise, $[x]^{M,g,t,c,i} \in [1,0]$ exclusive of 1.
 - I3. If $\phi = \neg\varphi$, then $[\phi]^{M,g,t,c,i} = 1$ if $[\varphi]^{M,g,t,c,i} = 0$.
Otherwise, $[\phi]^{M,g,t,c,i} \in [1,0]$ exclusive of 1.
Similarly, for other connectives ($\vee, \wedge, \rightarrow, \leftrightarrow$).
 - I4. If FQ is a quantifier and X is a predicate,
then $[FQ(X)]^{M,g,t,c,i} = 1$, iff $[FQ]^{M,g,t,c,i} \ni [X]^{M,g,t,c,i}$.
Otherwise, $[FQ(X)]^{M,g,t,c,i} \in [1,0]$ exclusive of 1.
- (7.15)

According to the rules set in (7.15), an intensional model can be formally defined as follows. For all $t \in T$ (a set of times), $c \in C$ (a set of contexts), $i \in I$ (a set of individuals),

$$[\phi]^{M,g,t,c,i} \in [1,0]. \quad (7.16)$$

In this model, time, context and individual form an index. Also, a degree of truth is introduced which is meant to represent fuzziness. Let us now see how this model works. For instance, *Many students went to the party* may be interpreted by Mary as, say 100 students, but by John as 150 students, although both may agree that the truth condition of the sentence is *a significant number* compared to a norm. As far as the truth value of the proposition is concerned, 100 and 150 are both true to a certain degree.

Another example, *There are a few lazy students*; suppose in 1993 *a few* was actually three, while in 1994 it was five. The extension of *a few* might differ over years, but the very same intension can be consistently applicable to determine either three or five belongs to *a few*, if an accessibility, iRj , can be established between the intension and an extension appropriate in a given year. Otherwise, if say in 1995, the number of lazy students increased to 20, then we may feel that 20 belongs to *a few* to a degree, where the accessibility would only be partially established.

Membership function or truth value represent the degree of accessibility from one time to another, one context to another, and one individual to another. As an illustration, for $ca \in C$, such that ca may be accessible to cb to an extent. Take *nearly n* as an example. We may interpret it as in a particular context there is a possible interpretation similar to the actual interpretation in which n is true. Also, *about 200 students* may be said as in a particular context there is a possible interpretation approximate to the actual interpretation in which 200 students is true.

The formal semantics of FQs is an extremely complex matter, for it raises a number of issues that question our conventional ideas. One is that intension determines a unique extension for any formulas. In fact, it is evident that the relation between intension and extension is not necessarily determinate, i.e. the function of intension is fuzzy in the sense that it may not specify a unique extension². Also, any parameters which affect the interpretation may be added into the index defined in (7.16). For example, we may represent the difference between *at least 20 students* and *at most 20 students* in terms of attitude of the speaker. The former indicates a positive attitude and a negative attitude is implied by the latter. This can be represented as:

$$[\phi]^{M,g,t,c,i,a} \in [1,0], \text{ where } a \text{ denotes the speaker's attitude by uttering a sentence.} \quad (7.17)$$

Within an opaque context, two propositions formed with FQs may have same truth values, but different intensions. On the other hand, it is also possible that one proposition is assigned different truth values corresponding to a different context, i.e. the same intension does not license the same extension or the same truth value. This feature can be formally represented as:

²However, if it is with respect to a context and an individual, then the relation between intension and extension may be determinate. It is just that here we have gone beyond Montague's (1975) defining intensionality, and so it is no longer true that extensions are unique with respect to $\langle w, i \rangle$. However, that is true of all standard models where an index does not specify all possible parameters.

$$|\phi| = |\varphi| \nleftrightarrow \text{int}[\phi] = \text{int}[\varphi] \quad (7.18)$$

It says that the same truth value does not guarantee the same intension, and vice versa.

It is noted that in a sentence like *Many students went to the party*, although a precise extension of *many students* is not accessible, there is still existential entailment here (*Some students went to the party*). The point is that in opaque contexts the substitution of equivalents fails, i.e. the NP *many students* may not be totally extensional, but the common noun may be (see Section 7.3.4 for further discussion of this point).

It must be noted that FQs are not intensional. Expressions like *unicorn* in English and *fenghuang* 'phoenix' in Chinese are intensional, since they have no extension at all, at most some kind of imaginary extension. On the contrary, FQs like *many students* can have real extensions, although their extensions could be indeterminate. Of course, FQs differ from expressions like *all students* and *25 students* in the way that the former have no precise extension as the latter would have.

Lakoff (1973a) states that one can get a fuzzy modal logic by adding operators " \Box " and " \Diamond " and giving the following valuations.

$$\begin{aligned} |\Box P|w &= \min\{|P|w'\} \text{ for all } w' \text{ such that } Rww' \\ |\Diamond P|w &= \max\{|P|w'\} \text{ for all } w' \text{ such that } Rww' \end{aligned} \quad (7.19)$$

where R is the alternativeness relation over possible worlds.

Note that the value of " $\Box P$ " will be equal to some α , such that $0 < \alpha < 1$, just in case the value of the P never falls below α in any alternative world. If " $\Box P$ " is interpreted as meaning that P is a necessary truth, then in fuzzy modal logic we will have degrees of necessary truth, since $|\Box P|$ may fall between zero and one. This raises the question of whether there are such things as propositions which are necessarily true to a degree. The one type of

possible example is an arithmetic statement that contains an approximation. Consider the following sentence,

Approximately half of the prime numbers are of the form of $4N + 1$.
(7.20)

Is (7.20) true? We would say it has a high degree of necessary truth, rather than say it is absolutely true.

It is shown in this section that FQs can be treated in an intensional model, where degree of truth is introduced. Intensional models make it definable in a formal theory—the modification of pragmatic factors on a basic membership function of FQs. This enhances the claim that a formal semantics of FQs is feasible.

7.3 Entailment with FQs

Conventional entailment with logical quantifiers is relatively straightforward: *If all students went to the lecture, then some student went to the lecture.* Here *all* entails *some*, but the reverse is not valid. However, a proposition containing an FQ is more complex to talk about entailment. In this section, we will explore entailment patterns that are not normally considered, and attempt to provide accounts for these patterns.

7.3.1 Degree of entailment

A general characteristic of the entailment with FQs is a degree of entailment. For instance, *About 20 students are ginger-haired* may entail *23 students are ginger-haired* to a degree of 0.9, rather than absolutely entailed or absolutely not entailed.

Applying FST to entailment, Lakoff (1973a) suggests: suppose " \rightarrow " were to take on intermediate values, then one would like to generalize the notion of entailment to the notion of entailment to a degree α (written $\vdash \alpha$), with the following holding:

$$P \vdash \alpha Q \text{ iff } \vdash \alpha P \rightarrow Q \quad (7.21)$$

He points out in such a system we will want to talk about concepts such as "degree of validity" and "degree of theoremhood", which are natural concomitants of the notion "degree of necessary truth". It can be demonstrated by the following examples.

$$\begin{array}{ll} a. & x \text{ is a bird.} \\ b. & x \text{ flies.} \end{array} \quad (7.22)$$

We know that not all birds fly, but we might well want to say that once an individual has a certain degree of bird-likeness, say 0.9, then we could possibly assume that it flies. We might then want to say that (a) entails (b) to degree 0.9.

Also, Lakoff points out that modus ponens in FST is not only a valid form of inference, but it can be generalized so that it preserves a degree of truth. This can be represented in (7.23):

$$\begin{array}{l} \vdash \alpha P \\ \vdash P \rightarrow Q \\ \text{therefore } \vdash \alpha Q \end{array} \quad (7.23)$$

What (7.23) means is that if P is true at least to degree α , $P \rightarrow Q$ is true; then Q is true at least to degree α . That is, P entails Q to degree α iff $|P \rightarrow Q|$ never falls below α in any valuation.

In addition, monotonicity of FQs indicates some significant inferential patterns of FQs (see Section 5.1.2 for the details). What follows is a discussion of basic patterns of fuzzy entailment.

7.3.2 Entailment pattern with FQs in Type I

In terms of fuzzy entailment with FQs in Type I, one may assume that $\langle \text{all, almost all, most, many, some, few} \rangle$ is ordered, provided that the quantifier on the right-hand denotes a smaller proportion than the left-hand one. A general idea is if *many* denotes x amount, *few* denotes y amount, the x entails y , i.e. if 10 then 2. The negative aspect is quite a separate issue. We may not say *not many* entails *not few*, because the two might be equivalent, or the entailment might be in reverse.

In terms of *very*, a proposition with *very* tends to entail the one without. For example, *Very few students are ginger-haired* entails *few students are ginger-haired*. Also, *Very many students are ginger-haired* entails *Many students are ginger-haired*. As for *quite*, the entailment would be different between Chinese and English (see Section 4.3.2 for the details). In English, *Quite a few students are ginger-haired* entails *A few students are ginger-haired*. On the contrary, *A lot of students are ginger-haired* entails *Quite a lot of students are ginger-haired*. However, in Chinese any propositions with *quite* entails the one without, much like *very* entailment.

Moreover, because *many* does not necessarily imply *more than half* (see page 75 for examples), therefore it is shown in (7.24) that (a) and (b) are fine, but not (c). The failure of (c) is due to the fact that *many* does not necessarily imply *more than half*.

- a. $\text{most}'(\text{student}') = \{X \subseteq A \mid |X \cap \text{student}'| > |\text{student}' \cap (A - X)|\}.$
 - b. $\text{more_than_half}'(\text{student}') = \{X \subseteq A \mid |X \cap \text{student}'| > |\text{student}' \cap (A - X)|\}.$
 - c. $*\text{many}'(\text{student}') = \{X \subseteq A \mid |X \cap \text{student}'| > |\text{student}' \cap (A - X)|\}.$
- (7.24)

It seems that the first two in (7.24) are truth-conditionally equivalent, but there is no such equivalent for (c).³

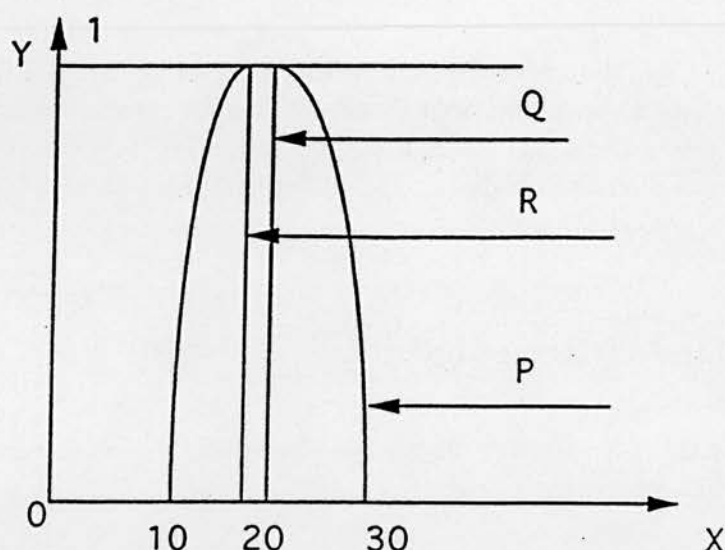
7.3.3 Entailment pattern with FQs in Types II and III

With FQs in Types II and III it is generally assumed, such as in Lakoff (1973a), that a proposition containing a non-FQ entails its equivalent containing an FQ. For example, *Sam is exactly six feet tall* entails *Sam is approximately six feet tall*. This can be formally represented as follows:

If P contains a non-FQ and Q contains an FQ, then $P \rightarrow Q$. (7.25)

However, a condition for the validity of this formula is that the numeral n in for example *about n* , *n -odd* and *more than n* , must be included in the interval of FQs that contain it, as illustrated in Fig. 7.2. Otherwise, the formula would be invalid.

³However, in terms of paraphrase between (a) and (b), we might feel that the sentence *Most students left* implies a greater proportion of students left than students who did not. Then, *Most students left* and *More than half of students left* are not semantically equivalent. Suppose there were 1,000 students, and 550 of them left. Truth-conditionally, both (a) and (b) could convey this information, but pragmatically (b) is more appropriate than (a). Here, it is a matter of cooperative principles (Grice, 1975), i.e. choosing *more than half* is determined in terms of appropriateness in a pragmatic sense rather than truth-conditions, as pointed out by Cann (1993). That is, if we know that there are 550 out of 1,000 students who left, we may prefer to use *more than half* rather than *most*.

FIGURE 7.2: An example for entailment

where P = "About 20 students are ginger-haired"; Q = "Exactly 20 students are ginger-haired"; R = "Nearly 20 students are ginger-haired".

It is shown in Fig. 7.2 that at the point of 20 the truth value of P equals that of Q , then the latter entails the former. However, Q does not entail R . So, the entailment in (7.25) may not be valid with FQs like *nearly* n or *fewer than* n , where n is not in the interval denoted by the FQ. That is, the entailment would not work well with an FQ exclusive of n as its member.

The entailment for n or m type is more complex. It is suspected that n or m may be a bimodal FQ. If so, we may say that *20 students are ginger-haired* and *30 students are ginger-haired* both entail *20 or 30 students are ginger-haired*. But, a number in between, say *25 students are ginger-haired* may entail *20 or 30 students are ginger-haired* to a certain degree, rather than absolutely (see Fig. 3.8 for an illustration).

As for FQs in Type III, apart from what has been said above, it appears that if we take FQs in Type III as generating a monotonic membership function, then all the numbers in the interval would be semantically equivalent. For example, suppose *more than 20 students* means an interval from 20 to a

positive infinity, then all members in the interval receive total membership 1. Thus, there is no entailment involved here, only equivalence. The same applies to *fewer than n* and other FQs in Type III. However, if we take FQs in Type III as having a single-peaked curve, then they should behave as other types of FQs, as discussed above.

7.3.4 Deductive or intuitive

The propositions involving FQs behave like *modal statements* in the way that they may be not logically true. Let us consider,

- a. In a warm spring, flowers may bloom.
- b. John must be happy. (7.26)
- c. Many students are happy.

In (a), the proposition expressed by the sentence is true if and only if some world is consistent with the knowledge of a warm spring, flowers and bloom, and whether flowers do bloom in a warm spring. The truth of (a) is associated with worlds consistent with general knowledge. On the other hand, the proposition conveyed by (b) is interpreted as true with all worlds that an individual, speaker or hearer, knows. It may not be logically true in every world, because John cannot be happy every single minute of his entire life. However, it may be interpreted as a true proposition according to what the speaker knows, e.g. that John is indeed happy, because he has just married Mary with whom he is deeply in love. Finally, the truth of the proposition expressed by (c) can be determined if we know that in a given context there is a set of students who are happy, and its size is larger than a normal expectation that we would have. The three examples are similar in the sense that they are not deductive entailments. However, (c) differs from (a) and (b) in that (c) does not need to be true in some/all accessible worlds, only in this one. That is, (c) is true iff the cardinality of the set of happy students in this world is greater than a norm.

Then, let us look at the inference pattern in (7.27), quoted from Cann (1993:

263).

- a. The Morning Star is the Planet Venus.
 - b. The Evening Star is the Morning Star.
 - c. Therefore, the Evening Star is the Planet Venus.
- (7.27)

It is the rule called *Leibniz's Law* or the *Law of Substitution* that guarantees the validity of the inference in (7.27). The rule asserts that the truth value of a formula can be maintained, if the expressions substituted are extensionally equivalent to one another. In (7.27), since all three terms, *the Morning Star*, *the Evening Star* and *the Planet Venus*, denote the same entity, they are substitutable for each other according to the Law of Substitution. A formal definition of the law is given in (7.28),

$$(m = n) \rightarrow [\psi \leftrightarrow \psi^{n/m}] \quad (7.28)$$

which says that if two entities m and n are extensionally equivalent, then the truth value of a formula ψ is equivalent to the formula formed from ψ by substituting an instance of n for every instance of m (see Cann (1993: 263) for more discussion). However, the Law of Substitution may collapse in inferences involving FQs. For instance,

- a. Many students are New Zealanders.
 - b. Many New Zealanders are friendly.
 - c. Therefore, many students are friendly.
- (7.29)

Since the two expressions in (7.29), *many students* and *many New Zealanders* may not be extensionally equivalent, the inference in (7.29) is not necessarily valid. However, if the set of New Zealanders (common noun) and the set of students (common noun) are identical, then the inference goes through. It appears that FQs are extensional to the extent that one can substitute extensionally identical common nouns and maintain the truth. The problem

is normally we do not know whether or not the two sets are extensionally equivalent, i.e. it is not simply a matter of syntax.

The claim that FQs are extensional to a certain degree is related to a question that whether or not the axiom of extensionality works with FQs. The axiom asserts: where S and S' are any sets, S is identical with S' iff for any object x , x belongs to S iff x belongs to S' . Tye (1990) has some doubt about the axiom, and says: "The statement schema—call it ' A '—that x belongs to S if, and only if, x belongs to S' has assignments under which it is not true, since there are objects that are borderline members of S and S' . However, A is not false under these assignments. Rather, by the truth-table for \leftrightarrow , it is neither true nor false. So, the universally quantified statement $(x)A_x$ is neither true nor false. So, the statement, $S = S' \leftrightarrow (x)A_x$, has an indefinite right hand side in the above case. So, the Axiom of Extensionality comes out as *indefinite* under the proposed semantics."

However, if the set of, for example, students is the same as the set of, for example, ginger-haired people, then if *Many students are happy* is true then so is *Many ginger-haired people are happy*. So, the axiom of extensionality holds for FQs. This is expected to be valid for all FQs. What is significant is that FQs are (or may be) epistemically inconsistent, i.e. their precise interpretation is tied to the speaker and also controlled by the context⁴.

Moreover, with logical quantifiers which are independent of context a syllogism such as "All Xs are Ys and all Ys are Zs, therefore all Xs are Zs" is deductively valid. If we use a deductive syllogism with FQs which are dependent of context, it would not work, as shown in (7.29) above, or it would work only if a certain condition is fulfilled, such as if common nouns involved are extensionally equivalent. The problem with the syllogistic reasoning of nonlogically quantified propositions is that truth conditions are satisfied with respect to the situation in question. FQs do not allow conventional syllogism, i.e. reasonings with FQs are non-deductive. For more discussion, see Altham (1971), Johnson-Laird (1983) and Moxey and Sanford (1993b)⁵.

⁴This is probably also true for all or most expressions in the language.

⁵Moxey and Sanford (1993b: 112) suggest that FQs and the like allow simple heuristic procedures. For instance, should one make an inquiry about the failure rate for a course

In conclusion, it appears that logical reasoning has limited application in the entailment of FQs, and that an intuitive reasoning is more commonplace.

7.4 Conclusion

The introduction of FST into GQT models shows that the membership function provides a more intuitive treatment. The intensional treatment here shows that the extension of FQs is not part of objective meaning, and is not accessible to language users in general. It is possible that even a fixed context would not determine the truth value for a proposition with an FQ. On the other hand intension is, relatively speaking, more public and objective. The relation between intension and extension of FQs is not determinate. It is demonstrated in Section 7.2 that the relation between intension and extension, contextual factors, and individual differences can be represented by intensional models involving a membership function. Finally, the investigation on fuzzy entailment deepens our understanding of inferential patterns of FQs.

In general, the work of formal semantics of FQs discussed in the last three chapters is significant in the sense that it enables us to identify and explain linguistic phenomena associated with FQs in a formal system.

and be told that *Many students failed*, one may think that the chance of failure is great. If the quantifier is *few* instead, then the chance of success is great. This kind of reasoning is to derive a conclusion with respect to available possibilities.

8

Implications and Conclusions

Quantifiers play an important role in linguistics. From the above discussion, it appears that semantic imprecision of FQs does exist and can be represented by a degree of membership. It is suggested that fuzziness is an inherent property of language, and consequently any adequate linguistic theories must account for it.

An important conclusion from my work is that FQs are compositional and motivated. This is also expected to be applicable, in principle, to other kinds of semantic imprecision. In this thesis we have dealt exclusively with FQs that are countable. It is expected that the same argument would apply to other similar expressions, like FQs that are not countable, probability and frequency items. Some parallels are obvious, e.g. *many* vs *much*, *occasionally* vs *a few*, *rarely* vs *few*, *often* vs *many*. This is an interesting area for further investigation.

As stated in the introduction, FQs are explored here in terms of denotation and membership function, i.e. a fuzzy semantic approach is used. It is semantic patterns of FQs we aim for, rather than try to solve the fuzziness itself, which is something we can hardly achieve. In other words, what is being focused on here is not the fine-grained scaling, but overall semantic trends. The membership function represents semantic patterns of FQs, and

indicates the ways in which FQs are interpreted. The semantic patterns explored give rise to ways in which we can predict the semantic tendency of FQs.

The formal models given in this thesis provide a mechanism for how a membership function can be defined with respect to FQs, using FST. The formal semantics of FQs is represented here by a degree of membership and a degree of truth, particularly by dealing with inference where truth is fewer than 1 and greater than 0. A distinctive characteristic of fuzzy quantification is that the number of elements in a domain which must satisfy a predicate is not precisely given by an FQ and so a proposition containing one may be more or less true depending on how closely the elements approximate to a given norm.

This chapter concludes our semantically oriented discussion of FQs. In this final chapter, I will discuss some interdisciplinary implications in areas that are relevant to my work, since it is shown that the matter of the semantics of FQs does not stand alone, but interfaces with other fields. Finally, I will draw up some general conclusions.

8.1 Semantic implications

It is claimed in my work that an expression can be considered as a fuzzy set with uncertain meaning boundary. An entity as a meaning component of an expression might belong to that expression to a degree. What does this indicate in semantics then?

8.1.1 The properties of compositionality and motivation

As discussed in Chapter 4, compositionality says that the meaning of an expression is a function of the meaning of its parts. Whether or not a semantic theory is adequate depends on its ability to sustain compositionality. The importance of the Principle of Compositionality is that the information contained in an expression is not lost during its interpretation, and information not connected with the subparts of the expression does not arbitrarily contribute to the meaning of that expression. A fundamental assumption in semantics is that if compositionality is out, then so is the provision of any sensible semantic theory (see Section 4.2 for further discussion of this point). My work on FQs here supports the assumption. The important thing is how we come to the conclusion, taking the fuzziness of FQs into consideration. The argument is that an FQ, its core meaning in particular, licenses the same way by which its basic membership function is derived. Also, FQs are motivated. That is, even if one cannot maintain that once a membership function is defined for an FQ then it is always applicable, one should and must be able to say that given a basic membership function it will vary only with certain limits defined by context. For example, *about n* is normally expected to have a bell-shaped curve. Over context, it may get thinner or have the ends cut off or skewed a bit, but *about n* can never be interpreted as having a monotonic curve.

Compositionality of FQs provides a mechanism by which a certain type of approximator has the same impact towards expressions being modified. Therefore, FQs are not idiomatic. An idiom has at least two characteristics (Channell 1983: 176): its meaning is not accessible from the sum of its parts and there is some degree of constraint on replaceability of items within it. FQs do not actually have the two characteristics. For instance, in *about 20* and *about 200*, *about* is accessible as an approximator to bring the same semantic pattern (i.e. *approximate to n*) to both FQs. That is, it functions towards 20 and 200 in the same way. Also, *about* in *about n* is replaceable by *approximately* or *around*, i.e. they differ little semantically. So, in the light of the two factors aforementioned, FQs do not fit in the category of *idiom*. They are compositional.

Consequently, it is shown that although no sharp line is found in the meaning boundaries of FQs and most likely in fuzzy expressions of any kind, the heterogeneity is systematic and definable. To take the core/peripheral system as an example, the core in my definition lays a standard for the interpretation of FQs, a peripheral member which can be selected by how closely they conform to the core. The importance of the core is to make a paradigm for peripheral members. The functional heterogeneity is also shown by the membership function. The membership function should not be regarded as a tool just for designing a precise interval of an FQ using exact numbers, which is too simplistic (see Section 5.2.7 for more discussion). As for the significance of the membership function, Newstead (1988: 66) states: "The complexity of quantifiers can perhaps be captured by assuming that they can be represented as fuzzy sets which can vary along an analogue scale. To illustrate this, in the absence of context, *often* might have a membership function corresponding to a normal curve with a mean of 65% and a standard deviation of $\pm 10\%$. The effect of context would be to alter the mean value, but the membership function itself might be relatively unchanged." The conclusion from my work supports Newstead's claim in principle (see Chapter 4).

8.1.2 Internal structure

Following the claim of compositionality on FQs, there is also an implication on the internal structure in FQs. The significance of the study of FQs is not only the issue of degree of truth itself, but also the issue of internal structure. There are two structural characteristics of FQs: prototypicality and membership function.

In terms of prototypicality of FQs, a set of ordering numbers are selected depending on how close each number in the set approximates to a norm—a prototype. This can be captured by the membership function. For instance, suppose a norm for *about 20* is the number 20, then we may assume that 19 belongs to *about 20* more than 15. That is, the membership of 19 would be higher than that of 15. The same principle is suspected to work for any other type of semantic imprecision. An immediate example that comes to mind is Channell's work in 1983. She gives an explicit analysis of fuzzy

conceptual categories, like *fruit and something like that* and *car or something*, apart from the analysis of number approximations. She claims empirically that the same general principle of manipulation is at work in both cases. Heider's (1971) work on the processing and formation of natural categories also provides empirical evidence to show ranking of members based on their distances from a *prototype*.

This kind of internal structure of fuzzy expressions also reveals inferential behaviours by which people perceive a fuzzy expression: placing potential members according to their similarities to a prototype or best example. In other words, degree of membership is determined by degree of similarity to a norm. The point is that it is a mistake to regard natural language as something that is extremely incoherent and arbitrary, impossible to treat systematically.

8.1.3 Semantics and pragmatics

A study of the meaning of FQs would be more adequate if we explore both semantic and pragmatic factors. Since my work here is primarily semantics, so the issue of how exactly contexts affect the interpretation of FQs is left to pragmatics to solve.

From the discussion of FQs in this thesis (e.g. Chapter 4), it appears that contextual factors play a crucial role in interpreting FQs. This indicates that semantics and pragmatics are intermingled, and probably the study of semantics should not be totally independent from that of pragmatics.

However, we cannot say that pragmatics controls over semantics, and pragmatics theory can solve fuzziness alone. As claimed in Chapter 4, pragmatic factors only function effectively on the interpretation of FQs, but have no significant impact on general semantic pattern of FQs, core meaning/truth condition in particular. That is to say a semantic pattern for a certain type of FQs would remain the same with or without the consideration of pragmatic factors. The fact that the general patterns maintain the same over contexts and individuals is because the heterogeneity of FQs is functional, as argued

in the previous two sections.

8.2 Formal semantic and logic implications

There are two aspects in terms of formal semantics and logic that we will discuss in this section.

8.2.1 A generalization of conventional formal theories

As mentioned earlier, fuzziness is not well-represented in conventional formal semantics. For decades, we have contented ourselves with meaning glosses which are, as Labov (1973) says, labels but not descriptions. The reason is probably the problematic nature of fuzziness and the fact that the conventional theory cannot cope with fuzziness. However, if we want to represent our languages appropriately, precision is certainly not the only phenomenon which should be studied. Imprecision also plays a key role. The two interact with one another, and it is a challenge for us to work out how to define fuzziness in a formal framework. The formal treatment of FQs given in this thesis raises a number of difficulties for the bivalent truth-conditional semantics and logic¹. The conventional true-or-false approach has an inability to cope with the complex and subjective phenomena exemplified by FQs.

Let us consider the proposition *Some students are girls*. A conventional model of this proposition would require a representation of a logical quantifier. For instance, that at least one student is a girl is a logical necessity in this model, and the logical possibility is that there are students who are not girls and girls who are not students. However, what I deal with in this thesis is the concept of a natural language quantifier, i.e. *some* in *Some students are girls* is considered as a non-logical quantifier, the same as *many* and *about*

¹The Principle of Bivalence says that any statement is either true or false, and the Principle of Biexclusion says no statement can have more than one semantic value of either true or false.

200. Its meaning is something like *several* or *a few*, which is a more natural interpretation than *at least one*, because in day-to-day communication we would interpret *some students* as something similar to *several* rather than *at least one*.

We need much more information to describe a model which represents non-logical quantifiers. That is, fuzzy semantics reveals more than what the conventional semantics can offer. Fuzzy semantics defines expressions like *some* a membership function, which represents its characteristic in natural language. In fuzzy semantics, the degree of truth approach does a better job in describing gradations. Instead of saying anything is either true or false, it can say to what degree the statement is true. In addition, fuzzy semantics can also capture a changing linguistic reality affected by time or spatial factors; or tell how far it has changed in degrees. Apart from dealing with a fuzzy set, membership function can represent a non-fuzzy set, i.e. it would give 1 or 0 to a non-fuzzy set, if that is the case. Therefore, it is reasonable to say that FST is a generalization of conventional formal theories.

It is concluded in my work that semantic imprecision can be manipulated more naturally, and defined more precisely in the framework of FST. As Smithson (1987: 1) comments: "Like many intuitively appealing concepts, the idea of a fuzzy set is simple. In classical set theory, an element either belongs inside a set or it does not. In fuzzy set theory, on the other hand, an element may belong *partially* to a set. Fuzzy sets have gradations of set membership and blurred boundaries, and so they resemble at first glance the kinds of categories ordinary people use in natural thought or communication." McCawley (1981: 380) also comments that in FST truth values between 0 and 1 have been proposed as an appropriate device for dealing with inexact concepts. With intermediate values one is not forced to draw an arbitrary distinction between members and non-members of a fuzzy set.

Moreover, in terms of FST in the practical sense, areas in which FST can successfully be applied include fund management systems, robots, sorting machines, helicopter control (unmanned), nuclear power plants, pattern recognition, operation research, document retrieval systems and expert systems. It appears that problems like processing imprecise information (Negoita, 1984), and approximate reasoning (Zadeh, 1984) can be solved most effectively by

using FST. The usefulness and significance of fuzzy models in both consumer products (e.g. washing machines and air conditioners) and industrial systems are due to the fact that the employment of FST makes the systems have high MIQ (Machine Intelligence Quotient). This leads to a rapid growth in the number and variety of applications of the FST (Smithson 1987). For instance, computers tend to have extremely rigid systems, which makes them much less flexible than a human brain and severely limits their ability in an uncertain situation. Machines need to be told how to mimic human inferential behaviour. Fuzziness is an inherent characteristic of human thinking and language, and so naturally machines have to know how to deal with fuzziness. In the case of fuzzy information, computers need to be given some kind of formal mechanism to process fuzzy input. FST improves the ability of computers by employing techniques like membership function to deal with fuzziness².

It is finally realized that much, perhaps most, languages are not precisely presented. There is little evidence to show that we can tackle fuzziness within the framework of classical approaches. Our discussion here has shown that one need not throw up one's hands in despair when faced by the problem of fuzziness. Fuzziness can be explored seriously within the framework of FST in a satisfactory manner. Newstead (1988: 63-64) says: "The conclusion that quantifiers are best regarded as fuzzy concepts seems inescapable. Attempts to determine *the* meaning of a quantifier have been noticeably unsuccessful in all areas of research. Clearly quantifiers have a range of meanings and there are no clear-cut boundaries between one term and another; it is difficult to see how this could be captured other than by notions such as derived from fuzzy set theory."

In general, by adopting FST in semantics, it is feasible to measure fuzziness in terms of degree of membership and truth. By this kind of quantitative measurement, we can define various different types of FQs. The significance of this is to provide a mechanism to show the regularities of the semantics of

²Of course, FST is not the only way; approaches like connectionist (Dolan, 1989; Sharkey, 1990) can also be used to deal with fuzziness. Also, FST can be developed to take account of new information. For example, in my work the notion of percentage of curve is introduced when talking about membership function; an intermediate value is also given for fuzzy entailment (see Chapter 6 for the details).

FQs and their compositional functions on the items with which they interact.

8.2.2 Core meaning, truth condition and inference pattern

In my formal treatment of FQs, models are built with a degree of truth. These capture natural language properties and shorten the gap between the formal framework and natural languages. As shown, a precise numerical value of any FQ is context-specific. However, the core meaning of FQs is not, which can determine the relation between an FQ and the entities that it can be used to refer to.

Similarly, although we may not know a proposition is actually true or false, we do know the truth condition that must be obtained for it to be true. This idea forms the basis of truth-conditional semantics, and it assumes that the core of a proposition is its truth condition. That is, to know the core of a proposition is to understand the truth conditions under which it could be true. For instance, *Many people went to the party* has a truth condition, *a significant number of people who went to the party*, which can be figured out without necessarily requiring the truth of the proposition to be known or knowable in any particular situation, although the interpretation of *a significant number of people* depends upon things like our normal expectation of it with respect to the situation in question.

Furthermore, core meaning determines truth condition or inference pattern. That is, if we know the core meaning of an FQ, then the truth condition of a proposition with this FQ and its inference pattern are expected to be predictable. For example, from the core meaning of *about 20 students*, we can predict that the truth condition for *About 20 students went to the party*, is *a number of students that approximate to 20*. Accordingly, its inference pattern is *20 students went to the party* entails *About 20 students went to the party*. Also, the inference is monotone increasing, both subject and predicate (see Section 5.1.2 for the details). The same principle may apply to all FQs.

A predictable relation between core meaning/truth condition and inference pattern makes the formal semantics of fuzzy expressions sensible. Otherwise, it may be difficult to establish any feasible formal models for fuzzy expressions.

8.3 General linguistic implications

It appears that language is fuzzy in the sense that almost all linguistic terms have "hazy" denotations, such as *tallness* and *redness* (see Lakoff (1987), Comrie (1989), Givon (1984, 1990), Langacker (1987, 1991) and Taylor (1989)). That is, if one comes to think about expressions in our language, most expressions have a fuzzy boundary, even some seemingly clear ones like *a dead person*. It is less clear whether a person is dead or not in the situation where a person has been in coma (i.e. brain dead) for a long long time.

However, as Martin Haspelmath (Linguist List: Vol-5-190) points out, the issue of fuzziness has not been addressed by the dominating paradigm in linguistics, the Chomskyan school. He says: "They assume without argument that categories are clear-cut, and that gradience has no place in linguistics theory. Due to the enormous prestige of Chomsky and numerical weight, they can get away with that although there is overwhelming evidence for fuzziness, but of course, fuzziness is hard to deal with if you think of human language as being like a programming language. Connectionist thought is only gradually beginning to have impact in theoretical linguistics."

8.3.1 Inherent property of language

As Burns (1991: 4) comments, Frege (1970) regards the existence of fuzziness in language as a defect, on the ground that a concept that is not sharply defined is a wrongly termed concept. Let us examine the two reasons Frege gives. One is that fuzzy expressions lack meaning. It seems to me that fuzzy expressions do have meaning, although they are fuzzy. Fuzzy meaning

shall be considered as part of meaning, because they are used in natural languages and serve the communicative purpose just as well as other kinds of expressions.

The other reason Frege gives is that the laws of logic fail when applied to those fuzzy concepts. Take the Law of Excluded Middle as an example—it cannot represent the working of a fuzzy concept. It again appears to be inadequate to say that if fuzzy language cannot be dealt with in existing theoretical frameworks, then it must be excluded. It seems to me that we should consider the matter in reverse. If our theoretical framework cannot handle the fuzzy language, then we ought to do something about the framework. In fact, my work shows that the Law of Excluded Middle should be rejected because it fails to provide a satisfactory treatment for those fuzzy concepts which exist pervasively.

Thus, fuzzy language should not be considered as a defect. It appears evident, at least in the English and Chinese data that we have here, that in natural language the existence of fuzziness is inherent rather than accidental. It appears that context does not resolve fuzziness, i.e. fuzziness cannot be removed by context—at least we can say this in a theoretic sense³. Context can make disambiguation take place, but it cannot make defuzzification happen. This is due to the very fact that fuzziness is inherent in natural language. We cannot change it at all. More importantly, there is no need to do so.

8.3.2 Not necessarily categorical

Fuzziness is a challenge to the theory of language. Behind most, if not all, theories of linguistic structure there is a common set of assumptions about the nature of structural units which can be called *categorical view*. Under traditional categorical view, as Labov (1973) outlines, the categorization is

³However, Linda Moxey (personal communication) states that if we want to claim on empirical grounds that context does not resolve fuzziness, we have to have a way by which we can extract context—which is impossible. There is no such thing as no context, and we cannot control all aspects of context.

such a fundamental and obvious part of linguistic activity that the properties of categories are normally assumed rather than studied. One of the assumptions is that of the feature of *discrete*—which assumes that categories are separated from each other by clear-cut discontinuities of form or function. However, in many areas of linguistics, it is problematic to implement this view, because of fuzziness. The uncertainty of FQs, for instance, undermines the application of the categorical theory and creates the limitation of the categorizing process.

We tend to make linguistic categories more rigid and absolute than they really are. This is in fact not compatible with natural language. My study of FQs here has to systematically reject the traditional categorical view, to explore the fuzziness. Chomsky (1957) suggests that any unclear case in grammar can be settled by “letting the grammar decide”. However, Ross (1973) raises a very serious question by examining the fake NP squish—the category of NP is not precisely defined at all. He points out that if we insist that the grammar is based on clear cases, then this may have the effect of making such delicate phenomena invisible to our scrutiny. In other words, the research strategy of dealing with the clear cases only, although heuristically valuable in the initial stages of studying a language, cannot be viewed as being a theoretically neutral strategy. For in effect, such a strategy makes an empirical claim that such incremental or delicate processes do not exist. However, it is evident that they do exist, therefore they shall be treated as being of central importance in linguistic studies. Lakoff (1973b) points out that a comprehensive study of fuzziness is much needed. We should examine our languages and the world in a different way, as opposed to the conventional way that everything should be represented precisely.

Labov (1973: 347) points out correctly: “If we are to take seriously the categorical property of language, we must pass beyond the categorical view which takes it as given, and study the process of categorization itself by focusing on such discontinuities directly. By avoiding the categorical view, or some equally rigid principle of distribution, we are free to study the real properties of such boundaries, and deduce the higher level properties which govern the use of language.”

8.4 Psychological implications

It is shown in my work that the meanings of FQs vary over individuals, and a specific value for an FQ in a particular utterance is a business that interacts a number of fields, such as semantics, pragmatics, psychology, like a matter of problem solving.

FST provides a way of representing fuzziness in psychology, as it does for the semantics of FQs in my work. Smithson (1987: 75-76) comments that discussions on FST have contributed greatly to a general realization of fuzzy phenomena. The proper study of fuzzy phenomena has long been hindered by the lack of a workable vocabulary and conceptual framework, but the situation is changing partly because of the stimulus provided by FST. These changes represent rather more than the mere "suburbanization" of the knowledge domain. They are part of a widespread trend involving fundamental shifts in how people perceive and manage certitude, doubt, and risk. He lists the following points which imply the significance of FST in social sciences, including psychology:

1. The gradient thesis: People's natural categories behave like fuzzy sets.
2. Many concepts in the human sciences are unavoidably fuzzy, since they are graded.
3. The ways in which people manipulate and structure their categories correspond to the rules in FST.

Smithson (1987: 75-76) says: "In the human sciences we certainly need the flexibility to adapt and modify fuzzy set theories according to either substantive findings or our own conceptual requirements. Cognitive psychology and psycholinguistics could both contribute immensely to this portion of the dialogue ... For researchers in many fields where human thought and language are modeled only heuristically or indirectly, fuzzy set theory undoubtedly provides a more coherent language in which to operationalize a number of concepts. Many observers have said that the true test of fuzzy set theory lies

in its application to the solution of problems that conventional frameworks cannot handle."

The semantic and inference patterns discussed in this work give an indication of how people perceive and process FQs. In particular, it is shown that the meaning of an FQ is interpreted by some general rules, such as the processing of an element by relating it to a prototype appropriate to a particular FQ. This kind of pattern is also proved to be applicable to other fuzzy categories, such as *bird* in Rosch (1973). Similarly, Harrinton (1994) claims empirically that people act more quickly to a prototype than to a peripheral member. A general implication is that the approximative inference seems not unusual in everyday reasoning, and it plays an important part in the study of everyday reasoning. This phenomenon shall not be overlooked in cognitive and psychological studies.

It is also claimed in my work that the application of the actual numbers for an FQ's specific meaning is *motivated*—neither arbitrary nor predictable (see Section 4.2). Similarly, as Lakoff (1987: 346) points out, in the human conceptual system motivation is also an important phenomenon. He claims: "The reason is this: it is easier to learn something that is motivated than something that is arbitrary. It is also easier to remember and use motivated knowledge than arbitrary knowledge." A conclusion given by Lakoff (1987: 538) is that motivation is a global property of a conceptual system.

Harrinton (1994) comments: "Motivation was proposed in Lakoff (1987) as a cognitive structuring principle manifested in a range of lexicosemantic structures, particularly in polysemy (i.e. where multiple senses are linked within an individual lexical item). Although polysemy is a pervasive fact of natural language, a tenable cognitive account of the phenomenon has proved elusive. Thus, Lakoff's proposal has important implications for our understanding of the mental lexicon." Harrinton conducted three tests which were intended to prove the assumption that motivation plays a central role in the ontology and organization of sense relations and it is directly reflected in processing; also, it is assumed that motivated structures are learned faster and recalled better than non-motivated structures. He tested the assumption by examining performance on learning and recall tasks involving motivated and non-motivated polysemy senses. It was reported that the results were generally consistent

with the motivation construct, and responses on the motivated items tended to be faster and more accurate.

Also, Lakoff (1987: 9) suggests a shift from classical categories to prototype-based categories. It is a change that implies other changes: changes in the concepts of truth, knowledge, meaning, rationality. He lists some familiar ideas along the line of the classical categories, which will have to be left behind:

1. The mind is separate from, and independent of, the body.
2. Emotion has no conceptual content.
3. Reason is transcendental, in that it transcends—goes beyond—the way human beings, or any other kinds of beings, happen to think. Mathematics is a form of transcendental reason.
4. There is a correct, God's eye view of the world—a single correct way of understanding what is and is not true.
5. All people think using the same conceptual system.

Oden (1977a) believes that his work provides a beginning for the development of a model of what he calls fuzzy psycho-logic. He thinks that the standard logic does not characterize much of our subjective experience and knowledge. He states that humans are competent at processing fuzzy information, which has a very important implication for the nature of nearly all semantic information processing. Oden (1977b) discusses how degree-of-class-membership information in semantic memory may be used to determine the degree to which various interpretations of sentences are perceived to be sensible. Also, the preference of which interpretation of an ambiguous sentence may be determined by this sensible information. A theory of fuzzy semantic information processing is involved in a better understanding of the nature of the basic cognitive processes used in dealing with fuzzy information, which constitutes the primary goal of the fuzzy psycho-logic proposed by Oden.

Finally, Moxey and Sanford's (1993b) work shows great potential to explore natural language quantifiers psychologically. The significance of Moxey and Sanford's work is to explore natural language quantifiers from non-numerical perspectives. As an illustration, a theory of attentional focus⁴ on natural language quantifiers is developed in their work, which gives new insights in the study of natural language quantifiers. The general idea is that different natural language quantifiers manipulate different inferential patterns, and pick up different subsets, illustrated in (8.1).

- a. Few people went to the meeting. They went to the party instead.
 - b. A few people went to meeting. They enjoyed it.
- (8.1)

Few can be thought of as putting emphasis on what they term a compset, which is the set of people who did not go to the meeting; whereas *a few* puts focus on what they term a refset, the set of people who did go. The focus theory is not only relevant to Type I FQs in my terms, but also applicable to FQs of Types II and III. For instance:

- a. At least 20 people went to the meeting. They felt that it was an important meeting.
 - b. At most 20 people went to the meeting. They felt that it was not an important meeting.
- (8.2)

In (a), *at least n* generates a refset, while in (b) *at most* focuses a compset.

It is commonly stated that natural language quantifiers serve to identify numerical values. For instance, *few* identifies a smaller proportion than *many*.

⁴Moxey and Sanford (1993b: 57) define *attention* as: "one subset can be given priority in processing over others, and this subset will form the basis for inference that will be drawn."

However, a problem is that quantifiers may serve to identify similar proportions (e.g. *a few* and *not many*), yet produce somewhat different representations when they are used. Moxey and Sanford's work shows empirically that these expressions serve to put *focus* into different subsets of the superset upon which they operate. In their work, natural language quantifiers are treated as having a major function in manipulating attentions and patterns of inference. They are *mental operators*, and exert an influence on the patterns of inference which follow utterances containing such operators; that is, they are controllers of what is attended to.

Moreover, Moxey and Sanford integrate psychology with linguistics and logic, to capture far-reaching properties of quantifiers. Moxey and Sanford's approach is plausible in that it opens up a wide range of topics for the study of natural language quantifiers, such as natural language quantifiers manipulate attentions, inference patterns and provide complex information regarding the speaker's expectations (see Section 2.4 for more details). Their work also has some bearing on technology, like expert systems and AI. Any formalism for representing or interpreting natural language quantifiers, especially in decision-making technology, would be more adequate if taking non-numerical meaning into account, such as attentional focus.

It appears that a potential area for exploring natural language quantifiers is psychology of language. This kind of study may explore properties that linguistics would not normally cover. Certainly, an integration of linguistics and psychology is predicted to make the study of natural language quantifiers more significant.

8.5 Conclusions

The study of FQs in this thesis explores the question of how the semantic theory could be improved. It also raises a serious question challenging the assumption that languages should always be precise. The most important conclusion is that fuzziness is an inherent characteristic of language, rather some feature of performance or context-dependent result. That is to say

that "unfortunately" there is no invariable and idealized natural language in the world. Fuzziness appears to be an extremely pervasive phenomenon, invading almost every area of linguistics. Fine (1975) states that any type of expression that is capable of meaning is also capable of being fuzzy: names, name-operators, predicates, quantifiers, and even sentence-operators. We should become aware that lesser preoccupation with precise analyses and greater acceptance of fuzziness would result in more real progress in linguistic research and any other research involving fuzziness.

Consequently, we have to make our theories (and machines) capable of dealing with and accounting for fuzziness in natural languages. For example, the studies of fuzziness are expected to contribute (e.g. provide linguistic models) to the formalization of natural languages in areas such as soft computing which is tolerant of imprecision, as opposed to conventional (hard) computing. It is suggested that an integration of linguistics, logic, and psychology would be ideal to advance further research on fuzziness, and reach more powerful solutions.

Finally, it should be emphasized that the issues discussed in this thesis are complex and controversial. My work here has raised more questions than it has answered. It is hoped that this study can attract much more attention to fuzzy phenomena in natural languages, and will inspire more interest in fuzzy phenomena.

References

Aarsleff, H. (1982) *From Locke to Saussure: Essays on the Study of Language and Intellectual History*. London: Athlone.

Aldridge, M.V. (1982) *English Quantifiers*. England: Avebury Publishing Company.

Allen, A.D. (1973) 'A method for evaluating technical journals on the basis of published comments through fuzzy implications: a survey of the major IEEE transactions'. *IEEE Trans. Syst. Man Cybern.* 3: 422-425.

Allwood, J. (1981) 'On the distinction between semantics and pragmatics'. In W. Klein & W. Levelt (eds.) *Crossing the Boundaries of Linguistics*. Dordrecht: Reidel, 177-189.

Altham, J.E.J. (1971) *The Logic of Plurality*. London: Methuen.

Altham, J.E.J. & N.W. Tennant (1975) 'Sortal quantification'. In E. L. Keenan (ed.) *Formal Semantics of Natural Language*. London: Cambridge University Press, 46-58.

- Aho, A. V., B. W. Kerningham & P. J. Weinberger (1988) *The AWK Programming Language*. Addison: Wesley.
- Barton, S.B. & A.J. Sanford (1990) 'The control of attributional patterns by the focusing properties of quantifying expressions'. *Journal of Semantics* 7: 81-92.
- Barwise, J. & R. Cooper (1981) 'Generalized quantifiers and natural language'. *Linguistics and Philosophy* 4: 159-219.
- Belnap, N.D.Jr (1977) 'A useful four-valued logic'. In M. Dunn & G. Epstein (eds.) *Modern Uses of Multiple-valued Logic*. Dordrecht: Reidel, 5-37.
- van Benthem, J. (1982) 'The logic of semantics'. In F. Landman & F. Veltman (eds.) *Varieties of Formal Semantics* (Proceedings of the Fourth Amsterdam Colloquium). Dordrecht: Foris Publications, 55-80.
- van Benthem, J. (1984) 'Questions about quantifiers'. *Journal of Symbolic Logic* 49: 443-466.
- van Benthem, J. (1986) *Essays in Logical Semantics*. Dordrecht: D.Reidel Publishing Company.
- Black, M. (1937) 'Vagueness'. *Philosophy of Science* 4: 427-455.
- Black, M. (1949) *Language and Philosophy*. Ithaca: Cornell University Press.
- Black, M. (1963) 'Reasoning with loose concepts'. *Dialogue* 2: 1-12.
- Blockley, D.I. (1975) 'Predicting the likelihood of structural accidents'. *Proc. Inst. Civ. Eng.*, 59 (Part 2), 659-668.
- Blockley, D.I. (1978) 'Analysis of subjective assessments of structural failures'. *Int. J. man-Mach. Stud.* 10: 185-195.
- Bochvar, D.A. (1939) 'On a three-valued logic calculus and its application to the analysis of contradictories'. *Matematicheskii Sbornik* 4: 287-308.

Bolinger, D. (1961a) 'Syntactic blends and other matters'. *Language* 37: 366-381.

Bolinger, D. (1961b) *Generality, Gradience, and the all-or-none*. The Hague: Mouton & CO. N.V..

Bolinger D. (1972) *Degree Words*. The Hague: Mouton & CO. N.V..

Borges, M.A. & B.K. Sawyers (1974) 'Common verbal quantifiers usage and interpretation'. *Journal of Experimental Psychology* 102: 335-338.

Brown, J. (1979) 'Vocabulary: learning to be imprecise'. *Modern English Teacher* 7(1): 25-27.

Budescu, D.V. & T.S. Wallsten (1985) 'Consistency in interpretation of probabilistic phrases'. *Organizational Behaviour and Human Decision Processes* 36: 391-405.

Budescu, D.V., R. Zwick, T.S. Wallsten, & I. Erev (1990) 'Integration of linguistic probabilities'. *International Journal of Man-Machine Studies* 33: 657-676.

Budescu, D.V. & T.S. Wallsten (1987) 'Subjective estimation of precise and vague uncertainties'. In G. Wright & P. Ayton (eds.) *Judgemental Forecasting*. Chichester: John Wiley & Sons Ltd, 63-82.

Budescu, D.V. & T.S. Wallsten (1990) 'Dyadic decisions with numerical and verbal probabilities'. *Organizational Behaviour and Human Decision Processes* 46: 240-263.

Burns, L.C. (1991) *Vagueness—An Investigation into Natural Languages and the Sorites Paradox*. Dordrecht: Kluwer Academic Publishers.

Butnariu, D. (1977) 'L-fuzzy automata-description of a neural model'. In J. Rose & C. Bilciu (eds.) *Modern Trends in Cybernetics and Systems* (Vol 2). New York: Springer-verlag, 119-124.

Cann, R. (1993) *Formal Semantics*. Cambridge (U.K.): Cambridge University Press.

Carnap, R. (1952) 'Meaning Postulates'. *Philosophical Studies* 3: 65-73.

Chang, S.K. (1971) 'Picture-processing grammar and its application'. *Information Science* 3: 121-148.

Channell, J. (1980) 'More on approximations: a reply to Wachtel'. *Journal of Pragmatics* 4: 461-476.

Channell, J. (1983) *Vague Language: Some Vague Expressions in English*. Ph.D. thesis, Department of Language, University of York, U.K.

Channell, J. (1994) *Vague Language*. Oxford: Oxford University Press.

Chierchia, G. & S. McConnell-Ginet (1990) *Meaning and Grammar: an Introduction to Semantics*. Cambridge (Mass): The MIT Press.

Chomsky, N. (1957) *Syntactic Structures*. The Hague: Mouton and Company.

Clark, D.A. (1990) 'Verbal uncertainty expressions: a critical review of two decades of research'. *Current Psychology: Research and Review* 9(3): 203-235.

Clark, H.H. (1990) 'Comment on F. Mosteller, & C. Youtz "Quantifying probabilistic expressions"'. *Statistical Science* 5: 2-34.

Clark, H.H. (1991) 'Words, the world, and their possibilities'. In G.R. Lockhead & James R. Pomerantz (eds.) *The Perception of Structure: Essays in Honor of Wendell R. Garner*. Washington DC: American Psychological Association.

Cliff, N. (1957). 'Adverbs as multipliers'. *Psychological Review* 66: 27-44.

Cohen, B.L. (1986) 'The effect of outcome desirability of comparisons of linguistic and numerical probabilities'. MA thesis, University of North Carolina at Chapel Hill, U.S.A..

Coleman, L. & P. Kay (1981) 'Prototype semantics: the English verb lie'. *Language* 57: 26-44.

Collins, A. (1978) 'Fragments of a theory of human plausible reasoning' In D. Waltz (ed.) *TINLAP-2 Theoretical Issues in Natural Language Processing-2*. Urbana, IL: University of Illinois, U.S.A..

Comrie, B. (1989) *Language Universals and Linguistic Typology*. Oxford: Blackwell.

Copilowish, I.M. (1939) 'Border-line cases, vagueness, and ambiguity'. *Philosophy of Science* 6: 181-195.

Coppo, M. & L. Saitta (1976) 'Semantic support for a speech-understanding system, based on fuzzy relations'. *Proceedings of International Conference for Cybernetics Society*. Washington D.C., 520-524.

Crystal, D. & D. Davy (1975) *Advanced Conversational English*. London: Longman.

Crystal, D. (1991) *A Dictionary of Linguistics and Phonetics*. Oxford: Blackwell Publisher.

Danell, K.S. (1978) 'The concept of vagueness in linguistics: some methodological reflections of a non-specialist'. *Studia Neophilologica* 50: 3-24.

De Swarts, H.E. (1991) *Adverbs of Quantification: A Generalized Quantifier Approach*. PhD thesis. Department of Dutch, Rijksuniversiteit Groningen, Netherlands.

Deese, J. (1965) *Structure of Associations in Language and Thought*. Baltimore: John Hopkins Press.

- Deese, J. (1974) 'Towards a psychological theory of the meaning of the sentences'. In A. Silverstein (ed.) *Human Communication: Theoretical Explorations*. New Jersey: Lawrence Erlbaum Assoc. Hillsdale.
- Dimitrov, V. (1977) 'Social choice and self-organization under fuzzy management'. *Kybernetes* 6: 153-156.
- Dolan, M.G. (1989) *Tensor Manipulation Networks: Connectionist and Symbolic Approaches to Comprehension, Learning and Planning*. PhD thesis, Computer Science Department, University of California at Los Angeles.
- Dowty, D.R. et al (1981) *Introduction to Montague Semantics*. Dordrecht: D.Reidel Publishing Company.
- Dubois, D. (1978) 'An application of fuzzy sets theory to bus transportation network modification'. *Proceedings of Joint Automat. Control Conference*. Philadelphia, U.S.A..
- Dubois, D. & H. Prade (1980) *Fuzzy Sets and Systems: Theory and Applications*. New York: Academic Press.
- Fillenbaum, S., T.S. Wallsten, B. Cohen & J.A. Cox (1987) 'Some effects of available vocabulary and communication task on the understanding and use of non-numerical probability expressions' (Report No. 177). Chapel Hill, NC: University of North Carolina, L.L. Thurstone Psychometric Laboratory.
- Fillmore, C.J. (1977) 'Scenes-and-frames semantics'. In A. Zampolli (ed.) *Linguistic Structure Processing*. Amsterdam: North Holland.
- Fine, K. (1975) 'Vagueness, truth, and logic'. *Syntheses* 30: 265-300.
- Fodor, J.D. (1977) *Semantics: Theories of Meaning in Generative Grammar*. Hassocks: Harvester.
- van Fraassen, B. (1966) 'Singular terms, truth-value gaps, and free logics'. *Journal of Philosophy* 63: 481-495.

van Fraassen, B. (1969) 'Presuppositions, supervaluations and free logic'. In K. Lambert (ed.) *The Logical Way of Doing Things*. New Haven: Yale University Press.

Frege, G. (1970) 'Grundgesetze der Arithmetik'. In Geach and Black (eds.) *Selections from Philosophical Writings of Gotlob Frege* (Vol II, Section 56). Oxford: Blackwell.

Fuhrmann, G. (1988) 'M-fuzziness in brain/mind modeling'. In T. Zétényi (ed.) *Fuzzy Sets in Psychology*. Elsevier Science Publishers B.V., 155-202.

Gamut, L.T.F. (1990) *Logic Language and Meaning* (Vol. I). To appear in the University of Chicago Press, ms.

Gazdar, G. (1979a) 'A solution to the projection problem'. In C.K. Oh & D. Dinneen (eds.) *Syntax and Semantics: Presupposition* (Vol II). New York: Academic Press, 57-90.

Gazdar, G. (1979b) *Pragmatics: Implicature, Presupposition and Logical Form*. New York: Academic Press.

Givon, T. (1984/1990) *Syntax: a Functional/Typological Introduction* (Vols I and II). Amsterdam: John Benjaming.

Goguen, J.A. (1968/1969) 'The logic of inexact concepts'. *Syntheses* 19: 325-373.

Goguen, J.A. (1975) 'On fuzzy robot planning'. In L.A. Zadeh *et al* (eds.) *Fuzzy Sets and Their Applications to Cognitive and Decision Processes*. Academic Press: New York, 429-447.

Goocher, B.E. (1965) 'Effects of attitude and experience on the selection of frequency adverbs'. *Journal of Verbal Learning and Verbal Behaviour* 4: 193-195.

Gottinger, H.W. (1973) 'Competitive processes: application to urban structures'. *Cybernetics* 16 (3): 177-197.

- Grice, H.P. (1967) 'Logic and conversation'. ms.
- Grice, H.P. (1975) 'Logic and conversation'. In P. Cole & J.L. Morgan (eds.) *Syntax and Semantics 3: Speech Acts*. New York: Academic Press, 41-58.
- Hakel, M.D. (1968) 'How often is often?'. *American Psychologist* 23: 533-534.
- Harnish, R.M. (1979) 'Logical form and implicature'. In K. Bath & R.M. Harnish (eds.) *Linguistic Communication and Speech Acts* 19. Cambridge (Mass.): The MIT Press, 313-391.
- Harrinton, M. (1994) 'Motivation in the mental lexicon'. PhD thesis, Programme in Experimental Psychology, University of California, Santa Cruz, U.S.A..
- Heider, E.R. (1971) 'On the Internal Structure of Perceptual and Semantic Categories'. Berkeley: University of California, ms.
- Hempel, C.G. (1939) 'Vagueness and logic'. *Philosophy of Science* 6: 163-180.
- Hersh, H.M. & A. Caramazza (1976) 'A fuzzy set approach to modifiers and vagueness in natural language'. *Journal of Experimental Psychology* 105 (3): 254-276.
- Higginbotham, J. & R. May (1981) 'Questions, quantifiers and crossing'. *The Linguistic Review* 1: 41-49.
- Hopper, P.J. & S.A. Thompson (1985) 'The iconicity of the universal categories *noun* and *verb*'. In J. Heiman (ed.) *Iconicity in Syntax*. Amsterdam: John Benjaming.
- Hooper, R. (1994) 'Type and instance nominalizations in Tokelauan'. Paper presented at the Linguistic Circle (27th Jan.), the University of Edinburgh, U.K..

Hörmann, H. (1982) 'Hidden determinants of understanding'. In J-F. Le Ny & W. Kintsch (eds.) *Language and Comprehension*. Amsterdam: North-Holland.

Hörmann, H. (1983) 'The calculating listener or how many are einige, mehrere, and ein paar (some, several and a few)?'. In R. Bauerle, C. Schwarze, & A. von Stechow (eds.) *Meaning, Use and Interpretation of Language*. Berlin: Walter de Gruyter.

Hurford, J.R. (1974) 'Exclusive or inclusive disjunction'. *Foundations of Language* 11: 409-411.

Jain, R. & H.H. Nagel (1977) 'Analyzing a real-world scene sequence using fuzziness'. *Proceedings IEEE Conference on Decision Control*. New Orleans, 1367-1372.

Johnson-Laird, P.N. (1983) *Mental Models*. Cambridge (U.K.): Cambridge University Press.

Jones, A. (1974) 'Towards the right solution'. *Int. J. Math. Educ. Sci. Technol.* 5: 337-357.

Jones, L.V. & L.L. Thurstone (1955) 'The psychophysics of semantics: an experimental investigation' *The Journal of Applied Psychology* 39(1): 31-36.

Jones, W.T. (1976) 'A fuzzy set characterization of interaction in scientific research'. *J. Am. Soc. Inf. Sci.* (Sept./Oct.): 307-310.

Joyce, J. (1976) 'Fuzzy sets and the study of linguistics'. *Pac. Coast Philol.* 11: 39-42.

Kamp, H. (1981) 'A theory of truth and discourse representation'. In J. Groenendijk, T. Janssen, & M. Stokhof (eds.) *Formal Methods in the Study of Language*. Amsterdam: Mathematical Centre Tracts, 135.

Kamp, J.A.W. (1975) 'Two theories about adjectives'. In E. Keenan (ed.) *Formal Semantics of Natural Language*. Cambridge (U.K.): Cambridge University Press.

Kay, P. & C.K. McDaniel (1978) 'The linguistic significance of the meanings of basic colour terms'. *Language* 54: 610-646.

Keenan, E.L. & J. Stavi (1986) 'A semantic characterization of natural language determiners'. *Linguistics and Philosophy* 9: 253-326.

Kempson, R.M. (1977) *Semantic Theory*. Cambridge (U.K.): Cambridge University Press.

Klein, E. (1980) 'A semantics for positive and comparative adjectives'. *Linguistics and Philosophy* 4: 1-45.

Klopp, A. von (1993) *Negation: Implications for Theories of Natural Language Processing*. PhD thesis. Center for Cognitive Science, University of Edinburgh, U.K..

Klopp, A. von (under review) 'Polarity items: are the right questions being asked?' For *Language and Philosophy*.

Kochen, M. (1975) 'Applications of fuzzy sets in psychology'. In L.A. Zadeh *et al* (eds.) *Fuzzy Sets and Their Applications to Cognitive and Decision processes*. New York: Academic Press, 395-408.

Kong, A., G.O. Barnett, F. Mosteller, & C. Youtz (1986) 'How medical professionals evaluate expressions of probability'. *The New England Journal of Medicine* 315 (12): 740-744.

Labov, W. (1973) 'The boundaries of words and their meanings', In C. J. Bailey & R. W. Shuy (eds.) *New Ways of Analyzing Variation in English*. Washington DC: Georgetown University Press, 340-373.

Labov, W. (1978) 'Denotational structure'. *Papers from the Parasession on the Lexicon*. Chicago Linguistic Society, 220-260.

Labov, W. (1985) 'Several logics of quantification'. In M. Wiepokuj *et al* (eds.) *Proceedings of the 11th Annual Meeting of the Berkeley Linguistics Society*. Berkeley: Berkeley Linguistics Society.

Lakoff, G. (1970) 'Repartee, or a reply to negation, conjunction and quantifiers'. *Foundations of Language* 6: 389-422.

Lakoff, G. (1973a) 'Hedges: a study in meaning criteria and the logic of fuzzy concepts'. *Journal of Philosophical Logic* 2: 458-508.

Lakoff, G. (1973b) 'Fuzzy grammar and the performance/competence terminology game'. In P. Peranteau *et al* (eds.) *Papers from the Ninth Regional Meeting of Chicago Linguistic Society*. Chicago: University of Chicago, 271-291.

Lakoff, G. (1975) 'Pragmatics in natural logic'. In E. L. Keenan (ed.) *Formal Semantics of Natural Language*. Cambridge (U.K.): Cambridge University Press, 253-286.

Lakoff, G. (1987) *Women, Fire, and Dangerous Things: What Categories Reveal about the Mind*. Chicago: University of Chicago Press.

Landy, F.J. & J.L. Farr (1980) 'Performance rating'. *Psychological Bulletin* 87: 72-107.

Langacker, R.W. (1987) *Foundations of Cognitive Grammar* (Vols I and II). Stanford: Stanford University Press.

Langacker, R.W. (1991) *Concept, Image, and Symbol: the Cognitive Basis of Grammar*. Mouton de Gruyter.

- Langedoen, D.T. (1970) *Essentials of English Grammar*. New York: Holt, Rinehart & Winston, Inc.
- Lee, S.C. & E.T. Lee (1974) 'Fuzzy sets and neural networks'. *Journal of Cybernetics* 4 (2): 83-103.
- Lyons, J. (1981) *Language and Linguistics*. Cambridge (U.K.): Cambridge University Press.
- Lyons, J. (1986) *Language, Meaning and Context*. Suffolk: Fontana Paperbacks.
- Matthews, P.H. (1981) *Syntax*. Cambridge (U.K.): Cambridge University Press.
- Margalit, A (1976) 'Vagueness in vogue'. *Syntheses* 33: 211-221.
- May, R. (1985) *Logical Form—Its Structure and Derivation*. Cambridge (Mass.): The MIT Press.
- McCawley, J.D. (1981) *Everything that Linguists have always Wanted to Know about Logic*. Oxford: Basil Blackwell.
- McConnell-Ginet, S. (1989) 'The construction of meaning: formal semantics and vagueness'. Department of Modern Languages and Linguistics, Cornell University, U.S.A.. ms.
- Meyer, C.F. (1992) *Apposition in Contemporary English*. Chicago: University of Chicago Press.
- Mosier, C.I. (1941) 'A psychometric study of meaning'. *Journal of Social Psychology* 13: 123-140.
- Montague, R. (1974) 'The proper treatment of quantification in ordinary English'. In R.H. Thomason (ed.) *Formal Philosophy: Selected Papers of Richard Montague*. New Haven: Yale University Press, 247-270.

Moore, T. (1993) 'Private minds, public language: Locke's inquiry into meaning'. Paper presented at the seminar of the Center for Cognitive Science and HCRC, the University of Edinburgh, U.K.

Morgan, J. (1972) 'Verb agreement as a rule of English'. In P. Peranteau *et al* (eds.) *Papers from the Eighth Regional Meeting of Chicago Linguistic Society*. Chicago: University of Chicago, 278-286.

Morgan, C.J. & F.J. Pelletier (1977) 'Some notes concerning fuzzy logics'. *Linguistics and Philosophy* 1: 79-98.

Mosteller, F. & C. Youtz (1990) 'Quantifying probabilistic expressions'. *Statistical Science* 5: 2-34.

Mostowski, A. (1957) 'On a generalization of quantifiers'. *Fundamental Mathematics* 44: 12-36.

Moxey, L.M. (1986) *A Psychological Investigation of the Use and Interpretation of English Quantifiers*. Ph.D thesis, Department of Psychology, University of Glasgow, U.K.

Moxey, L.M. & A.J. Sanford (1987) 'Quantifiers and focus'. *Journal of Semantics* 5: 189-206.

Moxey, L.M., A.J. Sanford & S.B. Barton (1990) 'Control of attentional focus by quantifiers'. In K.J. Gilhooly, M.T.G. Keane, R.H. Logie & G. Erdos (eds.) *Lines of Thinking* (1). Chichester: Wiley.

Moxey, L.M. (1991a) 'Expectation and the interpretation of quantifiers'. Department of Psychology, University of Glasgow, U.K.. ms.

Moxey, L.M. & A.J. Sanford (1991b) 'Context effects and the communicative functions of quantifiers: Implications for their use in attitude research'. In N. Schwarz & S. Sudman (eds.) *Context Effects in Social and Psychological Research*. New York: Springer-Verlag.

Moxey, L.M. & A.J. Sanford (1992) *Communicating quantities—a psychological perspective*. Department of Psychology, University of Glasgow, U.K.. ms.

Moxey, L.M. & A.J. Sanford (1993a) 'Prior expectation and the interpretation of natural language quantifiers'. *European Journal of Cognitive Psychology* 5 (1): 73-91.

Moxey, L.M. & A.J. Sanford (1993b) *Communicating Quantities—A Psychological Perspective*. Hove (U.K.): Lawrence Erlbaum Associates Ltd.

Moxey, L.M., A.J. Sanford & G. Grant (1993) 'Scenario characteristics and the interpretation of quantified statement'. Department of Psychology, University of Glasgow, U.K.. ms.

Negoita, C.V. (1984) *Fuzzy Systems and Expert Systems*. New York: Benjamin Cummings.

Newmeyer, F.J. (1986) *Linguistic Theory in America*. New York: Academic Press.

Newstead, S.E. (1988) 'Quantifiers as fuzzy concepts'. In T. Zétényi (ed.) *Fuzzy Sets in Psychology*. Elsevier Science Publishers B.V., 51-72.

Newstead, S.E. & J.M. Collis (1987) 'Context and the interpretation of quantifiers of frequency'. *Ergonomics* 30: 1447-1462.

Newstead, S.E. & R.A. Griggs (1984) 'Fuzzy quantifiers as an explanation of set inclusion performance'. *Psychological Research* 46: 377-388.

Norwich, A.M. & I.B. Turksen (1982) 'The fundamental measurement of fuzziness'. In R.R. Yager (ed.) *Fuzzy Set and Possibility Theory*. New York: Pergamon Press, 49-60.

Norwich, A.M. & I.B. Turksen (1984) 'A model for the measurement of membership and the consequences of its empirical implementation'. *Fuzzy Sets and Systems* 12: 1-25.

- Nowakowska, M. (1976) 'Towards a formal theory of dialogues'. *Semiotica* 17: 291-313.
- Oden, G.C. (1977a) 'Integration of fuzzy logical information'. *Journal of Experimental Psychology: Human Perception and Performance* 3 (4): 565-575.
- Oden, G.C. (1977b) 'Fuzziness in semantic memory: Choosing exemplars of subjective categories'. *Memory and Cognition* 5: 198-204.
- Oden, G.C. & N.H. Anderson (1974) 'Integration of semantic constraints'. *Journal of Verbal Learning and Verbal Behaviour* 13: 138-148.
- Pal, S.K. & D.D. Majumder (1978) 'Effect of fuzzification and the plosive cognition system'. *International Journal of Systems Science* 9: 873-886.
- Parducci, A. (1968) 'Often is often'. *American Psychologist* 23: 828.
- Partee, B. (1979) 'Semantics — Mathematics or Psychology'. In R. Bäuerle, U. Egli & A. von Stechow (eds.) *Semantics from Different Points of View*. New York: Springer-Verlag, 1-14.
- Partee, B. (1984) 'Compositionality'. In F. Landman & F. Veltman (eds.) *Varieties of Formal Semantics* (Proceedings of the Fourth Amsterdam Colloquium). Dordrecht: Foris Publications, 281-311.
- Partee, B., A. Meulen & R. Wall (1990) *Mathematical Methods in Linguistics*. Dordrecht: Kluwer Academic Publishers.
- Paterson, K. (1993) 'The immediacy of situation-based inferences in quantified texts'. Paper presented at the *International Conference on the Psychology of Language and Communication*. University of Glasgow, U.K..
- Peirce, C.S. (1902) 'Vagueness'. In J.M. Baldwin (ed.) *Dictionary of Philosophy and Psychology* (Vol. II). New York: Macmillan.

Pepper, S. & L.S. Prytulak (1974) 'Sometimes frequently means seldom: context effects in the interpretation of quantitative expressions'. *Journal of Research in Personality* 8: 95-101.

Pepper, S. (1981) 'Problems in quantification of frequency expressions'. In D. Fiske (ed.) *New Direction for Methodology of Social and Behavioural Science* 9.

Pinkal, M. (1983) 'Towards a semantics of precization'. In T. Ballmer & M. Pinkal (eds.) *Approaching Vagueness*. Amsterdam: North-Holland.

Pinkal, M. (1984) 'Consistency and context change: the sorities paradox'. In F. Landman & F. Veltman (eds.) *Varieties of Formal Semantics*. Dordrecht: Foris.

Pinkal, M. (1987) 'Imprecise concept and quantification'. *Linguistics and Philosophy* 4: 1-45.

Poulton, E.C. (1973) 'Unwanted range effects from using within-subject experimental designs'. *Psychological Bulletin* 80 (2): 113-121.

Powell, M. J. (1985) 'Purposive vagueness: an evaluative dimension vague quantifying expressions'. *Journal of Linguistics* 21: 31-50.

Prince, E. F., J. Frader & C. Bosk (1980) 'On hedging in physician-physician discourse'. Paper presented at the AAAL Symposium on Applied Linguistics in Medicine. San Antonio, TX.

Putnam, H. (1975) *Mind, Language, and Reality*. Cambridge (U.K.): Cambridge University Press.

Quirk, R., S. Greenbaum, G. Leech, & J. Svartvik (1972) *A Grammar of Contemporary English*. London: Longman.

Rapoport A., T. Wallsten, & J. Cox (1987) 'Direct and indirect scaling of membership functions of probability phrases'. *Mathematical Modelling* 9: 397-417.

- Rescher, N. (1969) *Many-valued Logic*. New York: McGraw-Hill.
- Richards, J., J. Platt & H. Weber (1985) *Longman Dictionary of Applied Linguistics*. Essex: Longman.
- Rieger, B.B. (1981) *Empirical Semantics*. Bochum: Studienverlag Dr. N. Brockmeyer.
- Rosch, E. (1973) 'On the internal structure of perceptual and semantic categories'. In T.E. Moore (ed.) *Cognitive Development and the Acquisition of Language*. New York: Academic Press.
- Rosch, E. (1975a) 'Cognitive representations of semantic categories'. *Journal of Experimental Psychology* 104: 192-233.
- Rosch, E. (1975b) 'Cognitive reference points'. *Cognitive Psychology* 7: 532-547.
- Rosch, E. & C.B. Mervis (1975) 'Family resemblances: studies of the internal structure of categories'. *Cognitive Psychology* 7: 573-605.
- Rosch, E., C.B. Mervis, D.M. Johnson & P. Boyes-Braem (1976) 'Basic objects in natural categories'. *Cognitive Psychology* 8: 382-439.
- Rosch, E. & B. Lloyd (1978) *Cognition and categorisation*. New York: Lawrence Erlbaum.
- Ross, J.R. (1972) 'The category squish: endstation hauptwort'. In P. Peranteau et al (eds.) *Papers from the Eighth Regional Meeting of Chicago Linguistic Society*. Chicago: University of Chicago.
- Ross, J. R. (1973) 'A fake NP squish'. In C.J. Bailey & R.W. Shuy (eds.) *New Ways of Analyzing Variation in English*. Washington D. C: Georgetown University Press, 96-140.
- Routh, D. (1994) 'On representations of quantifiers'. *Journal of Semantics* 10: 199-214.

- Russell, B. (1923) 'Vagueness'. *Australian Journal of Psychology and Philosophy* 1: 84-92.
- Sadock, J.M. (1977) 'Truth and approximations'. *Berkeley Linguistic Society Papers* 3: 430-439.
- Sadock, J.M. (1981) *Almost*. In P. Cole (ed.) *Radical Pragmatics*. New York: Academic Press, 257-271.
- Sainsbury, R.M. (1991) *Concepts without boundaries*. Department of Philosophy, King's College, University of London. Monograph.
- Sanford, D.H. (1976) 'Competing semantics of vagueness: many values versus super-truth'. *Syntheses* 33: 195-210.
- Sanford, A.J., L.M. Moxey, K. Doing-Harris, & G. Grant (submitted) 'Base-rate effects in the interpretation of quantifier'.
- Sanford, A.J., L.M. Moxey & G. Grant (submitted) 'Situation-based expectations and the numerical interpretation of quantifiers'.
- Sanford, A.J., L.M. Moxey, & K. Paterson (1994) 'Psychological studies of quantifiers'. *Journal of Semantics* 10: 153-170.
- Sanford, A.J., L.M. Moxey, & M.T. McGinley (in prep) 'Quantifiers as inference generators and focus controllers'.
- Sanford, A.J. & L.M. Moxey (1995) 'The psychological treatment of quantifiers'. ms.
- Schmidt, C. (1974) 'The relevance to semantic theory of a study of vagueness'. *Chicago Linguistic Society Papers* 10: 617-630.
- Sharkey, N.E. (1990) 'A connectionist model of text comprehension'. In D.A. Balota, G.B. Flores d'Arcais, & K. Rayner (eds.) *Comprehension Processes in Reading*. Hillsdale, NJ: Lawrence Erlbaum Associates Inc.

Sjöberg, L. (1980) 'Similarity and correlation'. In E.J. Lantermann & H. Feger (eds.) *Similarity and Choice*. Bern: Hans Huber, 70-87.

Simpson, R.H. (1944) 'The specific meanings of certain terms indicating differing degrees of frequency'. *Quarterly Journal of Speech* 30: 328-330.

Smithson, M. (1987) *Fuzzy Set Analysis for Behavioral and Social Sciences*. New York: Springer-Verlag.

Smithson, M. (1988) 'Possibility theory, fuzzy logic, and psychological explanation'. In T. Zéteñyi (ed.) *Fuzzy Sets in Psychology*. Elsevier Science Publishers B.V., 1-50.

Sorensen, R.A. (1990) 'Process vagueness'. *Linguistics and Philosophy* 13: 589-618.

Taranu, C. (1977) 'The economic efficiency—a fuzzy concept'. In J. Rose & C. Bilciu (eds.) *Modern Trends in Cybernetics and Systems* (Vol II). New York: Springer-verlag, 163-173.

Taylor, J. (1989) *Linguistic Categorization*. Oxford: Oxford University Press.

Teigen, K.H. (1988a) 'When are low-probability events judged to be "probable"? Effects of outcome-set characteristics on verbal probability estimates'. *Acta Psychologica* 68: 157-174.

Teigen, K.H. (1988b) 'The language uncertainty'. *Acta Psychologica* 68: 27-38.

Tye, M. (1989) 'Supervaluationism and the law of excluded middle'. *Analysis* (June).

Tye, M. (1990) 'Vague objects'. *Mind* 99 (396): 535-557.

Ullmann, S. (1962) *Semantics*. Oxford: Blackwell.

- Uragami, M., M. Mizumoto, & K. Tanaka (1976) 'Fuzzy Robot Controls'. *Journal of Cybernetics* 6: 39-64.
- Vaina, L. (1978) 'Semiotics of "with"'. *Versus Quad. Studi Semiot.* 19.
- Wachtel, T. (1980) 'Pragmatic approximations'. *Journal of Pragmatics* 4: 201-211.
- Wachtel, T. (1981) 'Distinguishing between approximations'. *Journal of Pragmatics* 5: 311-322.
- Wallsten, T.S., D.V. Budescu, A. Rapoport, R. Zwick, & B. Forsyth (1986a) 'Measuring the vague meanings of probability terms'. *Journal of Experimental Psychology: General* 115: 348-365.
- Wallsten, T.S., S. Fillenbaum & J.A. Cox (1986b) 'Base rate effects on the interpretations of probability and frequency expressions'. *Journal of Memory and Language* 25: 571-587.
- Wallsten, T.S., D.V. Budescu, & I. Erev (1988) 'Understanding and using linguistic uncertainties'. *Acta Psychologica* 68: 39-52.
- Wallsten, T.S., D.V. Budescu (1990) 'Comment'. *Statistical Science* 5: 23-26.
- Webber, B.L. (1978) 'Jumping ahead of the speaker: on recognition from indefinite descriptions'. Paper presented at the Sloan Workshop on Indefinite Reference. University of Massachusetts (Amherst), U.S.A..
- Westerstahl, D. (1985) 'Logical constants in quantifier languages'. *Linguistics and Philosophy* 5: 387-413.
- Westerstahl, D. (1989) 'Quantifiers in formal and natural languages'. In D. Gabbay & F. Guenther (eds.) *Handbook of Philosophical Logic* 4: 1-131.
- Wierzbicka, A. (1986) 'Precision in vagueness: the semantics of English "approximative"'. *Journal of Pragmatics* 10: 597-614.

- Zadeh, L.A. (1965) 'Fuzzy sets'. *Information and Control* 8: 338-353.
- Zadeh, L.A. (1971) 'Quantitative fuzzy semantics'. *Information Sciences* 3: 159-176.
- Zadeh, L.A. (1972) 'A fuzzy-set-theoretic interpretation of hedges'. *Journal of Cybernetics* 2: 4-34.
- Zadeh, L.A. (1973a) 'Outline of a new approach to the analysis of complex systems and decision processes'. *IEEE Transactions on Systems, Man, and Cybernetics* SMC-3: 28-44.
- Zadeh, L.A. (1973b) 'A system-theoretic view of behaviour modification'. In H. Wheeler (ed.) *Beyond the Punitive Society*. San Francisco & California: Freeman, 160-169.
- Zadeh, L.A. (1975) 'The concept of a linguistic variable and its application to approximate reasoning (III)'. *Information Science* 9: 43-80.
- Zadeh, L.A. (1980) 'Fuzzy sets and probability'. *Proceedings of IEEE* 68: 421.
- Zadeh, L.A. (1982) 'A note on prototype theory and fuzzy sets'. *Cognition* 12: 291-297.
- Zadeh, L.A. (1983) 'A computational approach to fuzzy quantifiers in natural language'. *Computer and Mathematics with Applications* 9 (1): 149-184.
- Zadeh, L.A. (1984) 'Making computers think like people'. *IEEE Spectrum*: 26-32.
- Zhang, Q. (1982) 'A talk on fuzzy words in Chinese'. *Journal of Liaoning University* 1: 77-79.
- Zhang, Q. (1983) 'Fuzzy meaning of words'. *Fuzzy Mathematics* 1: 55-60.
- Zhang, Q. (1983) 'The functions of fuzzy meaning'. *Language Study* 6: 47-48.

- Zhang, Q. (1985) 'Fuzzy meaning of sentence in Chinese'. *Journal of Inner-Mongolia Teaching University* 2: 52-57.
- Zhang, Q. (1986) 'On the membership function of fuzzy meaning'. *Language Research* 1: 38-43.
- Zhang, Q. (1987) 'Fuzziness and poetry in Chinese'. *Monthly Lanzhou* 3: 7-11.
- Zhang, Q. (1987) 'Variability of fuzzy meaning'. In X.K. Li *et al* (eds) *Language and Philology*. Guangzhou: Guangdong People's Publisher, 12-20.
- Zhang, Q. (1991). 'The fuzzy quantifiers in Chinese and English'. Paper presented at the *Postgraduate Conference*, Department of Linguistics, University of Edinburgh, U.K.
- Zhang, Q. (1992) 'Fuzzy quantifiers'. Paper presented at the *Conference of Generalized Quantifiers and Negation*, Department of Psychology and Centre of Cognitive Science, University of Glasgow, U.K.
- Zhang, Q. (1995) 'The Causal factors for fuzziness'. Paper presented at the *Fourth Biennial Conference of Chinese Studies Association of Australia*, Macquarie University, Australia.
- Zimmer, A.C. (1983) 'Verbal versus numerical processing of subjective probabilities'. In R.W. Scholz (ed.) *Decision Making under Uncertainty*. Amsterdam: North-Holland.
- Zimmermann, H.J. (1985) *Fuzzy Set Theory and Its Applications*. Boston: Kluwer-Nijhoff Publishing.
- Zwarts, F. (1984) 'Determiners: a relational perspective'. In M. Cobler *et al* (eds.) *West Coast Conference on Formal Linguistics* (3). Stanford: The Stanford Linguistics Association.
- Zwarts, F. (1991) 'The syntax and semantics of negative polarity'. Centre for Behavioural, Cognitive and Neuro-sciences, University of Groningen. ms.

Zwarts, F. (in press) 'Polarity items'. To appear in R.E. Asher & J.M.Y. Simpson (eds.) *The Encyclopedia of Language and Linguistics*. Oxford: Pergamon Press.

Appendices

Appendix 1: Channell's (1983) elicitation test for English FQs

Table 1.1: List of items used in the number approximation elicitation test

- 1 "After the girl had rung up everything I'd bought, I suddenly realised I only had about 2.00 pounds on me"
- 2 "You'll need about 4lbs of oranges"
- 3 "We should be there around 6"
- 4 "He's producing about ten pages a week and they're all getting published"
- 5 "There were about 15 people there"
- 6 "... it's something around the 20 percent mark and it's never changed"

- 7 "We sent out two hundred questionnaires and had about 40 replies"
- 8 "It's going to cost about 500 pounds to fly there and back"
- 9 "I want to spend about 14,000 pounds"
- 10 "In a country with a population of around ten million, like Belgium, proportional representation makes much more sense"
- 11 "She was wearing a dress costing not less than 150 pounds"
- 12 "The repair bill certainly won't be less than 500 pounds"
- 13 "It was a good evening, we must have drunk a bottle or so of wine each"
- 14 "How much flour shall I put?" "Two spoonfuls or so"
- 15 "Six or so books will be enough for a week's reading"
- 16 "There are ten pounds or so of butter in the freezer"
- 17 "Ten or so litres of wine should be enough for the party"
- 18 "They hired the de Grey rooms and invited 200 or so people to a champagne lunch"
- 19 "It's okay, I've got 500 pounds or so in my account at the moment"
- 20 "It'll cost two thousand or so pounds to do this place up reasonably"
- 21 "There are 3,000 or so students at York"
- 22 "The Tower of London gets 30,000 or so visitors a year"
- 23 "You'll need three or four metres of rope"
- 24 "Yes, but it's still around that four or five regions"

- 25 "It takes six or seven hours to drive from Paris to Midi"
- 26 "The garden extends eight or nine feet beyond the true boundary of the property"
- 27 "Eight or ten students were waiting in the entrance"
- 28 "How many people will turn up for the meeting?" "We usually get fifteen or twenty"
- 29 "They had seventy or eighty people with broken bones over just one week-end"
- 30 "He's bought a stereo costing three or four hundred pounds"
- 31 "Two or three thousand people turned up to hear him speak"
- 32 "A burst water main in the Hull Road flooded neighbouring streets with two or three thousand gallons of water"

Table 1.2: The number approximation test: general results

about n

1 No	2 Item title	3 % subjects marking interval	4 Mean length	5 Mean center	6 Mode	7 Interval length as % of exemplar no
1	about 2.00 pounds	100.00	.30	1.97	2.00	15.0%
2	about 4lbs	73.1	.61	4.08	4.00	15.1%
3	about 6pm	96.2	.58	6.00	6.00	9.6%
4	about 10 pages	92.3	3.23	9.96	10.00	32.3%
5	about 15 people	100.00	4.65	15.05	14/15	31.0%
6	around the 20% mark	96.2	4.27	20.60	20.00	21.3%
7	about 40 replies	96.2	5.96	40.13	40/41	14.2%
8	about 500 pounds	84.6	51.92	508.00	500.00	10.4%
9	about 14,000 pounds	100.00	1067.30	14,033.65	14,000	7.6%
10	around 10 million	69.2	1.5 million	9,942,307.00	10 million	15.0%

Notes:

1. The Units in Columns 4, 5 and 6 are those specified in column 2 for each item.
2. Standard half-rounding, to two decimal places, has been applied, and for percentages, to one decimal place.

Table 1.2 continued: Not less than

1 No	2 Item title	3 % subjects marking interval	4 Mean length	5 Mean center	6 Mode	7 Interval length as % of exemplar no
11	not less than 150 pounds	96.2	24.42	162.44	160	16.1%
12	not less than 500 pounds	100.00	54.42	529.00	520	10.9%

N or so

1 No	2 Item title	3 % subjects marking interval	4 Mean length	5 Mean center	6 Mode	7 Interval length as % of exemplar no
13	a bottle or so	96.2	.568	1.08	1	56.8%
14	2 spoonfuls or so	96.2	.788	2.30	2.50	39.4%
15	6 or so books	100.00	2.35	6.21	6	39.1%
16	10 lbs or so	88.5	2.15	10.58	10	21.5%
17	10 or so litres	92.3	1.62	10.63	10	16.2%
18	200 or so people	100.00	46.15	208.46	200/210	23.1%
19	500 pounds or so	88.5	77.30	560.96	500	15.5%
20	2,000 pounds or so	100.00	550.00	2300.00	2200	27.5%
21	3,000 or so students	96.2	538.46	3117.30	3000	17.9%
22	30,000 or so visitors	96.2	4846.00	30923.00	30,000	16.1%

N or m

1 No	2 Item title	3 % subjects marking interval	4 Mean length	5 Mean center	6 Mode	7 Interval length as % of exemplar no
23	3 or 4 metres	96.2	1.10	3.65	3.50	31.4%
24	4 or 5 regions	100.00	3.00	4.71	4/5	66.6%
25	6 or 7 hours	100.00	1.51	6.70	7	23.3%
26	8 or 9 feet	96.2	2.23	8.61	9	26.2%
27	8 or 10 students	100.00	4.42	8.87	8	49.1%
28	15 or 20 people	100.00	8.23	17.61	16/17/18	47.0%
29	70 or 80 people	100.00	17.15	73.94	75	22.9%
30	3 or 4 hundred pounds	100.00	90.46	358.17	350/375	25.8%
31	2 or 3 thousand people	100.00	1260.60	2541.34	2250	50.4%
32	2 or 3 thousand gallons	100.00	1177.10	2596.00	2500	47.1%

Note:

3. For *n* or *m* approximations, the mean of the two exemplar numbers was taken as the comparison point for the Mean Interval Length (column 7)

Table 1.3 Special results for n or m approximations

Item No.	% subjects specifying interval cont. both n and m	% subjects specifying int. bounded by n and m	% subjects specifying nos below n	% subjects specifying nos above m
23	57.7	76.9	30.8	73.1
24	100	57.6	50.0	46.1
25	69.4	88.5	42.3	80.8
26	88.5	73.1	46.2	65.4
27	96.2	77.0	61.5	57.7
28	92.3	77.0	65.0	69.2
29	80.8	88.5	57.7	65.4
30	69.2	61.5	30.8	53.8
31	69.2	76.9	41.1	65.4
32	73.1	76.9	57.7	57.7
Mean over 10 items:	79.6	75.4	48.3	63.5

Table 1.4: Frequency distribution of informant responses for number approximations

Note: Y-axis stands for the total number of subjects (25) of Channell's test.

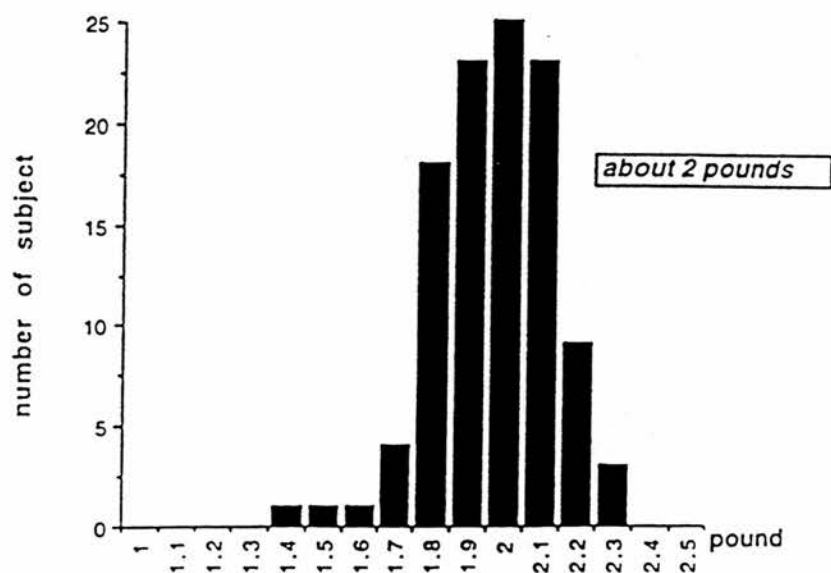


Fig. 1.1

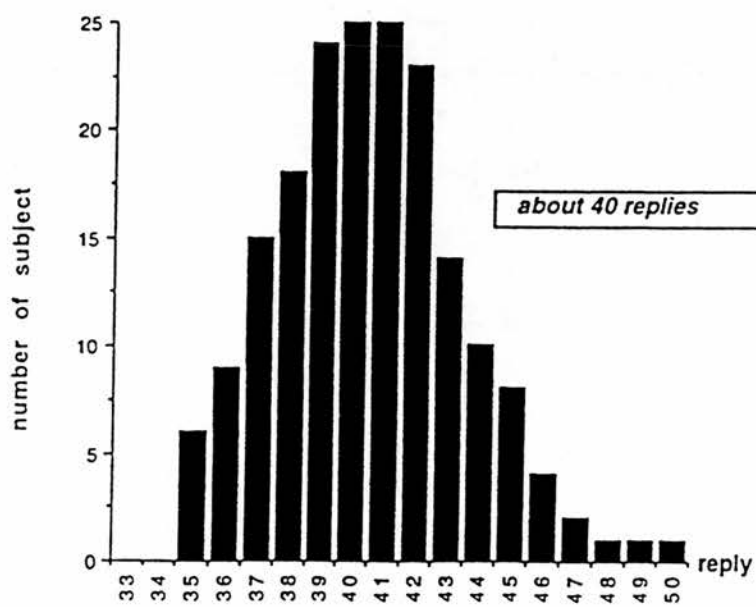


Fig. 1.2

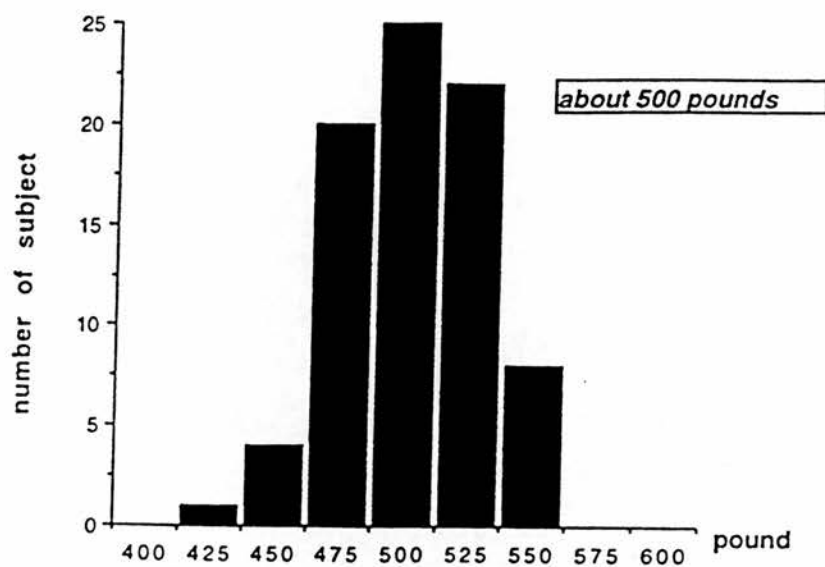


Fig. 1.3

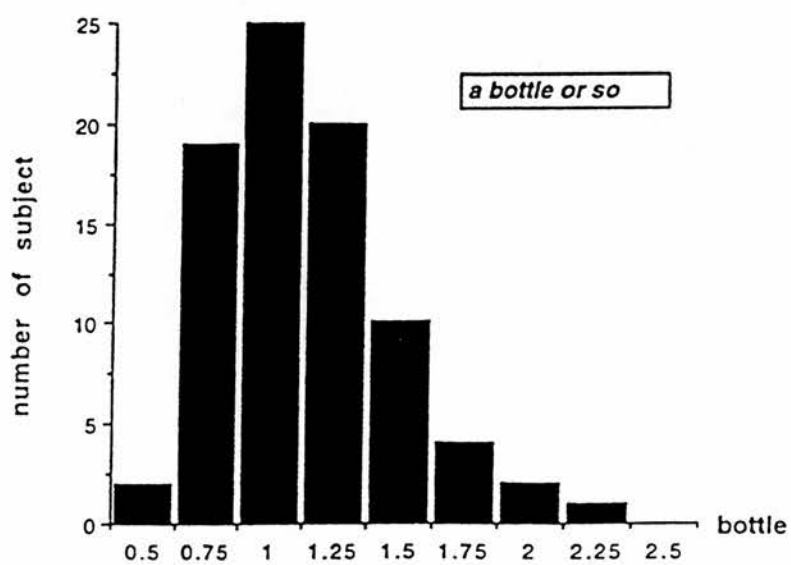


Fig. 1.4

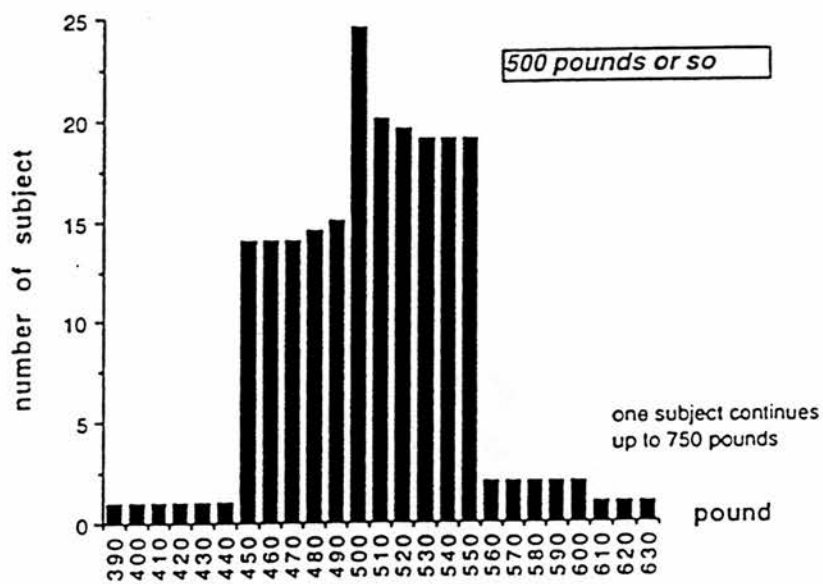


Fig. 1.5

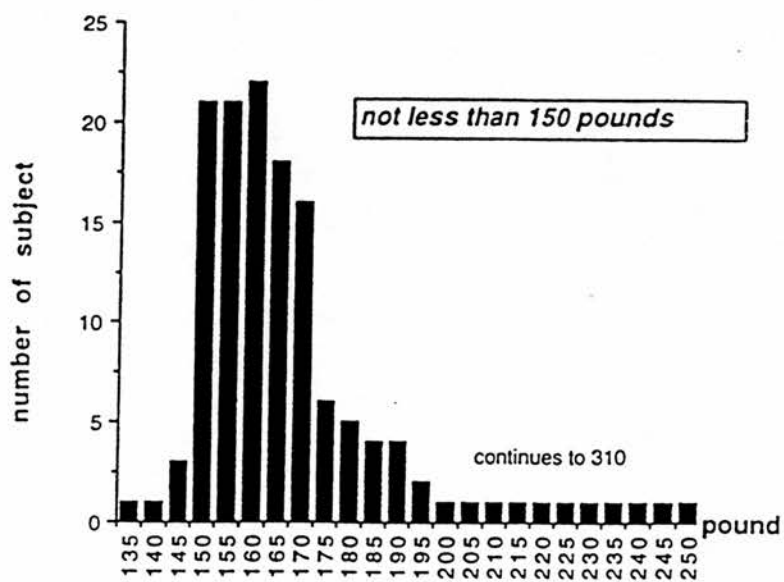


Fig. 1.6

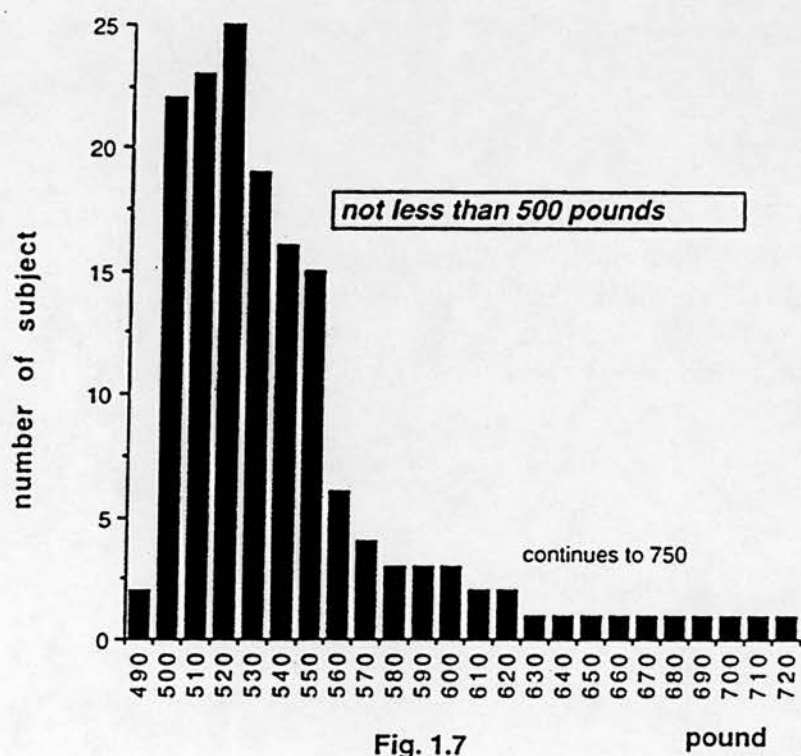


Fig. 1.7

Appendix 2: Zhang's questionnaire for Chinese FQs

Questionnaire (English translation)

Name:

Sex:

Subject:

First of all, I am extremely grateful for your valuable contribution. Please designate a numerical interval for the following phrases. Take *ershi yixia* 'under 20' as an example, it might be designated an interval, say 15-20, 1-20, or $-\infty$ -20.

Group A:

Given a range of 100, designate a numerical interval for the following phrases. For instance, out of 100 people, one might judge 30-50 as the interval of *yixie ren* 'quite a few people'; and 70-80 as the interval of *duoshu ren* 'majority of people'. Also, out of 100 (jin - Chinese weight measure unit, 1 jin = 0.5 kg), 70-80 might be considered as the interval of *xuduo (jin)* 'many (jin)'.

Phrases

Intervals

- 1 yixie ren 'quite a few people'
- 2 xuduo ren 'many people'
- 3 xuduo (jin) 'many (jin)'
- 4 duoshu ren 'majority of people'
- 5 shaoshu ren 'minority of people'

6 bushao ren 'many people'

7 haoxie ren 'many people'

8 haoxie (jin) 'many (jin)'

9 henduo (ge) 'very many'

10 youxie (ge) 'quite a few'

Group B

Here are some hypothetical examples:

Phrases	Intervals (hypothetical answers)
<u>dagai 20 ren</u> 'about 20 people'	17-23
<u>jiangjin 200 (jin)</u> 'nearly 200 (jin)'	180-199
<u>xiao 200 ren</u> 'a bit fewer than 200 people'	180-195
<u>liang, san (ge)</u> '2 or 3'	2-4
<u>qianbai (ge)</u> '100 or 1000'	100-1000

Phrases

Intervals

11 ji (ge) 'several'

- 12 haoji (ge) 'a bit more than several'
- 13 200 yishang 'more than 200'
- 14 200 yixia 'fewer than 200'
- 15 200 ren yishang 'more than 200 people'
- 16 200 ren yixia 'fewer than 200 people'
- 17 jiangjin 200 ren 'nearly 200 people'
- 18 jiangjin 200 (jin) 'nearly 200 (jin)'
- 19 jiangjin 2,000 ren 'nearly 2,000 people'
- 20 jiangjin 2,000 (jin) 'nearly 2,000 (jin)'
- 21 jiangjin 20,000 ren 'nearly 20,000 people'
- 22 jiangjin 20,000 (jin) 'nearly 20,000 (jin)'
- 23 200 duo ren '200-odd people'
- 24 200 duo (jin) '200-odd (jin)'
- 25 2,000 duo ren '2,000-odd people'
- 26 2,000 duo (jin) '2,000-odd (jin)'
- 27 20,000 duo ren '20,000-odd people'
- 28 20,000 duo (jin) '20,000-odd (jin)'
- 29 200 ren zuoyou 'about 200 people'
- 30 200 (jin) zuoyou 'about 200 (jin)'

- 31 2,000 ren zuoyou 'about 2,000 people'
- 32 2,000 (jin) zuoyou 'about 2,000 (jin)'
- 33 20,000 ren zuoyou 'about 20,000 people'
- 34 20,000 (jin) zuoyou 'about 20,000 (jin)'
- 35 xiao erbai ren 'a bit fewer than 200 people'
- 36 da erbai ren 'a bit more than 200 people'
- 37 erbai lai ren 'about 200 people'
- 38 baiba (ge) 'about 100'
- 39 200 ren shangxia 'about 200 people'
- 40 dayue 200 ren 'about 200 people'
- 41 shangbai (ge) 'about 100'
- 42 chengbai (ge) 'about 100'
- 43 200 (jin) duo yixie '200-odd (jin)'
- 44 200 (jin) shao yixie 'a bit fewer than 200 (jin)'
- 45 daban (ge) 'more than half'
- 46 xiaoban (ge) 'less than half'
- 47 duoban ren 'majority of people'
- 48 shaoban ren 'minority of people'
- (Note: From 45 to 48, a fraction or percentage should be used.)

49 liang, san (ge) '2 or 3'

50 san, wu (ge) '3 or 5'

51 shi'er, san (ge) '12 or 13'

52 baishi lai (ge) 'about 100'

53 qianbai (ge) '100 or 1000'

54 qianba bai (ge) '800 or 1000'

55 yiwan (ge) 'hundreds of millions/very many'

56 qianwan (ge) 'ten million/very many'

Group C

Please give a numerical interval for the phrases dotted.

Example	Phrase	Interval
	bantian (quite a while)	1-2 hours

Phrases	Intervals
---------	-----------

57 Ta yijing deng le	bantian	le.
he already wait (particle)	quite a while	(particle)
'He has already waited for quite a while.'		

58 Ta yijing deng le	dabantian	le.
he already wait (particle)	a very long time	(particle)
'He has already waited for a very long time.'		

59 Juchangzhong	daban ren	zai	guzhang.
in the theatre	majority of people	in process of	applaud
'A majority of people in the theatre were applauding.'			

60 Juchangzhong	shaoban ren	zai	shuijiao.
in the theatre	minority of people	in process of	sleep
'A minority of people in the theatre were sleeping.'			

(Note: from 59 to 60, a fraction or percentage should be used.)

I would be greatly appreciative if you could enclose your comments. Again, I

thank you sincerely for your contribution, and looking forward to your reply at your earliest convenience.

(The questionnaire ends here)

Notes:

1. *Group A: FQs in this group were estimated out of 100.*
2. *Group C: FQs in this group were tested in a given linguistic discourse, i.e. FQs were embodied in sentences. The aim was to find out the effect of context.*
3. *In the questionnaire, subjects were asked to tell their sex and occupation. This was meant to investigate whether the variables of sex and occupation would result in difference in performance. It appeared that more science students and professionals gave $+\infty$ and $-\infty$ to more than 200 and fewer than 200 than other people. Due to the limited length of this thesis, I haven't explored this issue; but it would be an interesting topic for a further discussion.*

Table 2.2: FQ approximation test: general results

Notes:

- 1 *There are two modes in Table 2.2, column 6. The mode without brackets represents a mode number; the other denotes a modal interval.*
2. *Standard Deviation: Standard deviation is the most frequently used measure of variability or dispersion of a distribution, i.e. of the degree to which scores vary from the mean. It is standard in the sense that it looks at the average variability of all the scores around the mean and all the scores that are taken into account. The standard deviation, s , is calculated by the formula in (10.1).*

$$s = [\sum (x_j - \bar{x})^2 / (N - 1)]^{1/2} \quad (10.1)$$

where x_1, x_2, \dots, x_n are the observed (accepted) values for a variable in the cases used in

an analysis. N is the sample size for the variable, and \bar{x} is the mean. Generally speaking, the larger a standard deviation, the more variability there is.

3. *Skewness*: it is an indication of asymmetry. The value should be zero or close to zero to indicate the normality of the distribution. Outliers may skew the distribution either to the right or to the left (notated with -). The BMDP programme used here computes skewness, g_1 , as

$$g_1 = \sum (x_j - \bar{x})^3 / (Ns^3) \quad (10.2)$$

4. *Kurtosis*: this refers to a measure of peakness or flatness. A peaked distribution will have a negative value of kurtosis; and a flat distribution will have a positive value of kurtosis. That is, if the kurtosis of a distribution is significantly greater than zero, then the distribution is longer-tailed than the normal. The BMDP programme used here computes kurtosis, g_2 , as

$$g_2 = \sum (x_j - \bar{x})^4 / (Ns^4) - 3 \quad (10.3)$$

5. Standard half-rounding, to two decimal places, has been applied. Percentage has also been calculated to two decimal places. In Table 2.2 and 2.3, most figures were calculated by computer, using BMDP.

6. Only 24 of 60 FQs tested are analyzed below, for the convenience of exposition. More details on Chinese data can be seen in Chapter 3.

Group 1:

1 No	2 Item title	3 % subjects designating interval	4 Range	5 Mean	6 Mode (Mode interval)
1	ji (ge) 'several'	100.0	10.00	4.95	4 (3-5)
2	yixie ren 'quite a few people'	100.0	60.00	26.26	20 (20-30)
3	xuduo ren 'many people'	100.0	95.00	60.97	70 (70-80)
4	xuduo (jin) 'many (jin)'	100.0	95.00	61.73	70 (70-80)

Group 1 continued:

No	7 Skewness	8 Kurtosis	9 Std dev	10 Maximum no.	11 Minimum no.
1	0.45	-0.46	2.13	10	0
2	0.36	-0.46	12.88	60	0
3	-0.34	-0.47	19.75	100	5
4	-0.50	-0.12	19.46	100	5

Group 2:

(Note: FQs in this group are represented in percentage.)

1 No	2 Item title	3 % subjects designating interval	4 Range	5 Mean	6 Mode (Mode interval)
5	daban ren 'majority of people'	81.48	70.00	68.75	70.00 (60-80)
6	shaoban ren 'minority of people'	81.48	60.00	29.37	30.00 (30-40)
7	daban ren 'majority of people' (in the context)	97.78	75.00	66.58	70.00 (60-80)
8	shaoban ren 'minority of people' (in the context)	97.78	45.00	26.52	30.00 (20-30)

Group 2 continued:

No	7 Skewness	8 Kurtosis	9 Std dev	10 Maximum no.	11 Minimum no.
5	-0.01	-0.07	12.14	100	30
6	-0.20	-0.52	11.35	60	0
7	-0.25	0.24	12.17	100	25
8	-0.002	-0.43	9.55	50	5

Group 3:

1 No	2 Item title,	3 % subjects designating interval	4 Range	5 Mean	6 Mode (Mode interval)
9	jiangjin 200 ren 'nearly 200 people'	100.0	100	188.88	195 (190-199)
10	jiangjin 2,000 ren 'nearly 2,000 people'	100.0	700	1,869.37	1,990 (1,900-2,000)
11	jiangjin 20,000 ren 'nearly 20,000 people'	100.0	20,000	18,766.68	19,500 (18,000-20,000)

Group 3 continued:

No	7 Skewness	8 Kurtosis	9 Std dev	10 Maximum no.	11 Minimum no.
9	-1.19	3.22	9.8	250	150
10	-1.13	1.11	115.66	2,200	1,500
11	-1.21	4.82	2,330.31	30,000	10,000

Group 4:

1 No	2 Item title	3 % subjects designating interval	4 Range	5 Mean	6 Mode (Mode interval)
12	200 duo ren '200-odd people'	99.23	125	214.17	210 (205-218)
13	2,000 duo ren '2,000-odd people'	99.26	1,350	2,096.02	2,100 (2,000-2,250)
14	20,000 duo ren '20,000-odd people'	99.26	15,000	20,482.39	20,500 (20,000-21,000)

Group 4 continued:

No	7 Skewness	8 Kurtosis	9 Std dev	10 Maximum no.	11 Minimum no.
12	2.55	10.64	14.45	305	180
13	2.81	16.64	124.18	3,000	1,650
14	2.85	0.38	2,626.64	31,000	16,000

Group 5:

1 No	2 Item title	3 % subjects designating interval	4 Range	5 Mean	6 Mode (Mode interval)
15	200 ren zuoyou 'about 200 people'	100.0	110	200.48	195/200/205 (190-210)
16	2,000 ren zuoyou 'about 2,000 people'	100.0	1,000	1,934.11	1,950/2,000/2,050 (1,900-2,100)
17	20,000 ren zuoyou 'about 20,000 people'	100.0	20,500	20,257.32	20,000 (19,000-21,000)

Group 5 continued:

No	7 Skewness	8 Kurtosis	9 Std dev	10 Maximum no.	11 Minimum no.
15	0.45	2.02	13.42	260	150
16	0.50	2.91	130.89	2,500	1,500
17	0.52	2.68	2,541.85	30,500	10,000

Group 6:

1 No	2 Item title	3 % subjects designating interval	4 Range	5 Mean	6 Mode (Mode interval)
18	liang, san (ge) '2 or 3'	100.0	6.00	2.94	3(2-3)
19	san, wu (ge) '3 or 5'	100.0	8.00	4.57	4(3-5)
20	shi'er, san (ge) '12 or 13'	100.0	9.00	12.85	13(12-13)

Group 6 continued:

No	7 Skewness	8 Kurtosis	9 Std dev	10 Maximum no.	11 Minimum no.
18	0.85	0.75	0.92	7	1
19	0.64	0.26	1.33	10	2
20	0.38	0.82	1.35	19	10

Group 7:

1 No	2 Item title	3 % subjects designating interval	4 Range	5 Mean	6 Mode (Mode interval)
21	200 yishang 'more than 200'	99.26	N/A	N/A	210(200- $+\infty$)
22	200 yixia 'fewer than 200'	99.26	N/A	N/A	190($-\infty$ -200)
23	200 ren yishang 'more than 200 people'	99.26	N/A	N/A	210(200-300)
24	200 ren yixia 'fewer than 200 people'	99.26	200	124.61	190(100-200)

Group 7 continued:

No	7 Skewness	8 Kurtosis	9 Std dev	10 Maximum no.	11 Minimum no.
21	N/A	N/A	N/A	$+\infty$	200
22	N/A	N/A	N/A	200	$-\infty$
23	N/A	N/A	N/A	$+\infty$	200
24	-0.58	-0.84	58.52	200	0

Table 2.3: Special results for n or m approximations

Item no.	% subjects specifying interval cont. both <i>n</i> and <i>m</i>	% subjects specifying int. bounded by <i>n</i> and <i>m</i>	% subjects specifying below <i>n</i>	% subjects specifying above <i>m</i>
18 liang, san (ge) '2 or 3'	100	50.36	0.01	49.63
19 san, wu (ge) '3 or 5'	99.99	42.96	0.01	55.56
20 shi'er, san(ge) '12 or 13'	99.99	30.37	34.81	63.7
Mean over 3 items:	99.99	41.23	11.61	56.3

Table 2.4: Special results for more than n approximations

Item no.	% subjects specifying u-bound as positive infinite	% subjects specifying l-bound as n	% subjects specifying l-bound below n	% subjects specifying l-bound above n
21 200 yishang 'more than 200'	28.36	70.15	0	29.85
23 200 ren yishang 'more than 200 people'	18.7	58.06	0	41.94
Mean over 2 items:	23.53	64.11	0	35.9

Table 2.5: Special results for fewer than n approximations

Item No.	% subjects specifying l-bound as negative infinite	% subjects specifying u-bound as n	% subjects specifying u-bound below n	% subjects specifying u-bound above n	% subjects specifying l-bound as 0/1
22 200 yixia 'fewer than 200'	23.88	54	46	0	16.42
24 200 ren yixia 'fewer than 200 people'	0	53.63	46.37	0	24.81
Mean over 2 items:	23.88	53.8	46.2	0	20.62

Table 2.6: Frequency distributions of informant responses

Note:

In the following histograms, the Y-axis indicates a domain of appropriate numbers, and corresponding counts (occurrences). The X-axis indicates a grade of membership using values in $[0, 1]$. For how the membership is calculated in the following histograms, see page 66 above.

Group 1:

1 ji (ge)
'several'

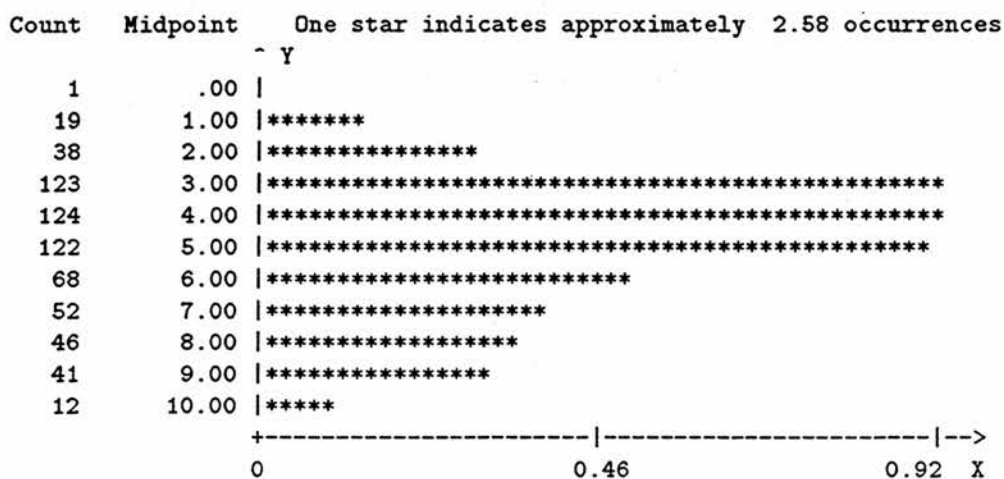


Fig. 2.1

Mode	4.00	Std dev	2.13
Skewness	.45	Range	10.00
Minimum	.00	Maximum	10.00

2 yizie ren
'quite a few people'

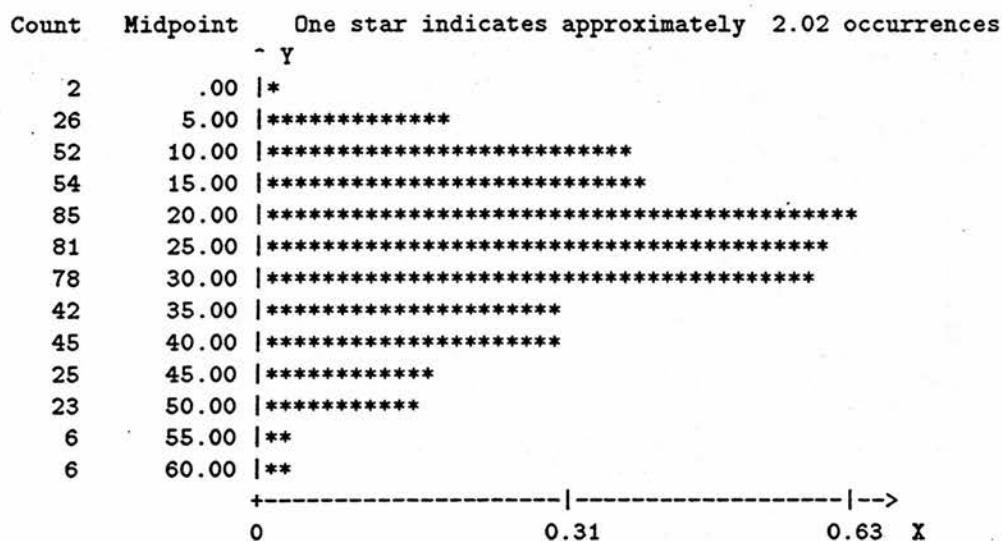


Fig. 2.2

Mode	20.00	Std dev	12.88
Skewness	.36	Range	60.00
Minimum	.00	Maximum	60.00

3 *xuduo ren*
'many people'

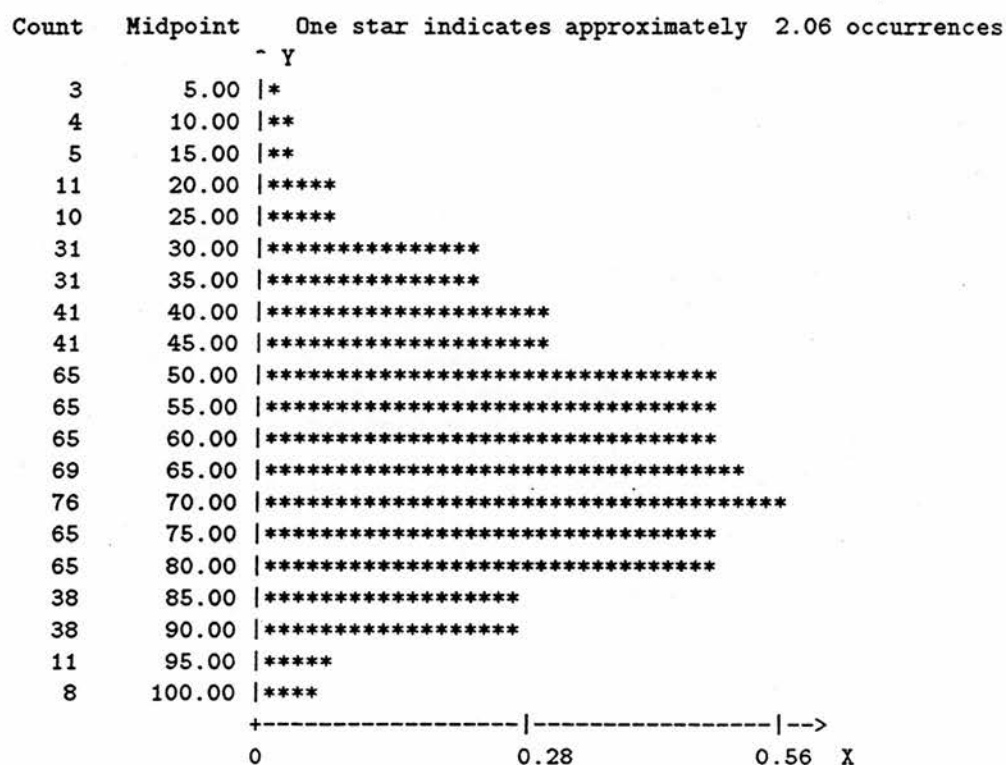


Fig. 2.3

Mode	70.00	Std dev	19.75
Skewness	-.34	Range	95.00
Minimum	5.00	Maximum	100.00

4 xuduo (jin)
'many (jin)'

Count Midpoint One star indicates approximately 2.06 occurrences

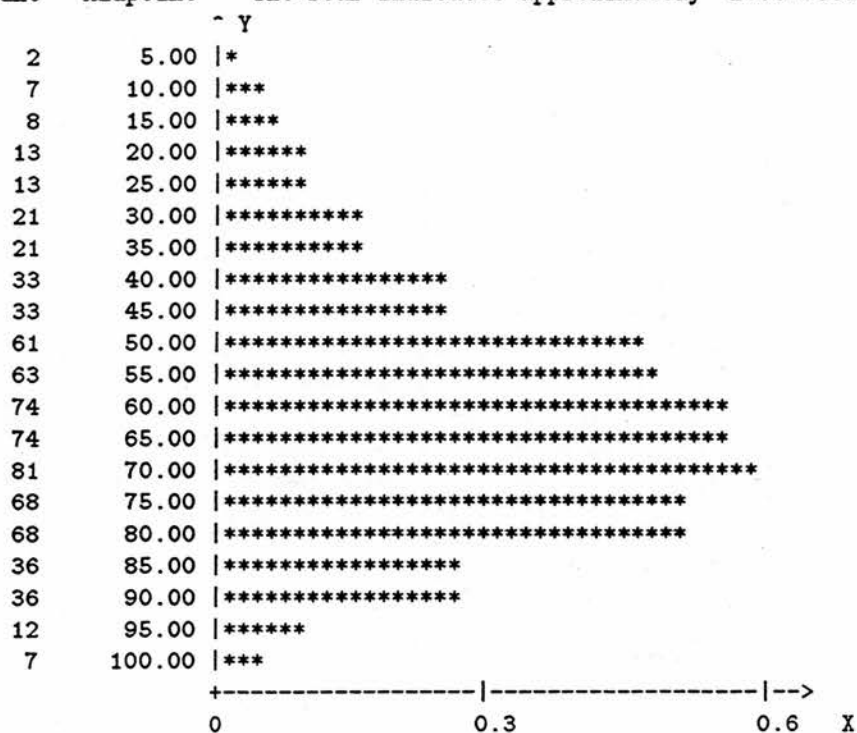
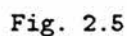


Fig. 2.4

Mode	70.00	Std dev	19.46
Skewness	-.50	Range	95.00
Minimum	5.00	Maximum	100.00

5 daban ren
'majority of people'



Mode	70.00	Std dev	12.14
Skewness	-.01	Range	70.00
Minimum	30.00	Maximum	100.00

'minority of people'

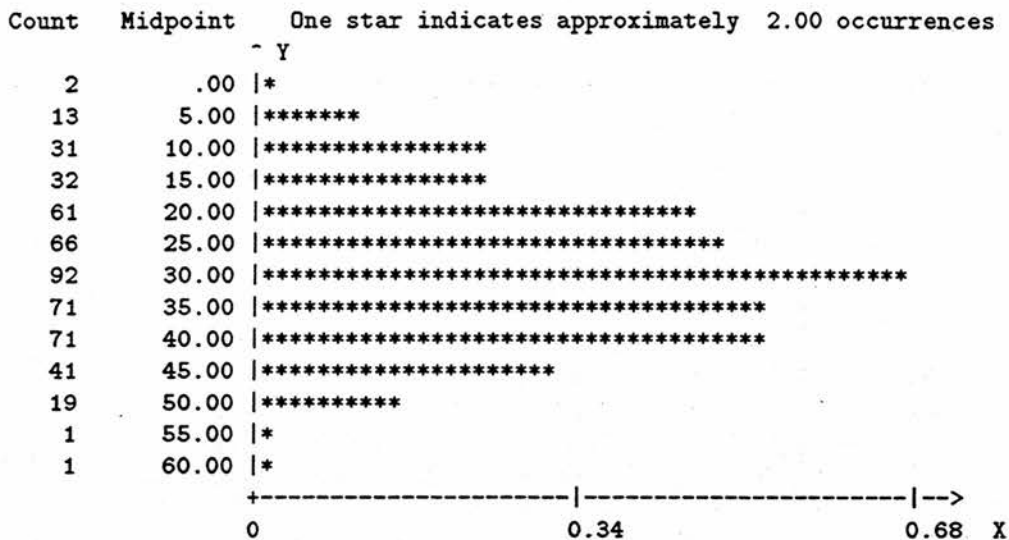
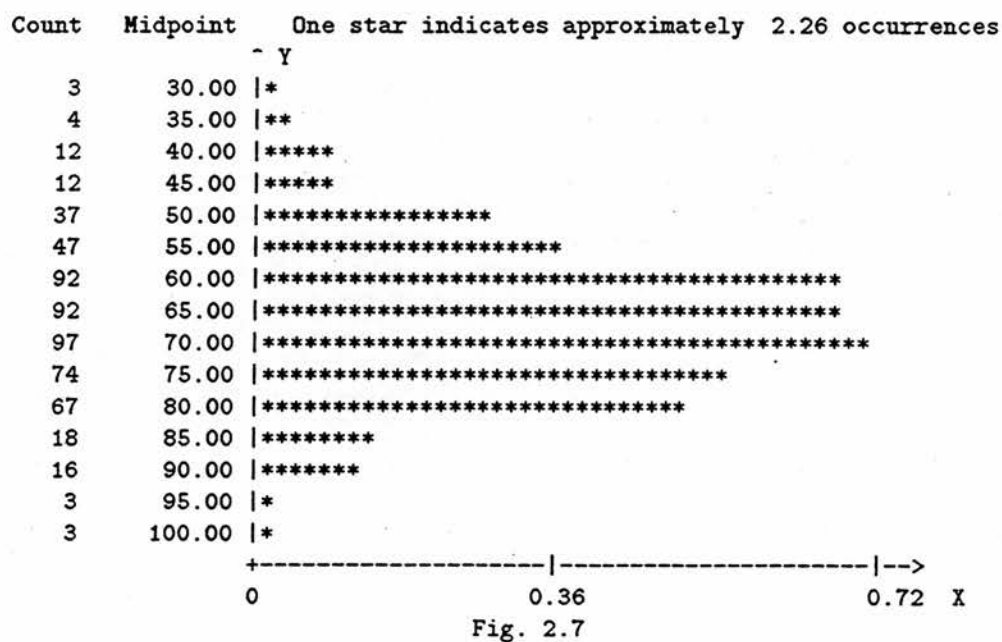


Fig. 2.6

Mode	30.00	Std dev	11.35
Skewness	-.20	Range	60.00
Minimum	.00	Maximum	60.00

7 daban ren

'majority of people'



Mode	70.00	Std dev	12.17
Skewness	-.25	Range	75.00
Minimum	25.00	Maximum	100.00

8 shaoban ren
'minority of people'

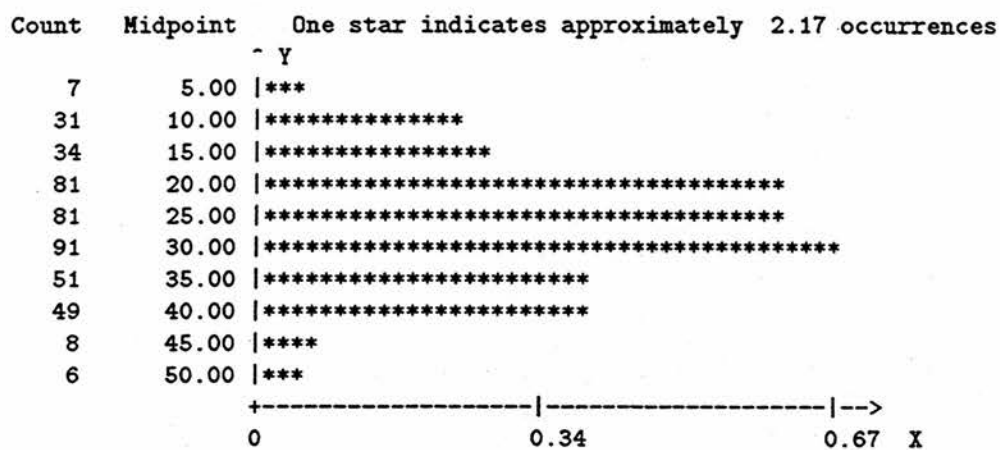
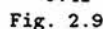


Fig. 2.8

Mode	30.00	Std dev	9.55
Skewness	-.02	Range	45.00
Minimum	5.00	Maximum	50.00

9 jiangjin 200 ren
'nearly 200 people'



10 *jiangjin* 2,000 *ren*
 'nearly 2,000 people'

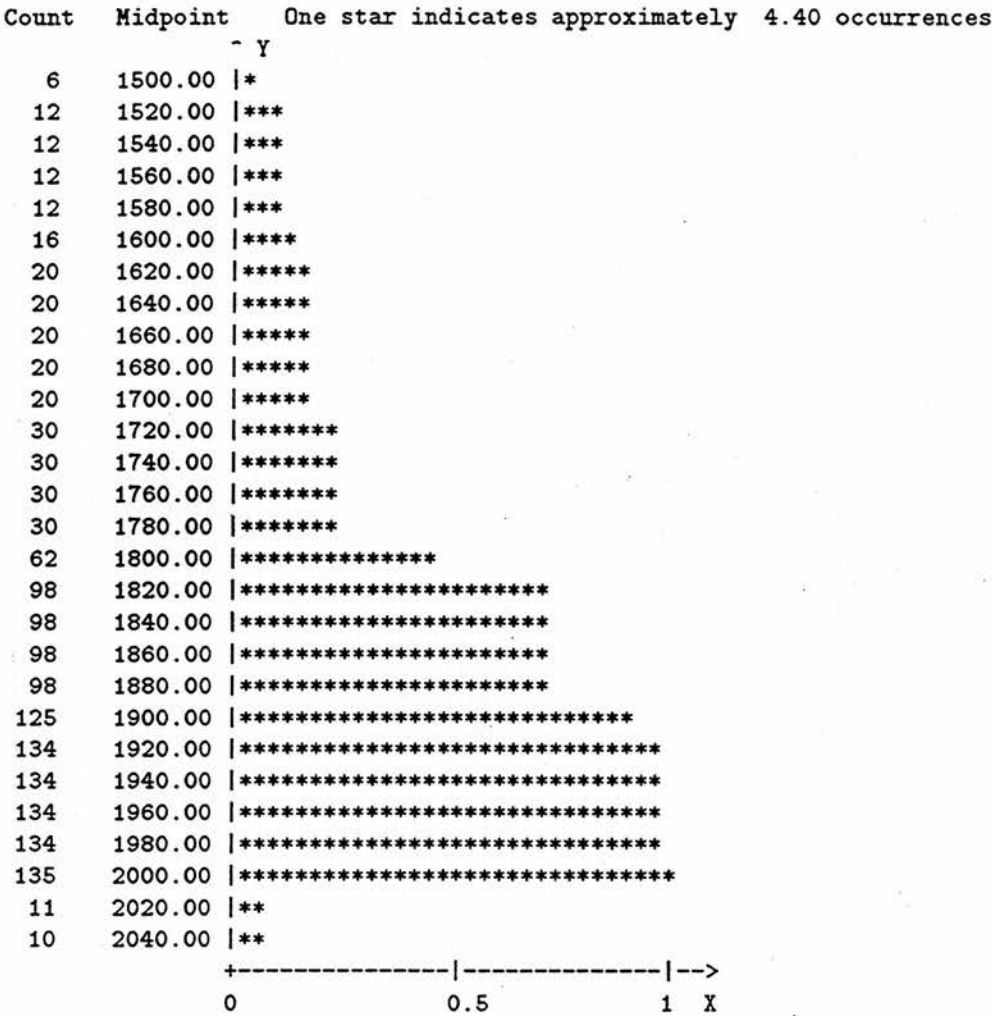


Fig. 2.10

Mode	1990.00	Std dev	115.66
Skewness	-1.13	Range	700.00
Minimum	1500.00	Maximum	2200.00

11 jiangjin 20,000 ren
'nearly 20,000 people'

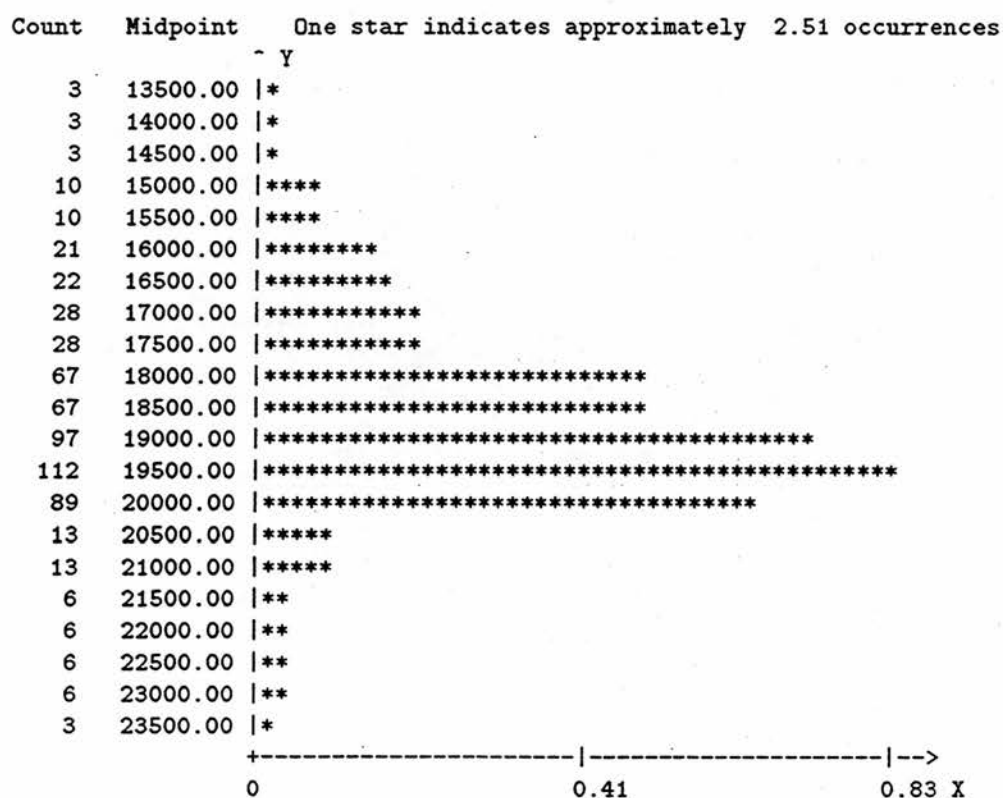


Fig. 2.11

Mode	19500.00	Std dev	2330.31
Skewness	-1.21	Range	20000.00
Minimum	10000.00	Maximum	30000.00

Group 4: n-odd approximations

12 200 duo ren
'200-odd people'

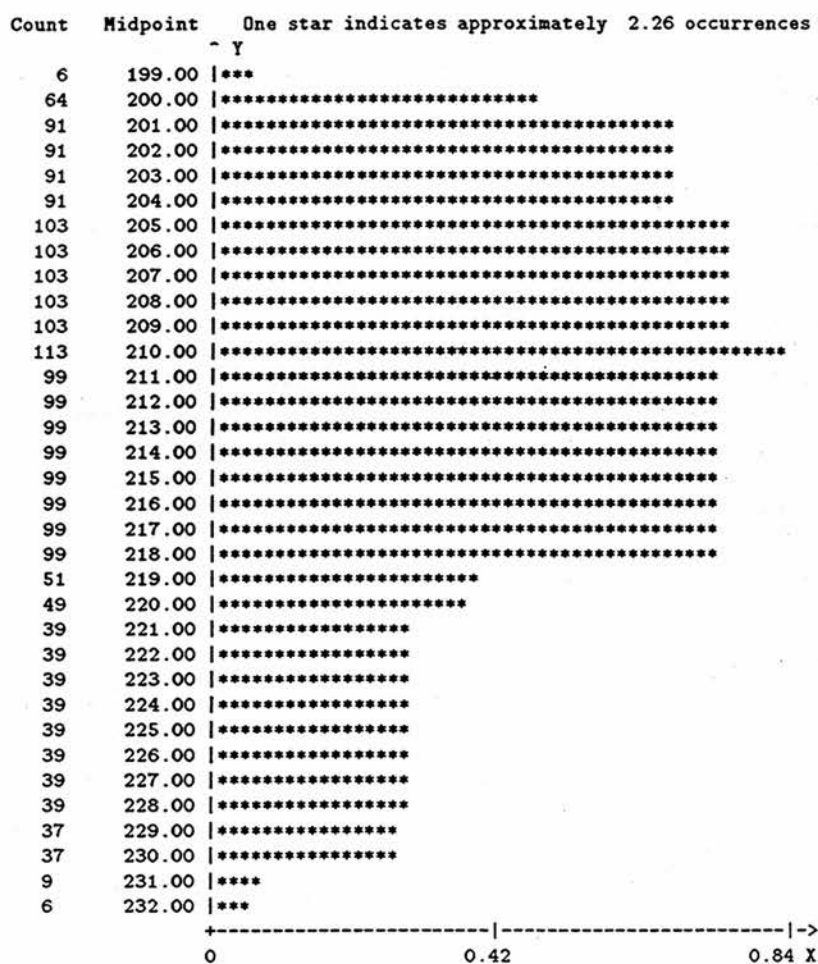


Fig. 2.12

Mode	210.00	Std dev	14.45
Skewness	2.55	Range	125.00
Minimum	180.00	Maximum	305.00

13 2,000 duo ren
'2,000-odd people'

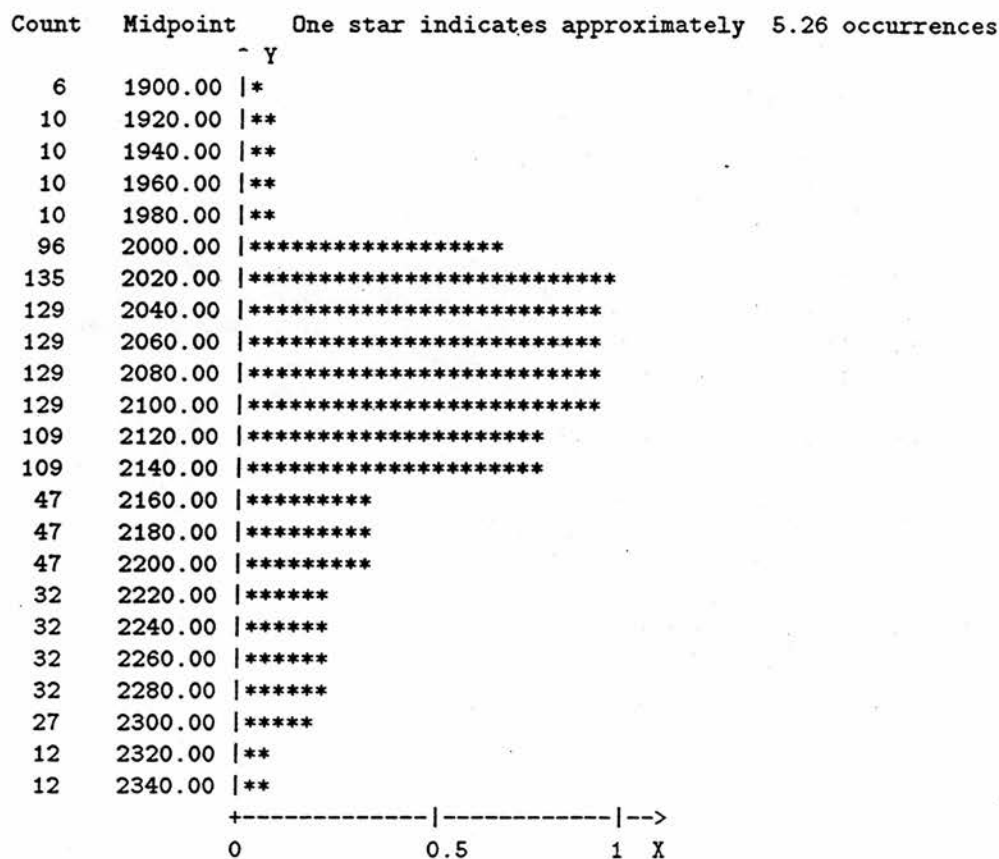


Fig. 2.13

Mode	2100.00	Std dev	124.18
Skewness	2.81	Range	1350.00
Minimum	1650.00	Maximum	3000.00

14 20,000 duo ren
'20,000-odd people'

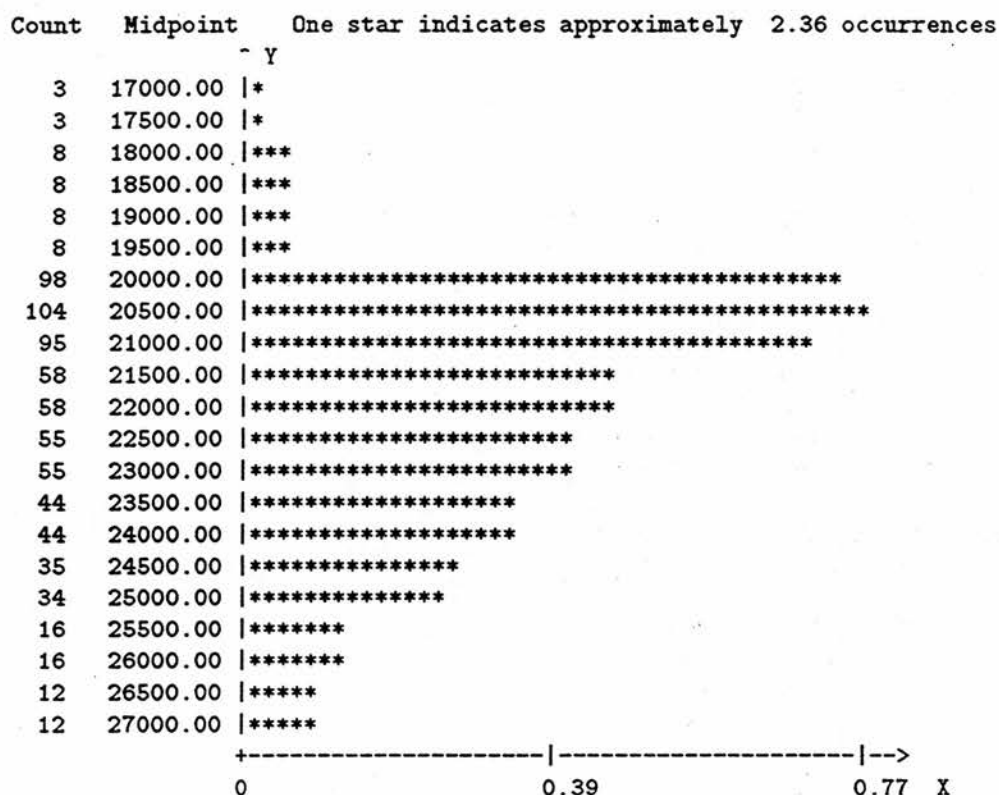


Fig. 2.14

Mode	20500.00	Std dev	2626.64
Skewness	2.85	Range	15000.00
Minimum	16000.00	Maximum	31000.00

Group 5: about n approximations

15 200 ren zuoyou 'about 200 people'

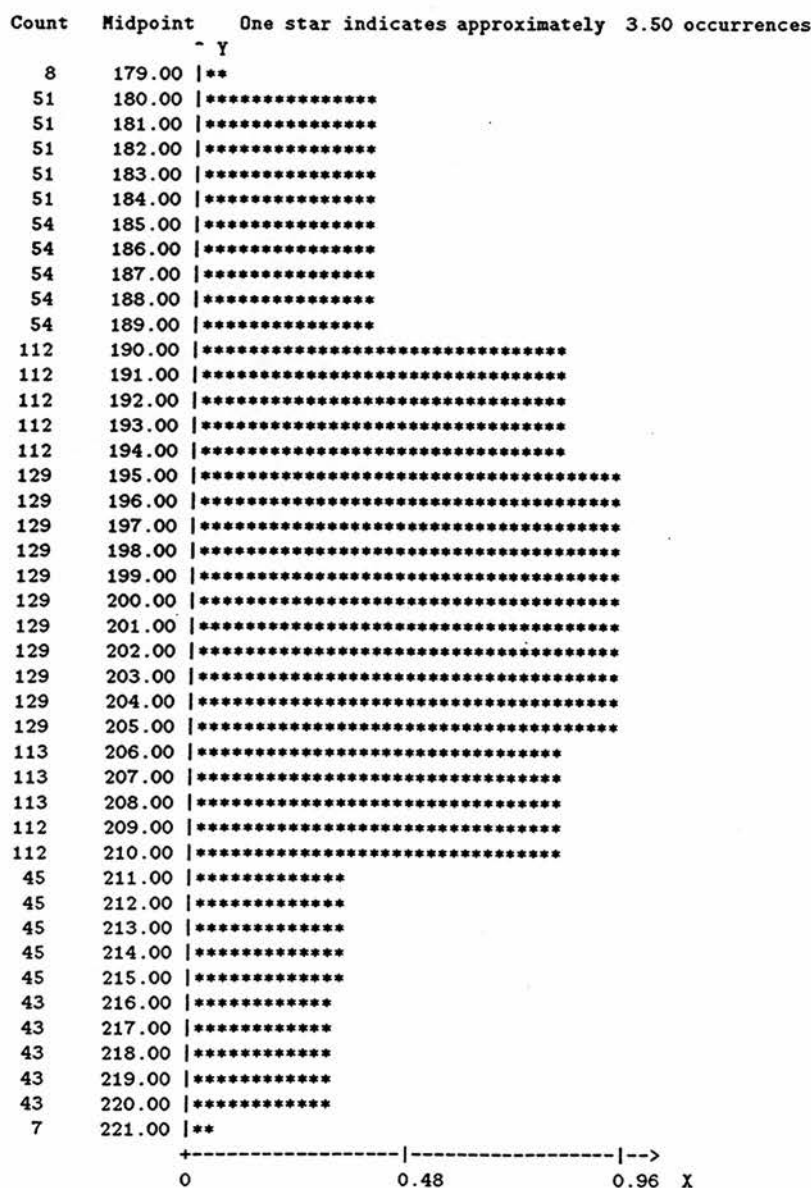


Fig. 2.15

Mode	195.00/200.00/205.00	Std dev	13.42
Skewness	.45	Range	110.00
Minimum	150.00	Maximum	260.00

16 2,000 ren zuoyou
'about 2,000 people'

Count Midpoint One star indicates approximately 9.54 occurrences

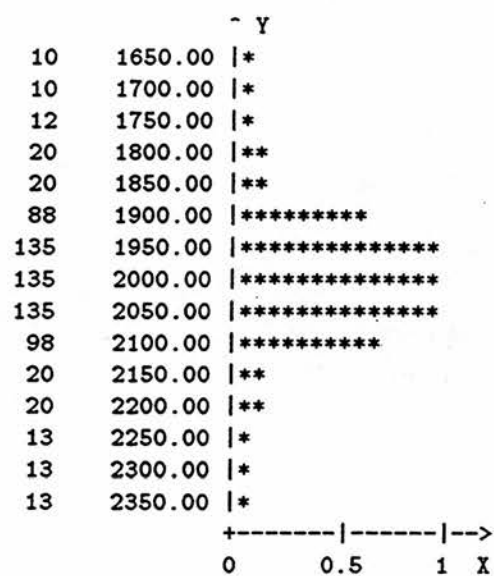


Fig. 2.16

Mode	1950.00/2000.00/2050.00	Std dev 130.89
Skewness	.50	Range 1000.00
Minimum	1500.00	Maximum 2500.00

17 20,000 ren zuoyou
 'about 20,000 people'

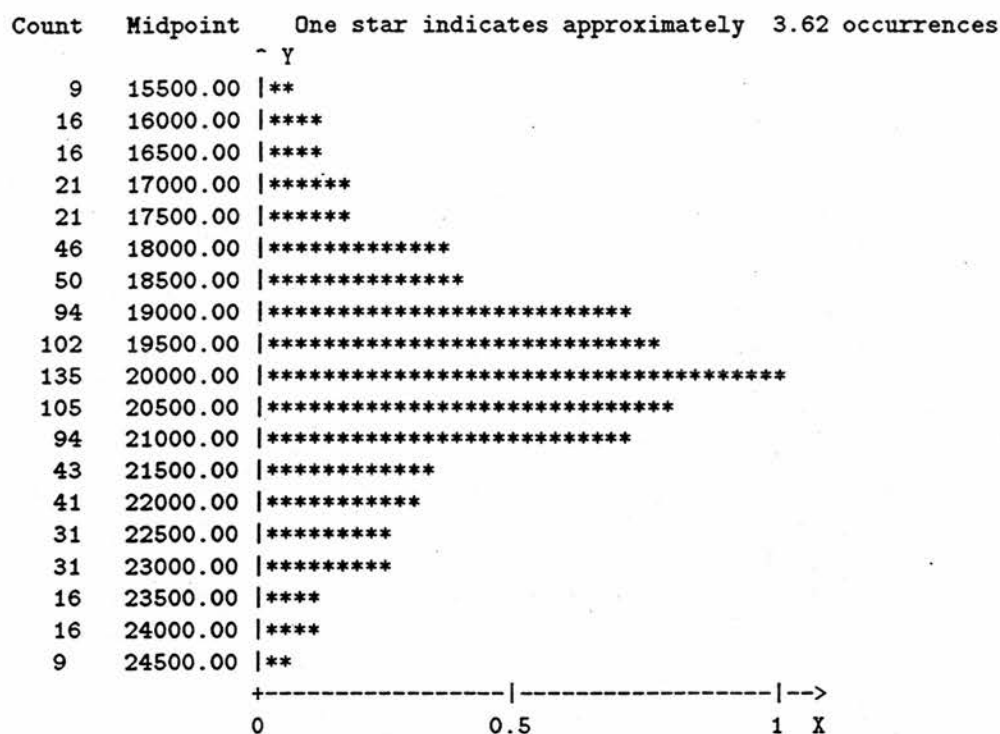


Fig. 2.17

Mode	20000.00	Std dev	2541.85
Skewness	.52	Range	20000.00
Minimum	10000.00	Maximum	30000.00

18 liang, san (ge)
'2 or 3'

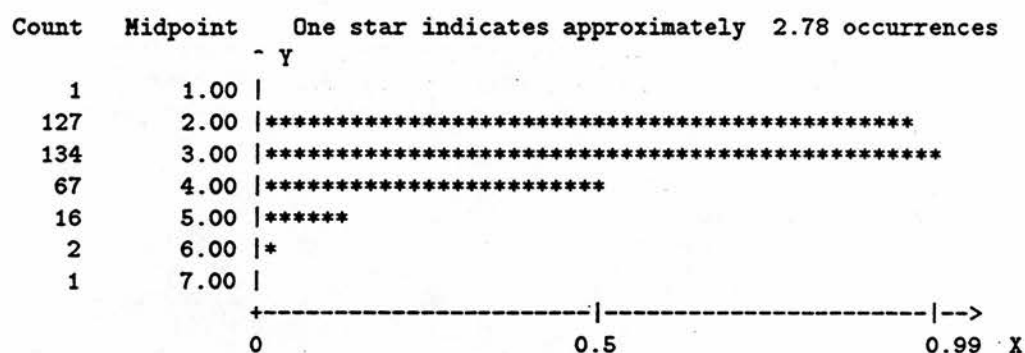


Fig. 2.18

Mode	3.00	Std dev	.92
Skewness	.85	Range	6.00
Minimum	1.00	Maximum	7.00

19 san, wu (ge)
'3 or 5'

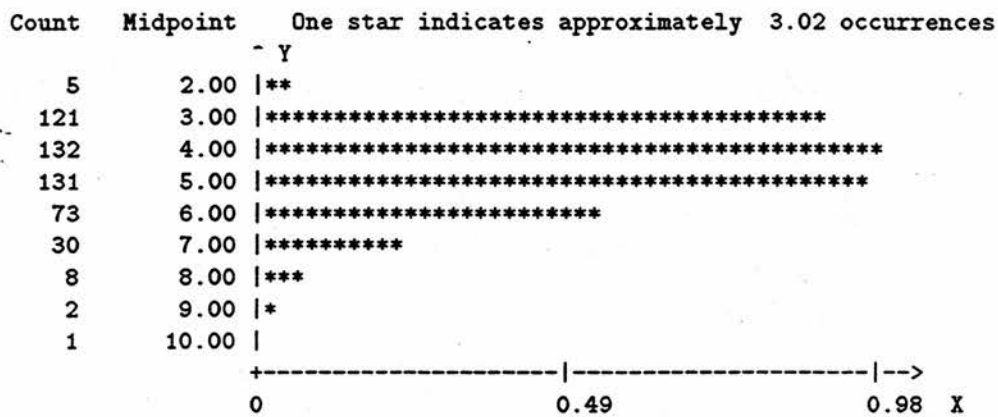


Fig. 2.19

Mode	4.00	Std dev	1.33
Skewness	.64	Range	8.00
Minimum	2.00	Maximum	10.00

20 shi'er, san (ge)
 '12 or 13'

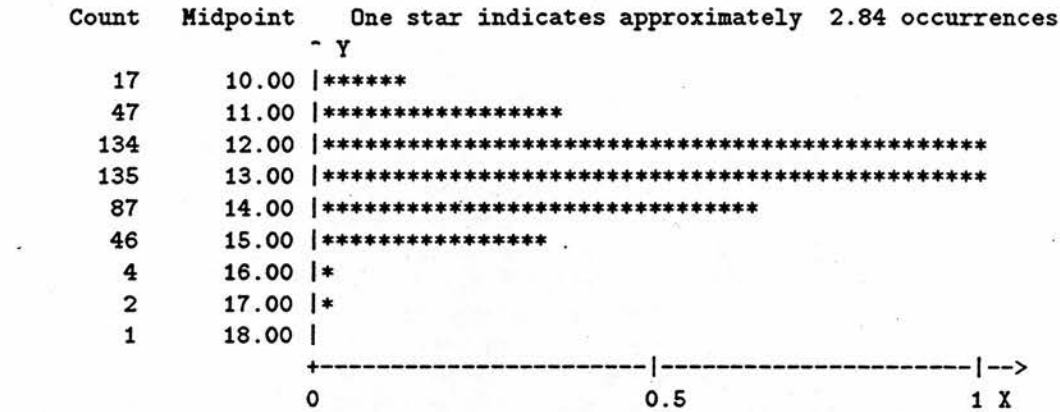
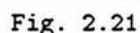


Fig. 2.20

Mode	13.00	Std dev	1.35
Skewness	.38	Range	9.00
Minimum	10.00	Maximum	19.00

21 200 yishang
'more than 200'



Mode	210.00	Std dev	N/A
Skewness	N/A	Range	N/A
Minimum	200.00	Maximum	+∞

22 200 yizia
'fewer than 200'

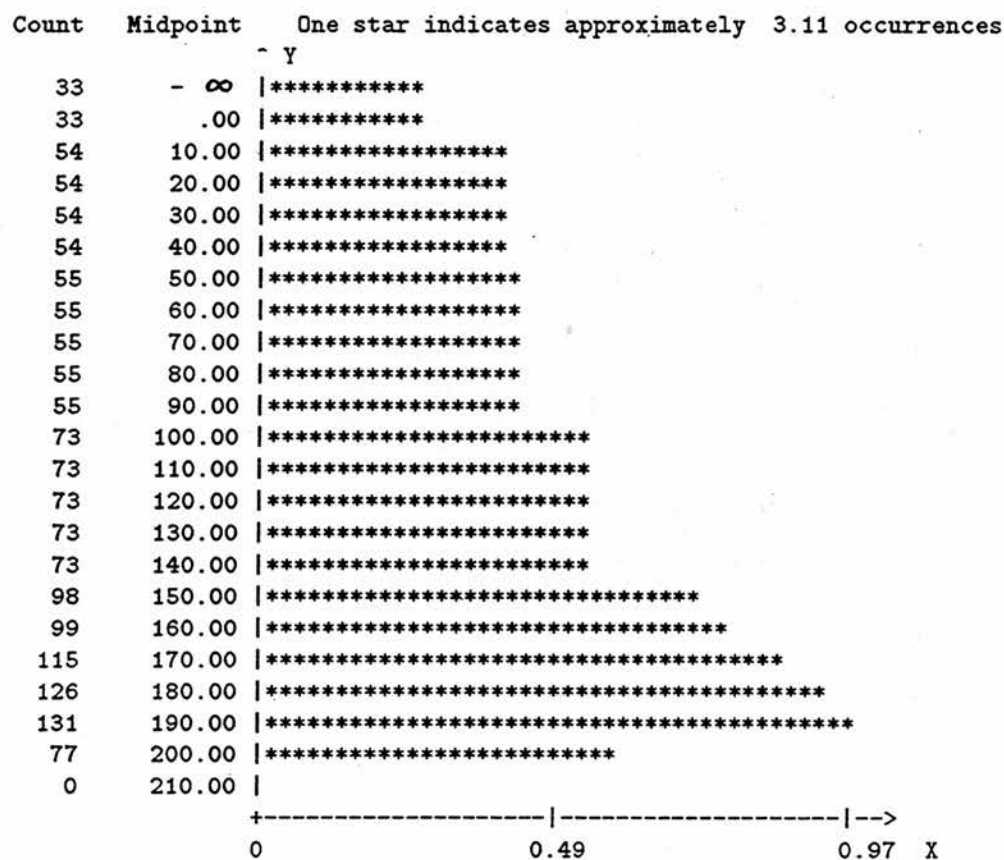


Fig. 2.22

Mode	190.00	Std dev	N/A
Skewness	N/A	Range	N/A
Minimum	- ∞	Maximum	200.00

'more than 200 people'

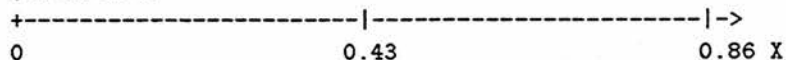


Fig. 2.23

Mode	210.00	Std dev	N/A
Skewness	N/A	Range	N/A
Minimum	200.00	Maximum	+∞

24 200 ren yixia
'fewer than 200 people'

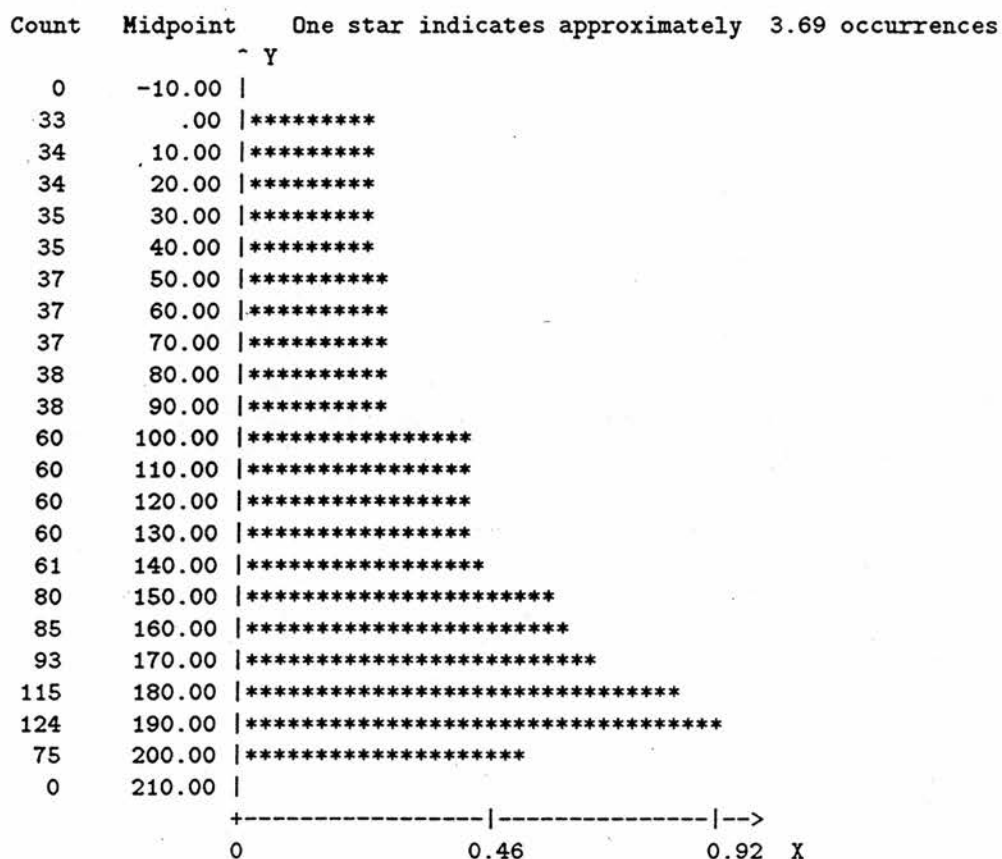


Fig. 2.24

Mode	190.00	Std dev	58.52
Skewness	-.58	Range	200.00
Minimum	.00	Maximum	200.00

Appendix 3: Zhang's Questionnaire in Chinese

学 术 统 计

姓名:

性别:

专业:

首先十分感谢您为此项学术研究将做的颇有价值的贡献。请您判断下列词组的数字区间。比如‘20以下’这个词组，有人会认为15-20是此词组意义所指的数字区间；有人则可能认为1-20；也许有人会判断为 $-\infty$ (负无穷)-20。

A组：请以100为参照数，判断出下列词组的数字区间。比如，在100（人）当中，我们可能判断30-50为‘一些（人）’的数字区间；70-80（人）为‘多数（人）’的数字区间。再如，以100（斤）为参照数，‘许多（斤）’的数字区间可能为70-80（斤）。

词组

数字区间

- | | |
|----|-------|
| 1 | 一些（人） |
| 2 | 许多（人） |
| 3 | 许多（斤） |
| 4 | 多数（人） |
| 5 | 少数（人） |
| 6 | 不少（人） |
| 7 | 好些（人） |
| 8 | 好些（斤） |
| 9 | 很多（个） |
| 10 | 有些（个） |

B组: 例子:

词组
大概20(人)
将近200(斤)
小二百(人)
两,三(个)
千百(个)

数字区间(假定答案)

17-23
180-199
180-195
2-4
100-1,000

词组

数字区间

- | | | |
|-----|-------------|--------------------------|
| 1 1 | 几(个) | |
| 1 2 | 好几(个) | |
| 1 3 | 200以上 | |
| 1 4 | 200以下 | |
| 1 5 | 200(人)以上 | |
| 1 6 | 200(人)以下 | |
| 1 7 | 将近200(人) | |
| 1 8 | 将近200(斤) | |
| 1 9 | 将近2,000(人) | |
| 2 0 | 将近2,000(斤) | |
| 2 1 | 将近20,000(人) | |
| 2 2 | 将近20,000(斤) | |
| 2 3 | 200多(人) | |
| 2 4 | 200多(斤) | |
| 2 5 | 2,000多(人) | |
| 2 6 | 2,000多(斤) | |
| 2 7 | 20,000多(人) | |
| 2 8 | 20,000多(斤) | |
| 2 9 | 200(人)左右 | |
| 3 0 | 200(斤)左右 | |
| 3 1 | 2,000(人)左右 | |
| 3 2 | 2,000(斤)左右 | |
| 3 3 | 20,000(人)左右 | |
| 3 4 | 20,000(斤)左右 | |
| 3 5 | 小二百(人) | |
| 3 6 | 大二百(人) | |
| 3 7 | 200来(人) | |
| 3 8 | 百把(个) | |
| 3 9 | 200(人)上下 | |
| 4 0 | 大约200(人) | |
| 4 1 | 上百(个) | |
| 4 2 | 成百(个) | |
| 4 3 | 200(斤)多一些 | } 用分数
或百分比表示
其数字区间 |
| 4 4 | 200(斤)少一些 | |
| 4 5 | 大半(个) | |
| 4 6 | 小半(个) | |
| 4 7 | 多半(人) | |
| 4 8 | 少半(人) | |
| 4 9 | 两,三(个) | |
| 5 0 | 三,五(个) | |
| 5 1 | 十二,三(个) | |
| 5 2 | 百十来(个) | |
| 5 3 | 千百(个) | |
| 5 4 | 千八百(个) | |
| 5 5 | 亿万(个) | |
| 5 6 | 千万(个) | |

C组: 判断下列带点词组的数字区间

例子	词组 半天	数字区间 1-2 (小时) (假定答案)
1	他已经等了半天了。	
2	他已经等了大半天了。	
3	剧场中的大半人在鼓掌。	
4	剧场中的少半人在睡觉。	
(用百分比描述句子三和四的数字区间)		

如您愿意提供任何有关此项研究的评论, 十二分欢迎一并附上。
最后, 再次表示衷心的感谢。如您方便的话, 请尽快回复。

英国 爱丁堡大学 语言学系

张乔

一九九零年十二月